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UCHINCHI TARTIBLI OPERATORLI MATRITSALAR OILASI MUHIM SPEKTRINING JOYLASHUV O‘RNI

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Annotatsiya: *Ushbu ishda uchinchi tartibli operatorli matritsalar oilasi uchun diskret spektri aniqlangan hamda muhim spektri va uning joylashuv o‘rni o’rganilgan.*

$\mathcal{H}$ orqali $\mathcal{H}_0 := \mathbb{C}$, $\mathcal{H}_1 := L_2(\mathbb{T}^3)$ va $\mathcal{H}_2 := L_2^s((\mathbb{T}^3)^2)$ fazolarning to‘g‘ri yig‘indisini belgilaymiz, ya’ni $\mathcal{H} := \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2$. Bunda $\mathcal{H}_0$, $\mathcal{H}_1$ va $\mathcal{H}_2$ fazolarga $L_2(\mathbb{T}^3)$ fazo yordamida qurilgan $\mathcal{F}_s(L_2(\mathbb{T}^3))$ bozonli Fok fazoning mos ravishda nol zarrachali, bir zarrachali va ikki zarrachali qism fazolari deyiladi.

$\mathcal{H}$ Hilbert fazosida ta’sir qiluvchi quyidagi

$$H(K) := \begin{pmatrix} H_{00}(K) & H_{01} & 0 \\ H_{01}^* & H_{11}(K) & H_{12} \\ 0 & H_{12}^* & H_{22}(K) \end{pmatrix} \tag{1}$$

uchinchi tartibli operatorli matritsalar oilasini qaraymiz. Bu yerda matritsaviy elementlar

$$H_{00}(K)f_0 = w_0(K)f_0, \quad H_{01}f_1 = \int_{\mathbb{T}^3} v_0(t)f_1(t)dt;$$

$$(H_{11}(K)f_1)(p) = w_1(K;p)f_1(p), \quad (H_{12}f_2)(p) = \int_{\mathbb{T}^3} v_1(t)f_2(p,t)dt;$$

$$(H_{22}(K)f_2)(p,q) = w_2(K;p,q)f_2(p,q), \quad f_i \in \mathcal{H}_i, \quad i = 0,1,2$$

kabi aniqlangan bo‘lib, H_{ij}^* $(i < j)$ orqali H_{ij} operatorga qo‘shma operator belgilangan. Bundan tashqari $w_0(\cdot)$ va $v_i(\cdot)$, $i = 0,1$ funksiyalar $\mathbb{T}^3$ da aniqlangan haqiqiy qiymatli chegaralangan funksiyalar, $w_1(\cdot;\cdot)$ va $w_2(\cdot;\cdot,\cdot)$ funksiyalar esa mos ravishda

$$w_1(K;p) := l_1\varepsilon(p) + l_2\varepsilon(K-p) + 1,$$
$$w_2(K;p,q) := l_1\varepsilon(p) + l_1\varepsilon(q) + l_2\varepsilon(K-p-q),$$

tengliklar yordamida aniqlanib, $l_1, l_2 > 0$ va

$$\varepsilon(q) := \sum_{i=1}^{3} (1 - \cos(nq^{(i)})), \quad q = (q^{(1)}, q^{(2)}, q^{(3)}) \in \mathbb{T}^3, \quad n \in \mathbb{N}.$$

$H(K)$ operatorli matritsalar oilasining spektral xossalarini o‘rganishda muhim sanalgan $\mathcal{H}_0 \oplus \mathcal{H}_1$ Hilbert fazosida

$$h(k) := \begin{pmatrix} h_{00}(k) & h_{01} \\ h_{01}^* & h_{11}(k) \end{pmatrix}$$

kabi aniqlangan va umumlashgan Fridrixs modellari oilasi deb ataluvchi $h(k)$, $k \in \mathbb{T}^3$ ikkinchi tartibli operatorli matritsalar oilasini qaraymiz. Bu yerda

$$h_{00}(k)f_0 = (l_2\varepsilon(k) + 1)f_0, \quad h_{01}f_1 = \frac{1}{\sqrt{2}}\int_{\mathbb{T}^3} v_1(t)f_1(t)dt,$$

$$(h_{11}(k)f_1)(q) = E_k(q)f_1(q), \quad E_k(q) := l_1\varepsilon(q) + l_2\varepsilon(k-q).$$

$h(k)$, $k \in \mathbb{T}^3$ operatorli matritsaning muhim spektri uchun

$$\sigma_{\text{ess}}(\mathrm{h}(k)) = [E_{\min}(k); E_{\max}(k)]$$

tenglik o'rinli

Bu yerda

$$E_{\min}(k) := \min_{q\in\mathbb{T}^3} E_k(q) \quad \text{va} \quad E_{\max}(k) := \max_{q\in\mathbb{T}^3} E_k(q).$$

Har qanday $k \in \mathbb{T}^3$ uchun $\mathbb{C}\backslash[E_{\min}(k); E_{\max}(k)]$ sohada analitik bo'lib,

$$\Delta(k\,; z) := l_2\varepsilon(k) + 1 - z - \frac{1}{2}\int_{\mathbb{T}^3} \frac{v_1^2(t)dt}{E_k(t) - z}$$

tenglik yordamida aniqlangan funksiyani qaraymiz. Odatda $\Delta(k\,;\cdot)$ funksiyaga $h(k)$ operatorli matritsaga mos Fredgolm determinanti deyiladi.

1-lemma. *Har bir* $k \in \mathbb{T}^3$ *uchun* $z \in \mathbb{C}\backslash[E_{\min}(k); E_{\max}(k)]$ *soni* $h(k)$ *operatorli matritsaning xos qiymati bo'lishi uchun* $\Delta(k\,; z) = 0$ *bo'lishi zarur va yetarlidir*.

Quyidagicha belgilashlar kiritamiz:

$$m_K := \min_{p,q\in\mathbb{T}^3} w_2(K; p, q), \quad M_K := \max_{p,q\in\mathbb{T}^3} w_2(K; p, q),$$

$$\sigma_K := \bigcup_{p\in\mathbb{T}^3} \{\sigma_{\text{disc}}(h(K-p)) + l_1\varepsilon(p)\boldsymbol{I}\}, \quad \Sigma_K := [m_K; M_K] \cup \sigma_K$$

bu yerda $\boldsymbol{I}$ orqali $\mathcal{H}_0 \oplus \mathcal{H}_1$ fazodagi birlik operator belgilangan.

Asosiy natijalarni bayon qilish maqsadida quyidagi belgilashlarni kiritamiz:

$$E_{\min}^{(l)}(K) := \min\{\sigma_K \cap (-\infty; m_K]\}, \quad E_{\max}^{(l)}(K) := \max\{\sigma_K \cap (-\infty; m_K]\},$$

$$E_{\min}^{(r)}(K) := \min\{\sigma_K \cap [M_K; +\infty)\}, \quad E_{\max}^{(r)}(K) := \max\{\sigma_K \cap [M_K; +\infty)\},$$

$$\sigma_{\text{two}}^{(l)}(K) := [E_{\min}^{(l)}(K); E_{\max}^{(l)}(K)], \quad \sigma_{\text{two}}^{(r)}(K) := [E_{\min}^{(r)}(K); E_{\max}^{(r)}(K)].$$

$\mathbb{C}\backslash[E_{\min}(K,k); E_{\max}(K,k)]$ sohada

$$I_K(k\,; z) := \int_{\mathbb{T}^3} \frac{v_1^2(t)dt}{w_2(K; k, t) - z}$$

yordamchi funksiyani qaraymiz.

1-teorema. *Faraz qilaylik, har bir tayinlangan* $K \in \mathbb{T}^3$ *uchun* $\min_{k\in\mathbb{T}^3}\Delta(K-k; m_K - l_1\varepsilon(k)) \geq 0$ *bo'lsin.*

(a) *Agar* $\max_{k\in\mathbb{T}^3}\Delta(K-k; M_K - l_1\varepsilon(k)) \leq 0$ *bo'lsa, u holda* $\sigma_{\text{ess}}(H(K)) = [m_K; M_K]$ *bo'ladi;*

(b) *agar* $\min_{k\in\mathbb{T}^3}\Delta\big(K-k; M_K - l_1\varepsilon(k)\big) \leq 0$ *va* $\max_{k\in\mathbb{T}^3}\Delta(K-k; M_K - l_1\varepsilon(k)) > 0$ *bo'lsa, u holda* $\sigma_{\text{ess}}(H(K)) = [m_K; E_{\max}^{(r)}(K)]$ *bo'ladi;*

(c) *agar* $\min_{k\in\mathbb{T}^3}\Delta(K-k; M_K - l_1\varepsilon(k)) > 0$ *bo'lsa, u holda*

$$\sigma_{\text{ess}}(H(K)) = [m_K; M_K] \cup \sigma_{two}^{(r)}(K)$$

munosabat o'rinli bo'ladi. Bundan tashqari, $E_{\min}^{(l)}(K) = m_K$ *tenglik o'rinli bo'ladi.*

2-teorema. *Faraz qilaylik, har bir tayinlangan* $K \in \mathbb{T}^3$ *uchun* $\min_{k\in\mathbb{T}^3}\Delta\big(K-k; m_K - l_1\varepsilon(k)\big) < 0, \quad \max_{k\in\mathbb{T}^3}\Delta(K-k; m_K - l_1\varepsilon(k)) \geq 0$ *bo'lsin.*

(a) *Agar* $\max_{k\in\mathbb{T}^3}\Delta(K-k; M_K - l_1\varepsilon(k)) \leq 0$ *bo'lsa, u holda* $\sigma_{\text{ess}}(H(K)) = \left[E_{\min}^{(l)}(K); M_K\right]$ *bo'ladi;*

(b) *agar* $\min_{k\in\mathbb{T}^3} \Delta(K-k; M_K - l_1\varepsilon(k)) \le 0$ *va* $\max_{k\in\mathbb{T}^3} \Delta(K-k; M_K - l_1\varepsilon(k)) > 0$ *bo'lsa, u holda* $\sigma_{\text{ess}}(H(K)) = \left[E_{\min}^{(l)}(K); E_{\max}^{(r)}(K)\right]$ *bo'ladi;*

(c) *agar* $\min_{k\in\mathbb{T}^3} \Delta(K-k; M_K - l_1\varepsilon(k)) > 0$ *bo'lsa, u holda*

$$\sigma_{\text{ess}}(H(K)) = \left[E_{\min}^{(l)}(K); M_K\right] \cup \sigma_{\text{two}}^{(r)}(K)$$

munosabatga ega bo'lamiz. Bundan tashqari, $E_{\min}^{(l)}(K) < m_K$ *tengsizlik o'rinli bo'ladi.*

3-teorema. *Faraz qilaylik, har bir tayinlangan* $K \in \mathbb{T}^3$ *uchun* $\max_{k\in\mathbb{T}^3} \Delta(K-k; m_K - l_1\varepsilon(k)) < 0$ *bo'lsin.*

(a) *Agar* $\max_{k\in\mathbb{T}^3} \Delta(K-k; M_K - l_1\varepsilon(k)) \le 0$ *bo'lsa, u holda* $\sigma_{\text{ess}}(H(K)) = \sigma_{\text{two}}^{(l)}(K) \cup [m_K; M_K]$ *bo'ladi;*

(b) *agar* $\min_{k\in\mathbb{T}^3} \Delta\big(K-k; M_K - l_1\varepsilon(k)\big) \le 0,$ $\max_{k\in\mathbb{T}^3}\Delta(K-k; M_K - l_1\varepsilon(k)) > 0$ *bo'lsa u holda*

$\sigma_{\text{ess}}(H(K)) = \sigma_{\text{two}}^{(l)}(K) \cup \left[m_K; E_{\max}^{(r)}(K)\right]$ *bo'ladi;*

(c) *agar* $\min_{k\in\mathbb{T}^3} \Delta(K-k; M_K - l_1\varepsilon(k)) > 0$ *bo'lsa, u holda*

$$\sigma_{\text{ess}}(H(K)) = \sigma_{\text{two}}^{(l)}(K) \cup [m_K; M_K] \cup \sigma_{\text{two}}^{(r)}(K)$$

bo'ladi.Bundan tashqari, $E_{\max}^{(l)}(K) < m_K$ *munosabat o'rinli bo'ladi.*

Ta'kidlash joizki, 1–3-teoremalarning birinchi tasdiqlarida $E_{\max}^{(r)}(K) = M_K$, ikkinchi tasdiqlarida $E_{\max}^{(r)}(K) > M_K$ uchinchi tasdiqlarida esa $E_{\min}^{(r)}(K) > M_K$ munosabat o'rinli bo'ladi.

FOYDALANILGAN ADABIYOTLAR

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INTEGRAL FORMULALAR TASNIFI HAQIDA.

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Annotatsiya: *Bizga ma'lumki integral formulalar kompleks o'zgaruvchili funksiyalar nazariyasining asosiy tushunchalaridan biri bo'lib, matematikadagi bir qancha masalalarni yechimini topishda, balki hayotimizdagi boshqa sohalarda ham muhim ahamiyatga ega. Shu kungacha bir qancha olimlar integral formulalar ustida ish olib borishgan. Bu maqolada integral formulalarning o'zaro bog'liqligi va farqlari haqida qisqacha keltirilgan.*