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**MATHEMATICAL ANALYSIS AND ITS
APPLICATIONS IN MODERN
MATHEMATICAL PHYSICS**

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2. *Shoyimardonov S.K.* Neimark-Sacker bifurcation and stability analysis in a discrete phytoplankton-zooplankton system with Holling type II functional response, arXiv:2207.01961 [math.DS]. P.1.16.

Threshold analysis for the family of generalized Friedrichs models

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Operators known as generalized Friedrichs model [1] appear in a series of problems in analysis, mathematical physics and probability theory. In the present note we discuss the threshold analysis for the family of generalized Friedrichs models corresponding to a system of quasi-particles, where their number is finite, but not fixed.

Let \mathbb{C} be the field of complex numbers and $L_2(\mathbb{T}^3)$ be the Hilbert space of square integrable (complex) functions defined on the three-dimensional torus \mathbb{T}^3 . Denote by \mathcal{H} the direct sum of spaces $\mathcal{H}_0 := \mathbb{C}$ and $\mathcal{H}_1 := L_2(\mathbb{T}^3)$, that is, $\mathcal{H} := \mathcal{H}_0 \oplus \mathcal{H}_1$.

In the present note we consider a family of generalized Friedrichs models $h(k)$, $k \in \mathbb{T}^3$, which acts in \mathcal{H} as

$$h(k) := \begin{pmatrix} h_{00}(k) & h_{01} \\ h_{01}^* & h_{11}(k) \end{pmatrix},$$

where

$$\begin{aligned} h_{00}(k)f_0 &= (l_2\varepsilon(k) + 1)f_0, & h_{01}f_1 &= \int_{\mathbb{T}^3} v(t)f_1(t)dt, \\ (h_{11}(k)f_1)(q) &= E_k(q)f_1(q), & E_k(q) &:= l_1\varepsilon(q) + l_2\varepsilon(k - q) \end{aligned}$$

with $l_1, l_2 > 0$ and the dispersion function $\varepsilon(\cdot)$ is defined by

$$\varepsilon(q) := \sum_{i=1}^3 (1 - \cos(nq^{(i)})), \quad q = (q^{(1)}, q^{(2)}, q^{(3)}) \in \mathbb{T}^3, \quad n \in \mathbb{N}.$$

Here $f_i \in \mathcal{H}_i$, $i = 0, 1$; the function $v(\cdot)$ is either even or odd function on each variable and there exist all second order continuous partial derivatives of $v(\cdot)$ on \mathbb{T}^3 . Then the family of operators $h(k)$, $k \in \mathbb{T}^3$ is bounded and self-adjoint in \mathcal{H} .

We remark that the operators h_{01} resp. h_{01}^* are called annihilation resp. creation operators, respectively.

Let Λ be a subset of \mathbb{T}^3 given by

$$\Lambda := \left\{ (p^{(1)}, p^{(2)}, p^{(3)}) : p^{(i)} \in \left\{ 0, \pm \frac{2}{n}\pi; \pm \frac{4}{n}\pi; \dots; \pm \frac{n'}{n}\pi \right\} \cup \Pi_n, \quad i = 1, 2, 3 \right\},$$

where

$$n' := \begin{cases} n - 2, & \text{if } n \text{ is even} \\ n - 1, & \text{if } n \text{ is odd} \end{cases} \quad \text{and} \quad \Pi_n := \begin{cases} \{\pi\}, & \text{if } n \text{ is even} \\ \emptyset, & \text{if } n \text{ is odd} \end{cases}$$

Direct calculation shows that the cardinality of Λ is equal to n^3 and for any fixed $k \in \Lambda$ the function $E_k(\cdot)$ has the non-degenerate zero minimum at the points of Λ .

Using the Weyl theorem, for the essential spectrum of $h(k)$ we obtain the equality $\sigma_{\text{ess}}(h(k)) = [E_{\min}(k); E_{\max}(k)]$, where the numbers $E_{\min}(k)$ and $E_{\max}(k)$ by the rule

$$E_{\min}(k) := \min_{q \in \mathbb{T}^3} E_k(q) \quad \text{and} \quad E_{\max}(k) := \max_{q \in \mathbb{T}^3} E_k(q).$$

For any $k \in \mathbb{T}^3$ we define an analytic function $\Delta(k; \cdot)$ (the Fredholm determinant associated with the operator $h(k)$) in $\mathbb{C} \setminus [E_{\min}(k); E_{\max}(k)]$ by

$$\Delta(k; z) := l_2\varepsilon(k) + 1 - z - \int_{\mathbb{T}^3} \frac{v^2(t)dt}{E_k(t) - z}.$$

A simple consequence of the Birman-Schwinger principle and the Fredholm theorem imply that

$$\sigma_{\text{disc}}(h(k)) = \{z \in \mathbb{C} \setminus [E_{\min}(k); E_{\max}(k)] : \Delta(k; z) = 0\}.$$

Since for any $k \in \Lambda$ the function $E_k(\cdot)$ has non-degenerate zero minimum at the points of Λ and the function $v(\cdot)$ is a continuous on \mathbb{T}^3 , for any $k \in \mathbb{T}^3$ the integral

$$\int_{\mathbb{T}^3} \frac{v^2(t)dt}{E_k(t)}$$

is positive and finite. The Lebesgue dominated convergence theorem and the equality $\Delta(\mathbf{0}; 0) = \Delta(k; 0)$ for $k \in \Lambda$ yield

$$\Delta(\mathbf{0}; 0) = \lim_{k \rightarrow k'} \Delta(k; 0), \quad k' \in \Lambda,$$

where $\mathbf{0} := (0, 0, 0) \in \mathbb{T}^3$.

Since $\mathbf{0} \in \Lambda$ the definition of $h(k)$ imply the identity $h(\mathbf{0}) \equiv h(k)$ for all $k \in \Lambda$.

Moreover, $\sigma_{\text{ess}}(h(\mathbf{0})) = [0; 6(l_1 + l_2)]$.

Let us denote by $C(\mathbb{T}^3)$ and $L_1(\mathbb{T}^3)$ the Banach spaces of continuous and integrable functions on \mathbb{T}^3 , respectively.

Definition 1. *The operator $h(\mathbf{0})$ is said to have a zero energy resonance, if the number 1 is an eigenvalue of the integral operator given by*

$$(G\psi)(q) = \frac{v(q)}{l_1 + l_2} \int_{\mathbb{T}^3} \frac{v(t)\psi(t)}{\varepsilon(t)} dt, \quad \psi \in C(\mathbb{T}^3)$$

and at least one (up to a normalization constant) of the associated eigenfunctions ψ satisfies the condition $\psi(p') \neq 0$ for some $p' \in \Lambda$.

We notice that in Definition 1 the requirement of the existence of the eigenvalue 1 of G corresponds to the existence of a solution of $h(\mathbf{0})f = 0$ and the condition $\psi(p') \neq 0$ for some $p' \in \Lambda$ implies that the solution f of this equation does not belong to \mathcal{H} .

The following result establishes in which cases the bottom of the essential spectrum is a threshold energy resonance or eigenvalue.

Theorem 1. *The following statements are hold.*

(i) *The operator $h(\mathbf{0})$ has a zero eigenvalue if and only if $\Delta(\mathbf{0}; 0) = 0$ and $v(q') = 0$ for all $q' \in \Lambda$;*

(ii) *The operator $h(\mathbf{0})$ has a zero energy resonance if and only if $\Delta(\mathbf{0}; 0) = 0$ and $v(q') \neq 0$ for some $q' \in \Lambda$.*

Let I be an identity operator on \mathcal{H} .

Theorem 2. *If the operator $h(\mathbf{0})$ has either a zero energy resonance or a zero eigenvalue, then for any $k \in \Lambda$ and $p \in \mathbb{T}^3$ the operator $h(k - p) + l_1\varepsilon(p)I$ is non-negative.*

Set

$$\Lambda_0 := \{q' \in \Lambda : v(q') \neq 0\}.$$

Now we formulate a result (zero energy expansion for the Fredholm determinant, leading to behaviors of the zero energy resonance).

Theorem 3. *Let the operator $h(\mathbf{0})$ have a zero energy resonance and $k, p' \in \Lambda$. Then the following decomposition*

$$\begin{aligned} \Delta(k - p; z - l_1\varepsilon(p)) &= \frac{4\pi^2}{n^2(l_1 + l_2)^{3/2}} \left(\sum_{q' \in \Lambda_0} v^2(q') \right) \sqrt{\frac{l_1^2 + 2l_1l_2}{l_1 + l_2} |p - p'|^2 - \frac{2z}{n^2}} \\ &+ O(|p - p'|^2) + O(|z|) \end{aligned}$$

holds for $|p - p'| \rightarrow 0$ and $z \rightarrow -0$.

We remark that Theorems 1, 2 and 3 are play key role in the spectral analysis of the family of 3×3 operator matrices, associated with the lattice systems describing two identical bosons and one particle, another nature in interactions, without conservation of the number of particles.

Reference:

1. *S.N.Lakaev*. Some spectral properties of the generalized Friedrichs model, (Russian), Trudy Sem. Petrovsk. 11 (1986), pp. 210–238, Translation in J. Soviet Math. 45:6 (1989), pp. 1540–1565.

On invariant sets of a quadratic non-stochastic operator

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Non-linear dynamical systems arise in many problems of biology, physics and other sciences. In particular, quadratic dynamical systems describe the behavior of populations of different species with population models [1, 2, 3]. Let $E = \{1, 2, \dots, m\}$. A distribution on the set E is a probability measure $x = (x_1, \dots, x_m)$, i.e., an element of the simplex:

$$S^{m-1} = \{x \in R : x_i \geq 0, \sum_{i=1}^m x_i = 1\}.$$

In general, a quadratic operator V , $V : x \in R^m \rightarrow x' = V(x) \in R^m$ is defined by:

$$V : x'_k = \sum_{i,j=1}^m P_{ij,k} x_i x_j, \quad k = 1, \dots, m \tag{1}$$

In this talk we are interested to a non-stochastic quadratic mapping of simplex to itself, i.e. $V : S^{m-1} \rightarrow S^{m-1}$.

Definition. [3] A quadratic operator (1), preserving a simplex, is called non-stochastic (QnSO) if at least one of its coefficients $P_{ij,k}$, $i \neq j$ is negative.

Consider the following example of QnSO on the two-dimensional simplex S^2 .

$$\begin{cases} x' = \frac{1}{2}(z - y)^2 + \frac{3}{2}x(y + z) \\ y' = \frac{1}{2}(x - z)^2 + \frac{3}{2}y(x + z) \\ z' = \frac{1}{2}(y - x)^2 + \frac{3}{2}z(x + y). \end{cases} \tag{2}$$

Fixed points. The fixed points are solutions to the system (2)

$$\begin{cases} x = \frac{1}{2}(z - y)^2 + \frac{3}{2}x(y + z) \\ y = \frac{1}{2}(x - z)^2 + \frac{3}{2}y(x + z) \\ z = \frac{1}{2}(y - x)^2 + \frac{3}{2}z(x + y). \end{cases}$$

By full analysis this system one obtains the following family of fixed points:

$$a_1 = (0, \frac{1}{2}, \frac{1}{2}), \quad a_2 = (\frac{1}{2}, 0, \frac{1}{2}), \quad a_3 = (\frac{1}{2}, \frac{1}{2}, 0), \quad a_4 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}).$$

Thus a_1 , a_2 and a_3 are saddle, but a_4 is an attracting fixed point.