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PANJARADAGI CHEKLI O'LCHAMLI QO'ZG'ALISHGA EGA BIR ZARRACHALI HAMILTONIAN UCHUN BIRMAN-SHVINGER PRINSIPI

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Annotatsiya. *Maқolada panjaradagi bir zarrachali Hamiltonianning koordinata va impuls tasvirlari keltirilgan. Qo'zg'alish operatorining musbat ekanligi ko'rsatilgan. Tadqiq qilinayotgan Hamiltonianga mos Fredgolm determinanti qurilgan hamda uzluksiz spektrdan o'ngda yotuvchi xos qiymatlarga ega emasligi ta'kidlab o'tilgan. Bundan tashqari, Birman-Shvinger prinsipi bayon qilingan va berilgan operator hamda Fredgolm determinantining nollari orasidagi bog'lanish o'rnatilgan.*

Kalit so'zlar: *panjara, Hamiltonian, koordinata tasvir, impuls tasvir, Fredgolm determinanti, Birman-Shvinger prinsipi.*

ПРИНЦИП БИРМАНА-ШВИНГЕРА ДЛЯ ОДНОЧАСТИЧНОГО ГАМИЛЬТОНИАНА С КОНЕЧНОМЕРНЫМ ДВИЖЕНИЕМ НА РЕШЁТКЕ

Аннотация. *В статье приведены координатное и импульсное представления одночастичного гамильтониана на решётке. Показано, что оператор возмущения положителен. Построен определитель Фредгольма, соответствующий изучаемому гамильтониану, и отмечено, что он не имеет собственных значений, лежащих справа от непрерывного спектра. Кроме того, сформулирован принцип Бирмана-Швингера, и установлена связь между нулями определителя Фредгольма и собственными значениями данного оператора.*

Ключевые слова: *решётка, гамильтониан, координатное представление, импульсное представление, определитель Фредгольма, принцип Бирмана-Швингера.*

THE BIERMANN-SCHWINGER PRINCIPLE FOR A SINGLE-PARTICLE HAMILTONIAN WITH FINITE-DIMENSIONAL MOTION ON A LATTICE

Abstract. *The coordinate and momentum representations of an one-particle Hamiltonian on a lattice are given. It is shown that the perturbation operator is positive. The Fredholm determinant corresponding to the Hamiltonian under study is constructed, and it is noted that it does not have eigenvalues lying to the right of the continuous spectrum. In addition, the Birmann-Schwinger principle was formulated and a connection between the zeros of the Fredholm determinant and the eigenvalues of a given operator was established.*

Key words: *lattice, Hamiltonian, coordinate representation, momentum representation, Fredholm determinant, Birmann-Schwinger principle.*

Kirish. Bizga yaxshi ma'lumki, ikki zarrachali diskret Shryodinger operatida to'la kvazi-impuls fiksirlanganda bir zarrachali Hamiltonianga unitar ekvivalent operator hosil bo'ladi. Shu sababli bir zarrachali Hamiltonianning spektral xossalarini o'rganish panjaradagi ko'p zarrachali operatorlarning spektral nazariyasida muhim ahamiyat kasb etadi [1]. Maqolada uch o'lchamli panjaradagi bitta kvant zarracha harakatini ifodalovchi Hamiltonian qaralgan. Bu Hamiltonian xos qiymatlari soni va joylashuv o'rni tadqiq qilingan.

Bir zarrachali Hamiltonianning koordinata tasviri. $l^2\left(Z^d\right)$ – d – o‘lchamli butun sonli panjara Z^d fazoda aniqlangan kvadrati bilan jamlanuvchi funksiyalarning Hilbert fazosi bo‘lsin.

Koordinatali tasvirida Z^d panjarada harakatlanuvchi bir kvant zarrachaning erkin Hamiltoniani $l^2\left(Z^d\right)$ fazoda chegaralangan o‘z-o‘ziga qo‘shma operator sifatida quyidagi fopmula orqali aniqlanadi [1–3]:

$$(\hat{h}_0\hat{\varphi})(x) = \sum_{s\in Z^d}\hat{\varepsilon}(x-s)\hat{\varphi}(s), \quad \hat{\varphi}\in\ell^2(Z^d).$$

Bu yerda $\hat{\varepsilon}(\cdot)$ funksiya Z^d da aniqlangan dispersion munosabat va quyidagi ko‘rinishga ega

$$\hat{\varepsilon}(s) = \begin{cases} d, & \text{agar} \quad |s| = 0, \\ -\dfrac{1}{2}, & \text{agar} \quad |s| = 1, \\ 0, & \text{agar} \quad |s| > 1, \end{cases}$$

$$s = (s^{(1)},\ldots,s^{(d)}) \in Z^d, \qquad |s| = |s^{(1)}| + \cdots + |s^{(d)}|.$$

Koordinatali tasvirda $\hat{v}_{\mu\lambda}$ potensial maydondagi bir zarrachaning to‘la Hamiltoniani $\hat{h}_0$ erkin Hamiltonianning chegaralangan qo‘zg‘alishi sifatida quyidagicha aniqlanadi:

$$\hat{h}_{\mu\lambda} = \hat{h}_0 - \hat{v}_{\mu\lambda}.$$

Bu yerda $\hat{v}_{\mu\lambda}$ funksiya $l^2\left(Z^d\right)$ fazoda $\hat{v}_{\mu\lambda}(\cdot)$ funksiyaga ko‘paytirish operatori, ya’ni

$$(\hat{v}_{\mu\lambda}\hat{\varphi})(x) = \hat{v}_{\mu\lambda}(x)\hat{\varphi}(x), \quad \hat{\varphi}\in\ell^2(Z^d).$$

$\hat{v}_{\mu\lambda}(\cdot)$ funksiya Z^d da quyidagicha aniqlangan

$$\hat{v}_{\mu\lambda}(s) = \begin{cases} \mu, & \text{agar} \quad |s| = 0, \\ \dfrac{\lambda}{2}, & \text{agar} \quad |s| = 1, \\ 0, & \text{agar} \quad |s| > 1, \end{cases}$$

bunda $\mu \geq 0$ va $\lambda \geq 0$ bir vaqtda nolga teng bo‘lmagan sonlar.

Ta’kidlash joizki, $\hat{h}_{\mu\lambda}$ funksiya $l^2\left(Z^d\right)$ Hilbert fazosida chegaralangan o‘z-o‘ziga qo‘shma operator bo‘ladi.

Bir zarrachali Hamiltonianning impuls tasviri. $T^d - d -$o‘lchamli tor, ya’ni $(-\pi;\pi]^d$ mos qarama-qarshi tomonlari aynan teng bo‘lgan kub bo‘lsin. $T^d \equiv (-\pi;\pi]^d \subset P^d$ to‘plamdagi qo‘shish va haqiqiy songa ko‘paytirish amallari P^d fazodagi $(2\pi Z^1)^d$ modul bo‘yicha amallar sifatida tushuniladi.

$L^2(T^d)$ fazo T^d torda aniqlangan kvadrati bilan integrallanuvchi funksiyalarning Hilbert fazosi, $L_e^2(T^d) \subset L^2(T^d) -$juft funksiyalar qism fazosi bo‘lsin.

Ushbu

$$\Phi : \ell^2(Z^d) \to L^2(T^d) : (\Phi\hat{f})(p) = (2\pi)^{-\frac{d}{2}} \sum_{s\in Z^d} \hat{f}(s) e^{i(p,s)}$$

$$(p,s)=\sum_{j=1}^{d} p^{(j)}s^{(j)},\quad p=(p^{(1)},\ldots,p^{(d)})\in Z^d,\quad s=(s^{(1)},\ldots,s^{(d)})\in \mathrm{T}^d$$

standart Fur'e almashtirishini qaraymiz.

Fur'e almashtirishi yordamida $\Phi(\ell_e^2(\mathrm{Z}^d))\subset L_e^2(\mathrm{T}^d)$ munosabatni hosil qilish mumkin. Φ_e orqali Φ ning $\ell_e^2(\mathrm{Z}^d)$ dagi qismini belgilaymiz. Ko'rsatish mumkinki, ushbu $\Phi_e(\ell_e^2(\mathrm{Z}^d))=L_e^2(\mathrm{T}^d)$ tenglik o'rinli bo'ladi.

Impuls tasvirida $H_{\mu\lambda}=\Phi_e\hat{h}_{\mu\lambda}\Phi_e^{-1}$ Hamiltonian $L_e^2(\mathrm{T}^d)$ Hilbert fazosida chegaralangan, o'z-o'ziga qo'shma operator bo'lib, u quyidagi formula orqali aniqlanadi

$$H_{\mu\lambda}=H_0-V_{\mu\lambda}.$$

bunda h_0 ε funksiyaga ko'paytirish operatori:

$$(H_0 f)(p)=\varepsilon(p)f(p),$$

$$\varepsilon(p)=\sum_{i=1}^{d}(1-\cos p^{(i)}),\quad f\in L_e^2(\mathrm{T}^d),\quad p=(p^{(1)},p^{(2)},\ldots,p^{(d)})\in \mathrm{T}^d,$$

$V_{\mu\lambda}-$ integral operator va uning rangi $d+1$ dan oshmaydi va quyidagi tenglik o'rinli bo'ladi:

$$(V_{\mu\lambda}f)(p)=\frac{1}{(2\pi)^d}\int_{\mathrm{T}^d}(\mu+\lambda\sum_{i=1}^{d}\cos p^{(i)}\cos t^{(i)})f(t)dt,\quad f\in L_e^2(\mathrm{T}^d).$$

Bir zarrachali Hamiltonianning spektral xossalari: $V_{\mu\lambda}-$ rangi $d+1$ dan oshmaydigan integral operator bo'lganligi uchun Veyl teoremasiga ko'ra, $H_{\mu\lambda}$ operatorning uzluksiz spektri $\sigma_{cont}(H_{\mu\lambda})$, $\mu,\lambda\geq 0$ lardan bog'liqsiz va $\sigma(h_0)$ operatorning spektri $\sigma(h_0)$ bilan ustma-ust tushadi [4]. Shunday qilib, quyidagi tengliklar o'rinli

$$\sigma_{cont}(H_{\mu\lambda})=\sigma(H_0)=[0,2d].$$

$L_e^2(\mathrm{T}^d)$ da quyidagi ortonormal sistemani qaraymiz:

$$\alpha_0=\frac{1}{(2\pi)^{\frac{d}{2}}},\qquad \alpha_i(p)=\frac{\sqrt{2}}{(2\pi)^{\frac{d}{2}}}\cos p^{(i)},\quad i=\overline{1,d}.$$

$v_{\mu\lambda}$ operator quyidagi ko'rinishda tasvirlanadi:

$$V_{\mu\lambda}f=\mu\alpha_0(f,\alpha_0)+\frac{\lambda}{2}\sum_{i=1}^{d}(f,\alpha_i)\alpha_i,$$

bunda $(\cdot,\cdot)-$ $L_e^2(\mathrm{T}^d)$ dagi skalyar ko'paytma.

1-lemma. *$V_{\mu\lambda}$ nomanfiy operator, ya'ni ixtiyoriy $f\in L_e^2(\mathrm{T}^d)$ uchun $(V_{\mu\lambda}f,f)\geq 0$ tengsizlik o'rinli.*

$V_{\mu\lambda}\geq 0$ operatorning nomanfiyligidan, uning $V_{\mu\lambda}^{\frac{1}{2}}\geq 0$ kvadrat ildizi mavjud. $V_{\mu\lambda}^{\frac{1}{2}}$ operator $L_e^2(\mathrm{T}^d)$ fazoda quyidagi formula bo'yicha aniqlanadi:

$$(V_{\mu\lambda}^{\frac{1}{2}}f)(p)=(2\pi)^{-\frac{d}{2}}\int_{\mathrm{T}^d}v_{\mu\lambda}^{\frac{1}{2}}(p-q)f(q)dq,$$

bunda

$$V_{\mu\lambda}^{\frac{1}{2}}(p) = (2\pi)^{-\frac{d}{2}} \sum_{s\in Z^d} \hat{v}_{\mu\lambda}^{\frac{1}{2}}(s) e^{i(p,s)}$$

va $\hat{v}_{\mu\lambda}^{\frac{1}{2}}(\cdot)$ funsiya $\hat{v}_{\mu\lambda}(\cdot)$ funksiyaning musbat kvadrat ildizi.

$V_{\mu\lambda}$ integral operatorining aniqlanishidan uning kvadrat ildizi $V_{\mu\lambda}^{\frac{1}{2}}$ quyidagicha aniqlanadi

$$V_{\mu\lambda}^{\frac{1}{2}} f = \sqrt{\mu}\alpha_0(f,\alpha_0) + \sqrt{\frac{\lambda}{2}} \sum_{i=1}^{d} (f,\alpha_i)\alpha_i. \qquad (1)$$

$\mathrm{C}-$ kompleks tekislik va $r_0(z)$, $z \in \mathrm{C} \setminus [0,2d] - h_0$ operatorning rezolventasi bo‘lsin.

$\varepsilon(q) = \varepsilon(q^{(1)},\ldots,q^{(d)}) = \sum_{i=1}^{d}(1-\cos q^{(i)})$ funksiya $q^{(i)}$ va $q^{(j)}$ $i,j=\overline{1,d}$ o‘zgaruvchilarning o‘rnini almashtirishga nisbatan simmetrik funksiya bo‘lganligi uchun ushbu

$$\int_{\mathrm{T}^d} \frac{\cos q^{(i)} dq}{\varepsilon(q)-z}, \quad \int_{\mathrm{T}^d} \frac{\cos^2 q^{(i)} dq}{\varepsilon(q)-z}, \quad \text{va} \quad \int_{\mathrm{T}^d} \frac{\cos q^{(i)} \cos q^{(j)} dq}{\varepsilon(q)-z}$$

integrallar $i,j=\overline{1,d}, \quad i \neq j$ lardan bog‘liq emas.

Quyidagi belgilashlarni kiritamiz:

$$\begin{aligned}
a(z) &= (\alpha_0, r_0(z)\alpha_0) = \frac{1}{(2\pi)^d} \int_{\mathrm{T}^d} \frac{dq}{\varepsilon(q)-z}, \\
b(z) &= (\alpha_0, r_0(z)\alpha_i) = \frac{\sqrt{2}}{(2\pi)^d} \int_{\mathrm{T}^d} \frac{\cos q^{(i)} dq}{\varepsilon(q)-z}, \\
c(z) &= (\alpha_i, r_0(z)\alpha_i) = \frac{2}{(2\pi)^d} \int_{\mathrm{T}^d} \frac{\cos^2 q^{(i)} dq}{\varepsilon(q)-z}, \\
d(z) &= (\alpha_i, r_0(z)\alpha_j) = \frac{2}{(2\pi)^d} \int_{\mathrm{T}^d} \frac{\cos q^{(i)} \cos q^{(j)} dq}{\varepsilon(q)-z},
\end{aligned} \qquad (2)$$

$z<0, \quad i,j=\overline{1,d}, i \neq j.$

Ixtiyoriy fiksirlangan $\mu,\lambda \geq 0$ va $z \in \mathrm{C} \setminus [0,2d]$ lar uchun $L_e^2(\mathrm{T}^d)$ fazoda quyidagi formula bilan ta’sir qiluvchi chekli o‘lchamli Birman–Shvinger integral operatori $G_{\mu\lambda}(z)$ ni aniqlaymiz:

$$G_{\mu\lambda}(z) = V_{\mu\lambda}^{\frac{1}{2}} r_0(z) v_{\mu\lambda}^{\frac{1}{2}}.$$

$V_{\mu\lambda}^{\frac{1}{2}}$ operatorning (1) tenglik bilan aniqlanishiga ko‘ra, $G_{\mu\lambda}(z)$ operator quyidagi ko‘rinishda tasvirlanadi:

$$G_{\mu\lambda}(z) f = \left(\mu a(z)(f,\alpha_0) + \sqrt{\frac{\mu\lambda}{2}} b(z) \sum_{i=1}^{d} (f,\alpha_i) \right) \alpha_0 +$$

$$+\sum_{i=1}^{d}[\sqrt{\frac{\mu\lambda}{2}}b(z)(f,\alpha_0)+\frac{\lambda}{2}c(z)(f,\alpha_i)+\frac{\lambda}{2}d(z)\sum_{i\neq j=1}^{d}(f,\alpha_j)]\alpha_i. \qquad (3)$$

(3) tenglikdan $G_{\mu\lambda}(z)$ operatorning rangi $z\in \mathbb{C}\setminus[0,2d]$ dan bog‘liqmas va $d+1$ dan oshmaydi.

Ixtiyoriy fiksirlangan $\mu,\lambda\geq 0$ uchun $H_{\mu\lambda}-zI$ operatorning determinantini $I-G_{\mu\lambda}(z)$ operatorning Fredgolm determinanti kabi aniqlaymiz:

$$\Delta(\mu,\lambda;z):=det(H_{\mu\lambda}-zI):=det(I-G_{\mu\lambda}(z)). \qquad (4)$$

Ravshanki, ixtiyoriy $\mu,\lambda\geq 0$ uchun $\Delta(\mu,\lambda;\cdot)$ funksiya $\mathbb{C}\setminus[0,2d]$ sohada analitik bo‘ladi.

2-lemma. *Barcha* $\mu,\lambda\geq 0$ *va* $z\in \mathbb{C}\setminus[0,2d]$ *uchun quyidagi tengliklar o‘rinli*

$$\Delta_d(\mu,\lambda;z)=\Delta_d^{(1)}(\mu,\lambda;z)(\Delta_d^{(22)}(\lambda;z))^{d-1}, \qquad (5)$$

$$\Delta_d(\mu,0;z)=1-\mu a(z), \quad \Delta_d(0,\lambda;z)=\Delta_d^{(21)}(\lambda;z)(\Delta_d^{(22)}(\lambda;z))^{d-1}, \qquad (6)$$

bunda

$$\Delta_d^{(1)}(\mu,\lambda;z)=\Delta_d(\mu,0;z)\Delta_d^{(21)}(\lambda;z)-\frac{d\mu\lambda}{2}b^2(z), \qquad (7)$$

$$\Delta_d^{(21)}(\lambda;z)=1-\frac{\lambda}{2}(c(z)+(d-1)d(z)), \quad \Delta_d^{(22)}(\lambda;z)=1-\frac{\lambda}{2}(c(z)-d(z)). \qquad (8)$$

1-lemma isboti. Haqiqatan ham, ixtiyoriy $f\in L_e^2(\mathrm{T}^d)$ uchun quyidagiga ega bo‘lamiz:

$$(V_{\mu\lambda}f,f)=\int_{\mathrm{T}^d}(V_{\mu\lambda}f)(p)\overline{f(p)}dp=$$

$$\frac{1}{(2\pi)^d}\int_{\mathrm{T}^d}[\int_{\mathrm{T}^d}(\mu+\lambda\sum_{i=1}^{d}\cos p^{(i)}\cos t^{(i)})f(t)dt]\overline{f(p)}dp=$$

$$=\frac{1}{(2\pi)^d}[\mu\int_{\mathrm{T}^d}f(t)dt\overline{\int_{\mathrm{T}^d}f(p)dp}+\lambda\sum_{i=1}^{d}\int_{\mathrm{T}^d}\cos t^{(i)}f(t)dt\overline{\int_{\mathrm{T}^d}\cos p^{(i)}f(p)dp}]=$$

$$=\frac{1}{(2\pi)^d}[\mu|\int_{\mathrm{T}^d}f(t)dt|^2+\lambda\sum_{i=1}^{d}|\int_{\mathrm{T}^d}\cos t^{(i)}f(t)dt|^2]\geq 0.$$

$L_{d+1}\subset L_e^2(\mathrm{T}^d)-$ orqali 1 va $\cos p^{(i)}$ $i=\overline{1,d}$ funksiyalarga tortilgan $d+1-$ o‘lchamli qism fazoni belgilaymiz.

1-eslatma. $V_{\mu\lambda}$ operator $L_e^2(\mathrm{T}^d)$ Hilbert fazoni L_{d+1} qism fazoga akslantiradi.

2-eslatma. $V_{\mu\lambda}$ operatorning $L_e^2(\mathrm{T}^d)$ fazoda nomanfiyligi (qar. 1-lemma) va $\sup_{f\neq 0}(H_{\mu\lambda}f,f)\leq \sup_{f\neq 0}(H_0 f,f)$ tengsizlikdan $h_{\mu\lambda}$ operatorning uzluksiz spektri $[0,2d]$ dan o‘ngda yotuvchi xos qiymatga ega emas.

3-lemma. Ixtiyoriy $\mu,\lambda\geq 0$ uchun $z<0$ soni $H_{\mu\lambda}$ operatorning xos qiymati bo‘lishi uchun 1 soni $G_{\mu\lambda}(z)$ operatorning xos qiymati bo‘lishi zarur va yetarlidir.

Isboti. $z<0$ soni $h_{\mu\lambda}$ operatorning xos qiymati va $f\in L_e^2(\mathrm{T}^d)$ unga mos xos funksiya bo‘lsin, ya’ni

$$H_{\mu\lambda}f=zf \qquad \text{yoki} \qquad (H_0-z)f=V_{\mu\lambda}f.$$

Bu yerdan

$$f = r_0(z)V_{\mu\lambda} f$$

tenglikni hosil qilamiz. Bundan esa

$$V_{\mu\lambda}^{\frac{1}{2}} f = (V_{\mu\lambda}^{\frac{1}{2}} r_0(z) V_{\mu\lambda}^{\frac{1}{2}}) V_{\mu\lambda}^{\frac{1}{2}} f = G_{\mu\lambda}(z) V_{\mu\lambda}^{\frac{1}{2}} f$$

tenglikka ega bo‘lamiz.

Teskarisi. 1 soni $G_{\mu\lambda}(z)$ operatorning xos qiymati va $\varphi \in L_e^2(\mathrm{T}^d)$ unga mos xos funksiya bo‘lsin, ya’ni

$$\varphi = (v_{\mu\lambda}^{\frac{1}{2}} r_0(z) V_{\mu\lambda}^{\frac{1}{2}})\varphi.$$

Bu yerdan

$$\psi = V_{\mu\lambda} r_0(z)\psi$$

tenglik o‘rinli bo‘ladi, bunda $\psi = v_{\mu\lambda}^{\frac{1}{2}}\varphi$. $f = r_0(z)\psi$ belgilash orqali

$$(h_0 - z) f = V_{\mu\lambda} f$$

tenglikka ega bo‘lamiz, ya’ni $f-$ ushbu $H_{\mu\lambda}$ operatorning $z < 0$ xos qiymatiga mos xos funksiyasi bo‘ladi.

H Hilbert fazosida aniqlangan va $\beta \in \cdot$ nuqtadan o‘ngda (mos holda chapda) muhim spektrga ega bo‘lmagan A chegaralangan o‘z-o‘ziga qo‘shma operator uchun $n_+(\beta, A)$ (mos holda $n_-(\beta, A)$) sonni quyidagicha aniqlaymiz:

$$n_+(\beta, A) = \sup\{\dim L:\ L \subset H;\ (Af, f) > \beta,\ \|f\| = 1\}$$

$$\left(n_-(\beta, A) = \sup\{\dim L:\ L \subset H;\ (Af, f) < \beta,\ \|f\| = 1\}\right).$$

$n_+(\beta, A)$ (mos holda $n_-(\beta, A)$) soni A operatorning β dan o‘ngda (mos holda chapda) yotuvchi xos qiymatlari soniga mos keladi.

Xos qiymatlar muammosini kamaytirish Birman [5] va Shvinger [6] tomonidan umumqabul qilingan bir jinsli Lipman–Shvinger tenglamasi ko‘paytmasiga keltirilgan.

1-teorema (*Birman–Shvinger prinsipi*). *Ixtiyoriy* $\mu, \lambda \geq 0$ *va* $z \leq 0$ *uchun*

$$n_-(z, H_{\mu\lambda}) = n_+(1, G_{\mu\lambda}(z))$$

tenglik o‘rinli.

O’z-o‘ziga qo‘shma $H_{\mu\lambda}$ operatorining xos qiymatlari va $I - G_{\mu\lambda}(z)$ Fredholm determinanti $\Delta(\mu, \lambda; z)$ ning nollari o‘rtasidagi bog‘liqlik quyidagi teorema bilan o‘rnatiladi.

2-teorema. *Ixtiyoriy* $\mu, \lambda \geq 0$ *uchun* $z \in \mathrm{C} \setminus [0, 2d]$ *soni* $H_{\mu\lambda}$ *operatorning* $m-$ *karrali xos qiymati bo‘lishi uchun u* $\Delta(\mu, \lambda; z)$ *funksiyaning* $m-$ *karrali noli bo‘lishi zarur va yetarli.*

Xulosa. Maqolada butun sonli panjaradagi bir zarrachali Hamiltonianning koordinatali va impuls tasvirlari keltirilib o‘tilgan. Qo‘zg‘alish operatori deb ataluvchi potensial operatori, ya’ni integral operatorning musbat aniqlangan operator ekanligi isbotgan. Tadqiq qilingan Hamiltonianga mos Fredgolm determinanti deb ataluvchi regulyar funktsiya qurilgan. Berilgan Hamiltonian o‘zining uzluksiz spektridan o‘ngda yotuvchi xos qiymatlarga ega emasligi ta’kidlab o‘tilgan. Birman-Shvinger prinsipi bayon qilingan. O‘rganilgan Hamiltonian va Fredgolm determinantining nollari orasidagi bog‘lanish o‘rnatilgan.

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