



## ABSTRACTS

### OF THE IX INTERNATIONAL SCIENTIFIC CONFERENCE “ACTUAL PROBLEMS OF APPLIED MATHEMATICS AND INFORMATION TECHNOLOGIES AL-KHWARIZMI 2024”

Dedicated to the 630th anniversary of the birth of Mirzo Ulugbek



APAMIT-2024



22-23 October, 2024, Tashkent, Uzbekistan

MINISTRY OF HIGHER EDUCATION, SCIENCE AND INNOVATION  
OF THE REPUBLIC OF UZBEKISTAN

NATIONAL UNIVERSITY OF UZBEKISTAN  
TASHKENT STATE TRANSPORT UNIVERSITY  
INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKY

INTERNATIONAL SCIENTIFIC  
CONFERENCE

ACTUAL PROBLEMS OF APPLIED MATHEMATICS AND  
INFORMATION TECHNOLOGIES - AL-KHWARIZMI 2024

A B S T R A C T S

22-23 October 2024, Tashkent, Uzbekistan

<b>Mona E., Nidal E. T., Agaeb M. A., Manahil A. M. A., Manal Y. A. J.</b> <i>Ion Acoustic Solitary Wave Solutions in the Context of the Nonlinear Fractional KdV Equation</i> ..	177
<b>Mukhambetkaliyev M. B.</b> <i>Inverse source problem for pseudoparabolic equation with memory</i> ..	178
<b>Muratbekov M.B., Muratbekov M.M.</b> <i>Estimates of Approximation Numbers and Completeness of Root Vectors of a Singular Operator Generated by the Linear Part of the Korteweg-de Vries Operator</i> ..	178
<b>Mustapokulov Kh.Ya., Mamadaliev N.A.</b> <i>Construction of P-strategies in a simple evasion game with impulse control</i> ..	178
<b>Myrzakulova Zh. R., Yesmakhanova K R., Myrzakulov R.</b> <i>Solutions of Camassa-Holm and Generalized Heisenberg ferromagnet equations with self-consistent sources</i> ..	179
<b>Nurjanova A. O.</b> <i>On the approximate solution of a boundary value problem for systems of integro-differential equations of the Fredholm type</i> ..	180
<b>Oripov D. D.</b> <i>Nonlocal Initial-Boundary Value Problem for a Partial Differential Equation of High Even Order Degenerating on the Boundary of the Domain</i> ..	181
<b>Otakulov S., Haydarov T. T.</b> <i>On the non-smooth optimization problem for dynamic control system under conditions of uncertainty</i> ..	182
<b>Otakulov S., Rahimov B. SH.</b> <i>About the property of locally relative controllability of a differential inclusion</i> ..	183
<b>Rahmonov A. A.</b> <i>An inverse source problem for a fractional diffusion-wave equation</i> ..	183
<b>Ramazanov M. I., Gulmanov N. K., Omarov M. T.</b> <i>Solving a singular integral equation of the Volterra type for heat conduction problems</i> ..	184
<b>Rasulov M. S.</b> <i>The Stefan problem for competitive reaction-diffusion system</i> ..	185
<b>Rozimatov J. A., Murodova Sh. I.</b> <i>Conditional stability of the initial-boundary value problem for parabolic equation with changing direction of time</i> ..	186
<b>Saparbayev R. A.</b> <i>Cauchy problem for fractional telegraph equation with Caputo operator</i> ..	186
<b>Sartabanov Zh., Omarova B., Zhumagaziyev A.</b> <i>Multiperiodic solutions of linear Hamiltonian systems by a diagonal differentiation operator defined on a cylindrical manifold</i> ..	187
<b>Sattorov E. N., Ermamatova Z. E.</b> <i>On a three dimensional Cauchy problem for inhomogeneous Helmholtz equation in bounded domain</i> ..	188
<b>Sattorov E. N., Pulatov O. U.</b> <i>Analytical forms of the Cauchy-type representation of the gravity field and its gradients</i> ..	189
<b>Shakir A. G.</b> <i>Inverse problem for parabolic equation with <math>p</math>-Laplacian and damping term</i> ..	190
<b>Shazyndayeva M.</b> <i>An inverse problem for fractional nonlinear pseudoparabolic equation</i> ..	190
<b>Siddheshwar P.G.</b> <i>A new type of heat equation to study natural convection in a fluid-saturated porous medium</i> ..	191
<b>Smadiyeva A. G.</b> <i>Decay estimates for the linear and nonlinear time-fractional differential equations</i> ..	192
<b>Sulima A. M. Z., Hala M. E. A., Batul A. A. M.</b> <i>Exploring Multiple and Singular Soliton Solutions for Negative-Order Space-Time Fractional mKdV Equations</i> ..	192
<b>Suyarov T. R.</b> <i>Inverse problems of determining the right-hand side for a one-dimensional wave equation</i> ..	193
<b>Talipova M.Zh.</b> <i>Investigation of normal solutions for a nonhomogeneous system of partial differential equation</i> ..	193
<b>Tobakhanov N., Torebek B.</b> <i>On the critical behavior for inhomogeneous semilinear biharmonic heat equations on exterior domains</i> ..	194
<b>Torebek B. T.</b> <i>Global existence and blow-up of solutions to the porous medium equation</i> ..	195

\*\*\*

**About the property of locally relative controllability of a differential inclusion****Otakulov S.<sup>1</sup>, Rahimov B. SH.<sup>2</sup>**Jizzakh Polytechnic Institute <sup>1,2</sup>, Jizzakh, Uzbekistan  
raximovboyxoroz@gmail.com

Consider a control object whose dynamics in the  $n$ -dimensional state space  $R^n$  is described by differential inclusion (1,2)

$$\dot{x} \in A(t)x + B(t), t \geq t_0, \quad (1)$$

where  $A = A(t)$  - is a  $n \times n$  -matrix,  $B = B(t)$  - is a multi-valued mapping. We will assume that the following conditions are met: 1) the elements of the matrix  $A(t)$  are measurable on any  $T = [t_0, t_1] \subset [t_0, +\infty)$  and  $\|A(t)\| \leq a(t)$ , where  $a(\cdot) \in L_1(T)$ ; 2) for each  $t \geq t_0$  a set  $B(t) \subset R^n$  compact and multivalued  $t \rightarrow B(t)$  mapping is measurable on an arbitrary segment  $T = [t_0, t_1] \subset [t_0, +\infty)$  and  $\|B(t)\| \leq b(t)$ , where  $b(\cdot) \in L_1(T)$ . Let  $X(t_0, t_1, \xi, A, B) = \Phi_A(t_1, t_0)\xi + \int_{t_0}^{t_1} \Phi_A(t_1, \tau)B(\tau)d\tau$ , where  $\Phi_A(t, \tau)$  - is the fundamental matrix of solutions to equation  $\dot{x} \in A(t)x$ . Let  $M \subset R^n$  be a given set terminal states of the control object (1),  $M^\varepsilon = M + S_\varepsilon(0)$  - the  $\varepsilon$  - neighborhood of the set  $M$ .

**Definition.** We will say that a differential inclusion (1) is locally controllable relative to a given set  $M$  (or, briefly, locally-relatively to  $M$  controllable) if there is a number and a time interval  $T = [t_0, t_1]$ , such that for any starting point  $x_0 \in M^\varepsilon$  the relation  $X(t_0, t_1, x_0, A, B) \cap M \neq 0, \forall x_0 \in M^\varepsilon$  holds.

**Theorem 1.** Let  $M$  - be compact from  $R^n$ . Then, for locally relative  $M$ -controllability of differential inclusion (1), it is necessary that the condition

$$\inf_{\|\psi\|=1} \sup_{\xi \in M} \int_{t_0}^{t_1} c(\Phi_A(t_1, \tau)[A(\tau)\xi + B(\tau)], \psi)d\tau \geq 0,$$

was performed at some point in time  $t_1 > t_0$ .

**Theorem 2.** Let  $B(t) = C(t)U(t)$ , where  $C(t) = (c_{i,j}(t))$  -  $n \times m$  - is a matrix whose elements  $c_{i,j}(t)$  are Lebesgue integrable on each  $T = [t_0, t_1], t \rightarrow U(t)$ , is a measurable multivalued map,  $U(t) \in \Omega(R^m)$ ,  $\|U(t)\| \leq g(t), t \in T = [t_0, t_1], g(\cdot) \in L_1(T)$ . Then if the system (1) is locally null-controlled, then there exists  $t_1 > t_0$  such that for any  $\psi \in R^n, \|\psi\|=1$  the relation  $\mu E(\psi) > 0$  is valid, where  $\mu E(\psi)$  is the Lebesgue measure of the set  $E(\psi) = \{t \in T = [t_0, t_1] : C'(t)\Phi_A'(t_1, t)\psi \neq 0\}$ .

**Corollary.** Let  $A(t) = A$ ,  $B(t) = C \cdot U(t)$ , where  $C - n \times m$  - is a matrix,  $t \rightarrow U(t)$  is a measurable multivalued map,  $U(t) \in \Omega(R^m)$ ,  $\|U(t)\| \leq g(t), t \in T = [t_0, t_1], g(\cdot) \in L_1(T)$ . Then, if the system (1) is locally null-controlled, then  $\text{rank } K = n$ , where  $K = \{C, AC, A^2C, \dots, A^{n-1}C\}$ . The paper researches the problem of local relative controllability for a mathematical model of a control system in the form of a linear differential inclusion. The studied property of locally relative  $M$  controllability of the considered model generalizes the concept of relative controllability of dynamic systems

**References**

1. Polovinkin E.S. Multivalued analysis and differential inclusions. Moskow: Fizmatlit, 2015.
2. Otakulov S., Rahimov B. Sh. Haydarov T.T. On the property of relative controllability for the model of dynamic system with mobile terminal set. AIP Conference Proceedings, 2022, 2432, 030062. -p. 1–5.

\*\*\*

**An inverse source problem for a fractional diffusion-wave equation****Rahmonov A. A.<sup>1</sup>**V.I.Romanovskiy Institute of Mathematics, Uzbekistan Academy of Sciences, Tashkent, Uzbekistan  
a.raxmonov@mathinst.uz

Consider the following Cauchy-type problem with inverse time:

$$\begin{cases} \partial_t^\alpha u(t) + Au(t) = f(t), & 0 < t < T, \\ u(T) = \varphi, \quad \partial_t u(T) = \psi, \end{cases} \quad (1)$$

where  $\varphi$  and  $\psi$  are given functions. The operator  $\partial_t^\alpha$  in Eq. (1) denotes the Caputo fractional derivative of order  $\alpha \in (1, 2)$  with respect to  $t$  (see [1]):

$$\partial_t^\alpha y(t) = \frac{t_+^{1-\alpha}}{\Gamma(2-\alpha)} * y''(t),$$

where  $*$  denotes the convolution,  $A : X \rightarrow X$  be an arbitrary unbounded positive selfadjoint operator in separable Hilbert space  $X$  and  $A^{-1}$  is a compact operator. We will assume that the  $f(t) \equiv f$ , where  $f \in X$ , that is  $f$  does not depend on  $t$ .

*Inverse problem.* Given  $\alpha$ ,  $\varphi$  and  $\psi$  find a pair of functions  $\{u, f\}$  satisfying the problem (1) and the additional condition

$$u(\tau) = h, \quad 0 < \tau < T, \quad (2)$$

where  $h \in D(A) \subset X$  is a given element.

**Definition 1.** A pair  $\{u(t), f\}$  of functions  $u \in C([0, T]; X)$  and  $f \in X$  with the properties  $\partial_t^\alpha u(t), Au(t) \in C([0, T]; X)$  and satisfying conditions (1)-(2) is called the solution of the inverse problem.

We set

$$\Lambda = \Lambda(\alpha, A) := \bigcup_{n=1}^{\infty} \left\{ \left( \frac{\eta_1}{\lambda_n} \right)^{\frac{1}{\alpha}}, \dots, \left( \frac{\eta_N}{\lambda_n} \right)^{\frac{1}{\alpha}} \right\},$$

where,  $\lambda_n$  are an positive eigenvalues of  $A$  correspondingly orthonormal eigenfunctions  $\phi_n$ .

**Theorem 1.** Let  $\varphi, \psi, h \in D(A)$  and  $T \notin \Lambda$ . Then the inverse problem (1), (2) has a unique solution  $\{u(t), f\}$ .

#### References

1. Kilbas A.A., Srivastava H.M., Trujillo J.J. Theory and Applications of Fractional Differential Equations, Amsterdam: Elsevier, 2006.

\*\*\*

#### Solving a singular integral equation of the Volterra type for heat conduction problems

**Ramazanov M. I.<sup>1</sup>, Gulmanov N. K.<sup>2</sup>, Omarov M. T.<sup>3</sup>**

Karaganda Buketov University<sup>1,2,3</sup>, Karaganda, Kazakhstan  
madiomarovt@gmail.com

In the paper, a general solution to the singular integral equation of the Volterra type of the second kind is found

$$\mathbf{M}_\lambda \mu \equiv (I - \lambda \mathbf{M})\mu \equiv \mu(t) - \lambda \int_0^t M(t, \tau) \mu(\tau) d\tau = g(t),$$

where  $\lambda > 0$  and the kernel possesses the property

$$\lim_{t \rightarrow 0} \lambda \int_0^t M(t, \tau) d\tau = \frac{\lambda}{v}.$$

Integral equations of this type arise when solving heat conduction problems in non-canonical degenerate regions, where the boundaries change over time [1-2]. They also appear in the mathematical modeling of thermophysical processes in the electric arc of high-current circuit breakers [3-4]. The distinctive feature of the considered integral equation is that the integral of its kernel, as the upper limit approaches the lower one, is not equal to zero, meaning that the Picard method is not applicable. It is shown that the corresponding homogeneous integral equation has a non-zero solution, which is obtained explicitly.

#### References