International Scientific Conference

ABSTRACTS

OF THE IX INTERNATIONAL SCIENTIFIC CONFERENCE “ACTUAL PROBLEMS OF APPLIED MATHEMATICS AND INFORMATION TECHNOLOGIES AL-KHWARIZMI 2024”

Dedicated to the 630th anniversary of the birth of Mirzo Ulugbek

APAMIT-2024

22-23 October, 2024, Tashkent, Uzbekistan

https://apmath.nuu.uz

MINISTRY OF HIGHER EDUCATION, SCIENCE AND INNOVATION
OF THE REPUBLIC OF UZBEKISTAN

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INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKY

INTERNATIONAL SCIENTIFIC CONFERENCE

ACTUAL PROBLEMS OF APPLIED MATHEMATICS AND INFORMATION TECHNOLOGIES - AL-KHWARIZMI 2024

A B S T R A C T S

22-23 October 2024, Tashkent, Uzbekistan

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About the property of locally relative controllability of a differential inclusion

Otakulov S.[1], Rahimov B. SH.[2]
Jizzakh Polytechnic Institute [1,2],Jizzakh, Uzbekistan
raximovboyxoroz@gmail.com

Consider a control object whose dynamics in the n-dimensional state space R^n is described by differential inclusion (1,2)

$$\dot{x} \in A(t)x + B(t), t \geq t_0, \tag{1}$$

where $A = A(t)$ – is a $n \times n$ -matrix,$B = B(t)$ - is a multi–valued mapping. We will assume that the following conditions are met:1) the elements of the matrix $A(t)$ are measurable on any $T = [t_0, t_1] \subset [t_0, +\infty)$ and $||A(t)|| \leq a(t)$, where $a(\cdot) \in L_1(T)$;2) for each $t \geq t_0$ a set $B(t) \subset R^n$ compact and multivalued $t \to B(t)$ mapping is measurable on an arbitrary segment $T = [t_0, t_1] \subset [t_0, +\infty)$ and $||B(t)|| \leq b(t)$, where $b(\cdot) \in L_1(T)$.Let $X(t_0, t_1, \xi, A, B) = \Phi_A(t_1, t_0)\xi + \int\limits_{t_0}^{t_1} \Phi_A(t_1, \tau)B(\tau)d\tau$, where $\Phi_A(t, \tau)$ - is the fundamental matrix of solutions to equation $\dot{x} \in A(t)x$. Let $M \subset R^n$ be a given set terminal states of the control object (1),$M^\varepsilon = M + S_\varepsilon(0)$ – the ε - neighborhood of the set M.

Definition.We will say that a differential inclusion (1) is locally controllable relative to a given set M (or, briefly, locally-relatively to M controllable) if there is a number and a time interval $T = [t_0, t_1$, such that for any starting point $x_0 \in M^\varepsilon$ the relation $X(t_0, t_1, x_0, A, B) \cap M \neq 0, \forall\, x_0 \in M^\varepsilon$ holds.

Theorem 1. Let M – be compact from R^n . Then, for locally relative M-controllability of differential inclusion (1), it is necessary that the condition

$$\inf_{||\psi||=1} \sup_{\xi \in M} \int\limits_{t_0}^{t_1} c(\Phi_A(t_1, \tau)[A(\tau)\xi + B(\tau)], \psi)d\tau \geq 0,$$

was performed at some point in time $t_1 > t_0$.

Theorem 2. Let $B(t) = C(t)U(t)$, where $C(t) = (c_{i,j}(t))$ – $n \times m$ - is a matrix whose elements $c_{i,j}(t)$ are Lebesgue integrable on each $T = [t_0, t_1], t \to U(t)$, is a measurable multivalued map, $U(t) \in \Omega(R^m), |||U(t)|| \leq g(t), t \in T = [t_0, t_1], g(\cdot) \in L_1(T)$. Then if the system (1) is locally null-controlled, then there exists $t_1 > t_0$ such that for any $\psi \in R^n, ||\psi|| = 1$ the relation $\mu E(\psi) > 0$ is valid, where $\mu E(\psi)$ is the Lebesgue measure of the set $E(\psi) = \{t \in T = [t_0, t_1] : C^{'}(t)\Phi^{'}_A(t_1, t)\psi \neq 0\}$.

Corollary. Let $A(t) = A$, $B(t) = C \cdot U(t)$, where $C - n \times m$ – is a matrix, $t \to U(t)$ is a measurable multivalued map, $u(t) \in \Omega(R^m)$, $||U(t)|| \leq g(t)$,$t \in T = [t_0, t_1], g(\cdot) \in L_1(T)$. Then, if the system (1) is locally null-controlled, then $rankK = n$, where $K = \{C, AC, A^2C, ..., A^{n-1}C\}$. The paper researches the problem of local relative controllability for a mathematical model of a control system in the form of a linear differential inclusion. The studied property of locally relative M controllability of the considered model generalizes the concept of relative controllability of dynamic systems

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An inverse source problem for a fractional diffusion-wave equation

Rahmonov A. A.[1]
V.I.Romanovskiy Institute of Mathematics, Uzbekistan Academy of Sciences, Tashkent, Uzbekistan
a.raxmonov@mathinst.uz

Consider the following Cauchy-type problem with inverse time:

$$\begin{cases} \partial_t^\alpha u(t) + Au(t) = f(t), & 0 < t < T, \\ u(T) = \varphi, \quad \partial_t u(T) = \psi, \end{cases} \tag{1}$$

where φ and ψ are given functions. The operator ∂_t^α in Eq. (1) denotes the Caputo fractional derivative of order $\alpha \in (1, 2)$ with respect to t (see [1]):

$$\partial_t^\alpha y(t) = \frac{t_+^{1-\alpha}}{\Gamma(2-\alpha)} \star y''(t),$$

where $\star$ denotes the convolution, $A : X \to X$ be an arbitrary unbounded positive selfadjoint operator in separable Hilbert space X and A^{-1} is a compact operator. We will assume that the $f(t) \equiv f$, where $f \in X$, that is f does not depend on t.

Inverse problem. Given α, φ and ψ find a pair of functions $\{u, f\}$ satisfying the problem (1) and the additional condition

$$u(\tau) = h, \quad 0 < \tau < T, \tag{2}$$

where $h \in D(A) \subset X$ is a given element.

Definition 1. A pair $\{u(t), f\}$ of functions $u \in C([0, T]; X)$ and $f \in X$ with the properties $\partial_t^\alpha u(t), Au(t) \in C([0, T]; X)$ and satisfying conditions (1)-(2) is called the solution of the inverse problem.

We set

$$\Lambda = \Lambda(\alpha, A) := \bigcup_{n=1}^{\infty} \left\{ \left(\frac{\eta_1}{\lambda_n}\right)^{\frac{1}{\alpha}}, \dots, \left(\frac{\eta_N}{\lambda_n}\right)^{\frac{1}{\alpha}} \right\},$$

where, λ_n are an positive eigenvalues of A correspondingly orthonormal eigenfunctions ϕ_n.

Theorem 1. *Let $\varphi, \psi, h \in D(A)$ and $T \notin \Lambda$. Then the inverse problem* (1), (2) *has a unique solution* $\{u(t), f\}$.

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Solving a singular integral equation of the Volterra type for heat conduction problems

Ramazanov M. I.[1], Gulmanov N. K.[2], Omarov M. T.[3]

Karaganda Buketov University[1,2,3], Karaganda, Kazakhstan

madiomarovt@gmail.com

In the paper, a general solution to the singular integral equation of the Volterra type of the second kind is found

$$\mathbf{M}_\lambda \mu \equiv (I - \lambda \mathbf{M})\mu \equiv \mu(t) - \lambda \int_0^t M(t, \tau)\mu(\tau) d\tau = g(t),$$

where $\lambda > 0$ and the kernel possesses the property

$$\lim_{t \to 0} \lambda \int_0^t M(t, \tau) d\tau = \frac{\lambda}{v}.$$

Integral equations of this type arise when solving heat conduction problems in non-canonical degenerate regions, where the boundaries change over time [1-2]. They also appear in the mathematical modeling of thermophysical processes in the electric arc of high-current circuit breakers [3-4]. The distinctive feature of the considered integral equation is that the integral of its kernel, as the upper limit approaches the lower one, is not equal to zero, meaning that the Picard method is not applicable. It is shown that the corresponding homogeneous integral equation has a non-zero solution, which is obtained explicitly.

References