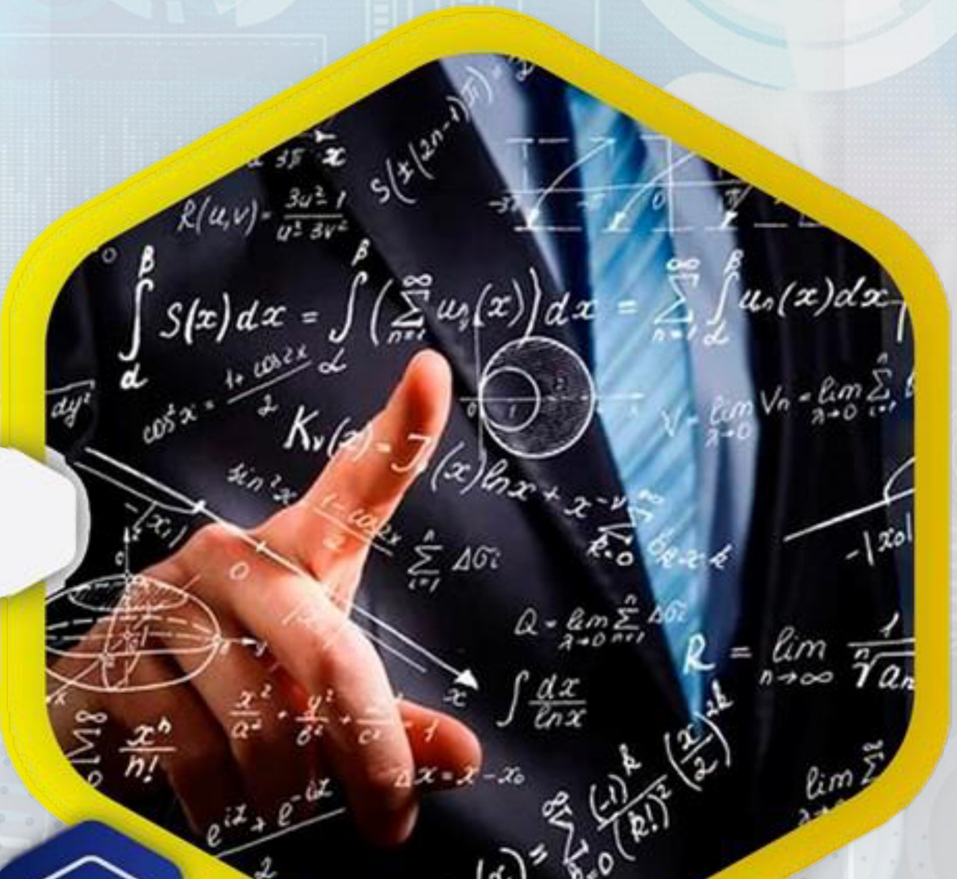
O'ZBEKISTON RESPUBLIKASI
OLIY TA'LIM, FAN VA INNOVATSIYALAR
VAZIRLIGI

BUXORO
DAVLAT
1930 UNIVERSITETI

"FIZIKA, MATEMATIKA VA SUN'IY INTELLEKT TEXNOLOGIYALARINING DOLZARB MUAMMOLARI"

XALQARO ILMIY-NAZARIY ANJUMAN MATERILLARI



BUXORO-2025

O'ZBEKISTON RESPUBLIKASI
OLIY TA'LIM, FAN VA INNOVATSIYALAR
VAZIRLIGI

BUXORO
DAVLAT
1930 UNIVERSITETI

CURRENT PROBLEMS OF PHYSICS, MATHEMATICS AND ARTIFICIAL INTELLIGENCE TECHNOLOGIES

INTERNATIONAL SCIENTIFIC AND THEORETICAL CONFERENCE

(May 16-17, 2025)

Bukhra-2025

UDC: 530+51+004.8
LBC: 22.1 + 22.16 + 32.81
A92 **ISBN: 978-6206-800-31-9**

SPECTRAL DEFERRED CORRECTION METHODS COLLOCATION FORMULATION AND EXPLICIT ITERATION

Akramov Ikrom Isroil o'g'li
Buxoro davlat universiteti
Fiz -mat va AT fakulteti o'qituvchisi
Email: ikrom960623@gmail.com

Abstract: Spectral Deferred Correction (SDC) methods are high-order iterative techniques for solving ordinary differential equations (ODEs) based on refining a low-order approximation using collocation nodes and correction sweeps. This paper outlines how to formulate the collocation problem central to SDC and describes the structure of an explicit SDC iteration. The flexibility and extensibility of SDC make it particularly suitable for solving both stiff and non-stiff initial value problems (IVPs) in scientific computing.

Аннотация: Методы спектральной отсроченной коррекции (SDC) — это итеративные методы высокого порядка для решения обыкновенных дифференциальных уравнений (ОДУ), основанные на уточнении приближения низкого порядка с использованием узлов коллокации и корректирующих проходов. В данной статье изложено, как сформулировать задачу коллокации, лежащую в основе SDC, и описана структура явной итерации SDC. Гибкость и расширяемость методов SDC делают их особенно подходящими для решения как жестких, так и нежестких задач Коши (IVP) в научных вычислениях.

Annotatsiya: Spektral kechiktirib tuzatish (SDC) usullari – bu oddiy differensial tenglamalarni (ODT) yechishda past tartibli taxminiy joylashish tugunlari va tuzatish aylanishlari yordamida aniqlashtirishga asoslangan yuqori tartibli iteratsion usullardir. Ushbu maqolada SDC usulining asosiy qismi bo'lgan joylashish muammosini qanday shakllantirish mumkinligi va aniq SDC iteratsiyasi tuzilmasi bayon qilinadi. SDC usullarining moslashuvchanligi va kengaytiriluvchanligi ularni ilmiy hisoblashlarda qattiq va qattiq bo'lmagan boshlang'ich qiymat masalalarini (IVP) yechish uchun ayniqsa mos qiladi.

Introduction

Traditional time-stepping schemes like Runge-Kutta and linear multistep methods struggle to balance accuracy, computational efficiency, and ease of implementation, especially when high-order accuracy is needed. Spectral Deferred Correction (SDC) methods offer a systematic framework to enhance time-stepping accuracy by correcting a low-order approximation using information from spectral collocation points. This idea was introduced by Dutt, Greengard, and Rokhlin [1] and further extended in subsequent [2,3].

Obtaining the Collocation Problem

Consider the initial value problem (IVP):

$$\frac{dy}{dt} = f(y,t), \quad y(t_0) = y_0, \quad t \in [t_0, t_0 + \Delta t]$$

Using Picard's formulation, the solution is expressed as: $y(t) = y_0 + \int_{t_0}^{t} f(y(s),s)ds$

To discretize the integral over $[t_0, t_0 + \Delta t]$, select a set of $M+1$ collocation nodes $\{t_0, t_1, \dots, t_M\}$, typically using Gauss-Lobatto or Gauss-Legendre points. The integral is approximated via spectral quadrature:

$$y(t_m) \approx y_0 + \sum_{j=0}^{M} Q_{mj} f(y_j, t_j)$$

Here, Q_{mj} are precomputed quadrature weights, and $y_j \approx y(t_j)$. The objective is to solve for $\{y_j\}$ that satisfy these integral approximations. Collectively, these form the collocation problem.

Explicit SDC Iteration

Instead of solving the collocation problem directly, SDC applies an iterative deferred correction approach, starting with a low-order method and refining it. Let $y_j^{(k)}$ denote the approximation at node t_j during the k^{th} sweep. The explicit SDC procedure is:

Step 1: Compute $y_j^{(0)}$ using forward Euler: $y_{j+1}^{(0)} = y_j^{(0)} + \Delta t_j f(y_j^{(0)}, t_j)$

Correction Sweeps

For $k = 0$ to $K-1$: Compute residual: $R_j^{(k)} = y_0 + \sum_{m=0}^{j} Q_{jm} f(y_m^{(k)}, t_m) - y_j^{(k)}$

Step 3: Update: $y_j^{(k+1)} = y_j^{(k)} + R_j^{(k)}$ and repeat until a desired residual reached

Conclusion

Spectral Deferred Correction methods reformulate an initial value problem into a collocation system and solve it iteratively. By using collocation points and correction sweeps, SDC enables high-order accuracy with modular implementations. Explicit SDC methods are particularly efficient for non-stiff systems and form a foundation for advanced implicit and parallel-in-time solvers.

References

1. Dutt, A., Greengard, L., & Rokhlin, V. (2000). Spectral Deferred Correction Methods for Ordinary Differential Equations. BIT Numerical Mathematics.
2. Minion, M. (2003). Semi-implicit spectral deferred correction methods for ordinary differential equations. Communications in Mathematical Sciences.
3. Emmett, M., & Minion, M. (2012). Toward an efficient parallel in time method for partial differential equations. Communications in Applied Mathematics and Computational Science.

ЗАДАЧА ДЛЯ ПСЕВДОПАРАБОЛИЧЕСКИХ ИНТЕГРО – ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ОДНОРОДНЫМИ ГРАНИЧНЫМИ УСЛОВИЯМИ

Б. Р. Абдуллаев
(докторант, Институт математики имени В. И. Романовского Академии наук Республики Узбекистан)

В области $D_T = \{(x,t): 0 < x < 1, 0 < t \le T\}$, рассмотрим псевдопараболического уравнения третьего порядка с классическими краевыми условиями:

$$u_t(x,t) - \alpha(t)u_{txx}(x,t) - \beta(t)u_{xx}(x,t) = \int_0^t K(t-\tau)\cdot u(x,\tau)d\tau + f(x,t) \quad (1)$$

начальным условием

$$u(x,0) = \varphi(x) \qquad (0 \le x \le 1), \qquad (2)$$

и граничными условиями

$$u(0,t) = u(1,t) = 0 \qquad (0 \le t \le T), \qquad (3)$$

где $\alpha(t) > 0, \beta(t) > 0, f(x,t), \varphi(x)$-заданные функции.

Определение. Под классическое решением прямой задачи (1) – (3) будем понимать функцию $u(x,t)$, из класса

$$u(x,t) \in C_{x,t}^{2,1}(D_T) \cap C(\overline{D_T}).$$

Для нахождения решения задачи (1) – (3) используем метод Фурье. Будем искать нетривиальное решение этой задачи в форме $u(x,t) = X(x)\cdot T(t)$. Подставляя эту формулу решения в уравнение и краевые условия, получим задачу для нахождения собственной функции $X(x)$:

$$X''(x) + \lambda X(x) = 0, \qquad 0 < x < 1, \qquad (4)$$

$$X(0) = X(1) = 0. \qquad (5)$$

Задача (4) – (5) имеет собственные значения

$$\lambda_k = (\pi k)^2, \qquad k = 0,1, \ldots,$$