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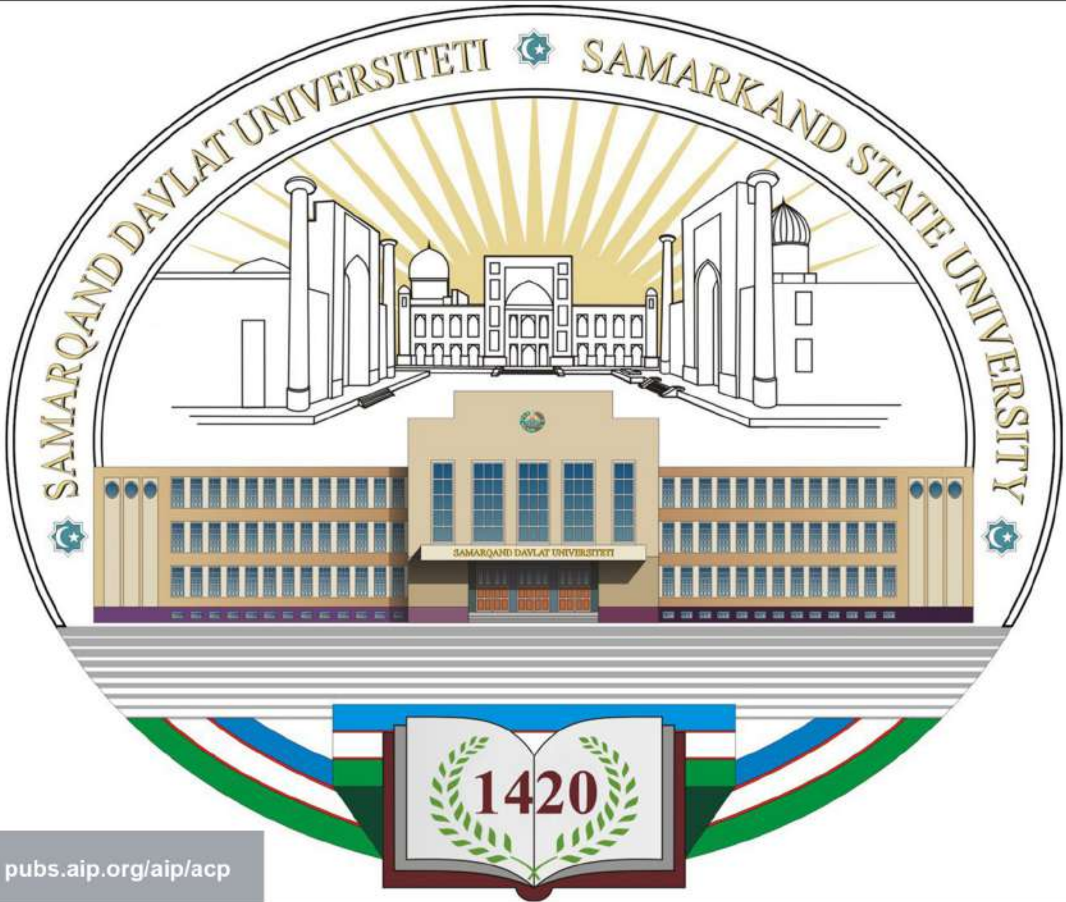

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Zukhro Adizova ; Istam Shadmanov

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Mathematical Modeling of Heat and Moisture Exchange Processes for Grain Storage

Zukhro Adizova [1, a)], Istam Shadmanov [1, b)]

[1]*Bukhara State University, Bukhara, Uzbekistan*

[a)]*Corresponding author: z.m.adizova@buxdu.uz*
[b)] *i.u.shadmanov@buxdu.uz*

Abstract. The modelling of heat and moisture transfer during the storage of grain products, monitoring, and forecast of temperature and moisture content are discussed in the article. In modern technologies of storage of grain crops and processed products, the most important factor is to control storage conditions to ensure preservation of product quality and minimize losses. The authors consider the main factors that affect heat and moisture transfer in storage facilities and present methods and means of their control and analysis. A mathematical model of the temperature changes and moisture content, occurring during the storage of porous bodies under the influence of solar energy, internal heat transfer, and ambient temperatures is developed. Wheat grains are taken as an example in the analysis of experimental work.
Keywords: mathematical model, temperature, moisture content, heat transfer, ambient temperature, grain.

INTRODUCTION

In the modern world, the problem of storing grain crops and the products of their processing plays a crucial role in ensuring food security. One of the key aspects of maintaining grain quality is the control of temperature and moisture content in storage facilities. Monitoring and forecasting these processes nowadays are essential parts of modern technologies in crop storage, making it possible to improve preservation conditions, reduce product losses, and ensure stable grain quality throughout the storage time. Food loss and spoilage worldwide have become a hot topic for research due to their significant impact on the environment and agricultural industry [1]. Every year, around one-third of the food produced worldwide is wasted [2]. Due to inadequate grain storage management [3], nutritional value and quality of grain deteriorate quickly. Insect activity and microbiota are the most crucial aspects of grain storage [4]. The rate of grain spoilage is mainly determined by the temperature and moisture distribution inside the grain mass [5]. That is, these factors affect the safety of the ecosystem of stored grain. Thus, even if the storage methods are appropriate, the safe storage of grain cannot be effectively guaranteed, and the volume of grain is then at risk of spoilage. In food technology, the main research has focused on the method of reducing drying time, for example, using additional heat sources (microwaves and radiation) to accelerate the freeze-drying process and increase productivity [6].

In [7], the authors used a model called the hysteresis cycle model (HCM) to describe the average temperature of a grain pile and daily fluctuations in air temperature, providing a prediction model obtained using a Fourier series based on the least squares method. There, the Fourier series model is used to predict the temperature of grain statistics based on the daily air temperature. By finding the relationship between air temperature and grain harvest temperature, called conversion coefficients, HCM determines the optimal order of model construction using Akaike information criteria. This method can reflect the time delay in changing the temperature of the grain pile when the air temperature changes and requires only one variable (that is, the value of the air temperature). The experiments conducted, using the model for predicting the temperature compared the results obtained by storing the grain for 425 days with the values observed in the experiments; the comparison showed satisfactory results.

In [8], information on the pattern of changes in environmental conditions (i.e. temperature, relative humidity) during the storage of wheat in silos made of corrugated galvanized iron, storage indoors, and in shelters is provided.

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In [9], the impact of changes in temperature and relative humidity in the storage area on the moisture content of stored maize grain and soya beans was examined. In [10], the authors developed a three-dimensional transient coupled model, which included thermal conductivity models for space, soil, and bulk grain, with the aim of predicting the temperature of grain stored in the granary. Different grids were used for grain temperature modelling (refinement of the grid over the entire region or boundary), including linear and hybrid (linear and quadratic) elements. The accuracy of temperature forecasts obtained from different grids was compared. Given the slight increase in accuracy due to the increased time spent cleaning the set at the boundary, the same set was preferable for predicting the temperature of grain of whole grain products along with cleaning over the entire area.

The development of a mathematical model, numerical algorithms, and software to monitor and forecast changes in temperature and moisture content during grain storage is vital for agriculture and food security. Heat and moisture transfer processes can significantly affect grain products' quality and safety during storage.

PROBLEM FORMULATION

Tanks designed for storing agricultural products in open space have the shape of a rectangular parallelopiped, which is communicates with the environment. The release of heat in drying agricultural products is a result of chemical and biological changes in live mass cells that take the shape of an exponential dependence. The thermal conductivity equation accounts for this component and describes the dynamics of the mass's thermal state [11-13]:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + g\left(x, y, z, t\right), \tag{1}$$

where T - is the temperature change of the mass over time t and in space; $a = \lambda / \left(\rho c\right)$; λ – coefficient of thermal conductivity of the mass; ρ - density; and c - heat capacity; $g\left(x, y, z, t\right) = be^{-\alpha t}$ – the intensity of the mass's heat release, decreasing with the expiration of time, b = q_0/c – the heat dissipation coefficient of the body; α – an empirical constant, l_x, l_y. - dimensions in coordinates.

The initial temperature value for the equation (1) is:

$$T\left(x, y, z, 0\right) = T_0\left(x, y, z\right) \tag{2}$$

and boundary conditions on the faces of a rectangular parallelepiped are:

$$\lambda \left.\frac{\partial T}{\partial x}\right|_{x=0} = -\beta\left(T_a\left(t\right) - T\left(0, y, z, t\right)\right), \tag{3}$$

$$\lambda \left.\frac{\partial T}{\partial x}\right|_{x=l_x} = -\beta\left(T_a\left(t\right) - T\left(l_x, y, z, t\right)\right), \tag{4}$$

$$\lambda \left.\frac{\partial T}{\partial y}\right|_{y=0} = -\beta\left(T_{oc}\left(t\right) - T\left(x, 0, \mathrm{z}, t\right)\right), \tag{5}$$

$$\lambda \left.\frac{\partial T}{\partial y}\right|_{y=l_y} = -\beta\left(T_a\left(t\right) - T\left(x, l_y, \mathrm{z}, t\right)\right), \tag{6}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = 0, \tag{7}$$

$$\lambda \left.\frac{\partial T}{\partial z}\right|_{z=l_z} = -\beta\left(T_a\left(t\right) - T\left(x, y, l_z, t\right)\right). \tag{8}$$

Here β - is the heat transfer coefficient; $T_a\left(t\right)$ – is the ambient temperature.

SOLUTION TO THE PROBLEM

We use an implicit finite difference approach to solve the problem (1–8):
The space-time grid is:

$$\Omega=\left\{\left(x_i=i\Delta x,\ y_j=j\Delta y,\ z_k=k\Delta z,\ \tau_n=n\ \Delta\tau\right);\ i=\overline{1,N_x};\ \ j=\overline{1,M_y},\ \ k=\overline{1,L_z},\ n=\overline{0,N_\tau},\ \ \Delta\tau=1/N_\tau\right\},$$

Equation (1) is approximated along the OX axis on grid Ω in the following form:

$$\frac{T_{i,j,k}^{n+\frac{1}{3}}-T_{i,j,k}^{n}}{\Delta\tau/3}=a\frac{T_{i+1,j,k}^{n+\frac{1}{3}}-2T_{i,j,k}^{n+\frac{1}{3}}+T_{i-1,j,k}^{n+\frac{1}{3}}}{\Delta x^2}+a\frac{T_{i,j+1,k}^{n}-2T_{i,j,k}^{n}+T_{i,j-1,k}^{n}}{\Delta y^2}+a\frac{T_{i,j,k+1}^{n}-2T_{i,j,k}^{n}+T_{i,j,k-1}^{n}}{\Delta z^2}+g_{i,j,k}^{n+\frac{1}{3}};$$

Let us introduce the notation:

$$A_{i,j,k}=\frac{a_t}{\Delta x^2},\qquad B_{i,j,k}=\frac{3}{\Delta\tau}+\frac{2a_t}{\Delta x^2},\qquad C_{i,j,k}=\frac{a_t}{\Delta x^2},$$

$$D_{i,j,k}=\frac{3}{\Delta\tau}T_{i,j,k}^{n}+a\frac{T_{i,j+1,k}^{n}-2T_{i,j,k}^{n}+T_{i,j-1,k}^{n}}{\Delta y^2}+a\frac{T_{i,j,k+1}^{n}-2T_{i,j,k}^{n}+T_{i,j,k-1}^{n}}{\Delta z^2}+g_{i,j,k}^{n+\frac{1}{3}},$$

we obtain:

$$A_{i,j,k}T_{i-1,j,k}^{n+\frac{1}{3}}-B_{i,j,k}T_{i,j,k}^{n+\frac{1}{3}}+C_{i,j,k}T_{i+1,j,k}^{n+\frac{1}{3}}=-D_{i,j,k}. \tag{9}$$

Now, by *X* axis, the boundary condition (3) is approximated:

$$\lambda_1\frac{-3T_{0,j,k}^{n+\frac{1}{3}}+4T_{1,j,k}^{n+\frac{1}{3}}-T_{2,j,k}^{n+\frac{1}{3}}}{2\Delta x}=-\beta_1T_a+\beta_1T_{0,j,k}^{n+\frac{1}{3}}. \tag{10}$$

For *i*=1 in system (9),

$$A_{1,j,k}T_{0,j,k}^{n+\frac{1}{3}}-B_{1,j,k}T_{1,j,k}^{n+\frac{1}{3}}+C_{1,j,k}T_{2,j,k}^{n+\frac{1}{3}}=-D_{1,j,k}. \tag{11}$$

Then, from the equations (11) and (10), we find $T_{0,j,k}^{n+\frac{1}{3}}$:

$$T_{0,j,k}^{n+\frac{1}{3}}=\frac{\lambda_1B_{1,j,k}-4\lambda_1C_{1,j,k}}{A_{1,j,k}\lambda_1-3C_{1,j,k}\lambda_1-2\Delta xC_{1,j,k}\beta_1}T_{1,j,k}^{n+\frac{1}{3}}+\frac{-D_{1,j,k}\lambda_1-2\Delta xC_{1,j,k}\beta_1T_a-2\Delta xC_{1,j,k}}{A_{1,j,k}\lambda_1-3C_{1,j,k}\lambda_1-2\Delta xC_{1,j,k}\beta_1}. \tag{12}$$

It follows from formula (12), that $\alpha_{0,j,k}$ and $\beta_{0,j,k}$ are the sweep coefficients:

$$\alpha_{0,j,k}=\frac{\lambda_1B_{1,j,k}-4\lambda_1C_{1,j,k}}{A_{1,j,k}\lambda_1-3C_{1,j,k}\lambda_1-2\Delta xC_{1,j,k}\beta_1}\ \text{ and }\ \beta_{0,j,k}=\frac{-D_{1,j,k}\lambda_1-2\Delta xC_{1,j,k}\beta_1T_a-2\Delta xC_{1,j,k}}{A_{1,j,k}\lambda_1-3C_{1,j,k}\lambda_1-2\Delta xC_{1,j,k}\beta_1}.$$

Approximating condition (4) by *X,* we obtain:

$$\lambda_1\frac{T_{N-2,j,k}^{n+\frac{1}{3}}-4T_{N-1,j,k}^{n+\frac{1}{3}}+3T_{N,j,k}^{n+\frac{1}{3}}}{2\Delta x}=-\beta_1T_a+\beta_1T_{N,j,k}^{n+\frac{1}{3}}. \tag{13}$$

Then, we find $T_{\mathrm{N}-1,j,k}^{n+\frac{1}{3}}$ and $T_{\mathrm{N}-2,j,k}^{n+\frac{1}{3}}$:

$$T_{N-1,j,k}^{n+\frac{1}{3}}=\alpha_{N-1,j,k}T_{N,j,k}^{n+\frac{1}{3}}+\beta_{N-1,j,k}, \tag{14}$$

$$T_{N-2,j,k}^{n+\frac{1}{3}}=\alpha_{N-2,j,k}\alpha_{N-1,j,k}T_{N,j,k}^{n+\frac{1}{3}}+\alpha_{N-2,j,k}\beta_{N-1,j,k}+\beta_{N-2,j,k}. \tag{15}$$

Substituting $T_{\mathrm{N}-1,j,k}^{n+\frac{1}{3}}$ from (14) and $T_{\mathrm{N}-2,j,k}^{n+\frac{1}{3}}$ from (15) in (13), we find $T_{\mathrm{N},j,k}^{n+\frac{1}{3}}$:

$$T_{N,j,k}^{n+\frac{1}{3}}=\frac{-\lambda_1\alpha_{N-2,j,k}\beta_{N-1,j,k}-\lambda_1\beta_{N-2,j,k}+4\lambda_1\beta_{N-1,j,k}-2\Delta x\beta_1T_a-2\Delta x}{3\lambda_1-2\Delta x\beta_1+\lambda_1\alpha_{N-2,j,k}\alpha_{N-1,j,k}-4\lambda_1\alpha_{N-1,j,k}}. \tag{16}$$

The temperature values for sequence $T_{\mathrm{N}-1,j,k}^{n+\frac{1}{3}}$, $T_{\mathrm{N}-2,j,k}^{n+\frac{1}{3}}$, ..., $T_{1,j,k}^{n+\frac{1}{3}}$ are ascertained by reducing the *i* sequence using the reverse sweep method:

$$T_{i,j,k}^{n+\frac{1}{3}}=\alpha_{i,j,k}T_{i+1,j,k}^{n+\frac{1}{3}}+\beta_{i,j,k},\ 1\le i\le N-1,\ 0\le j\le M,0\le k\le L$$

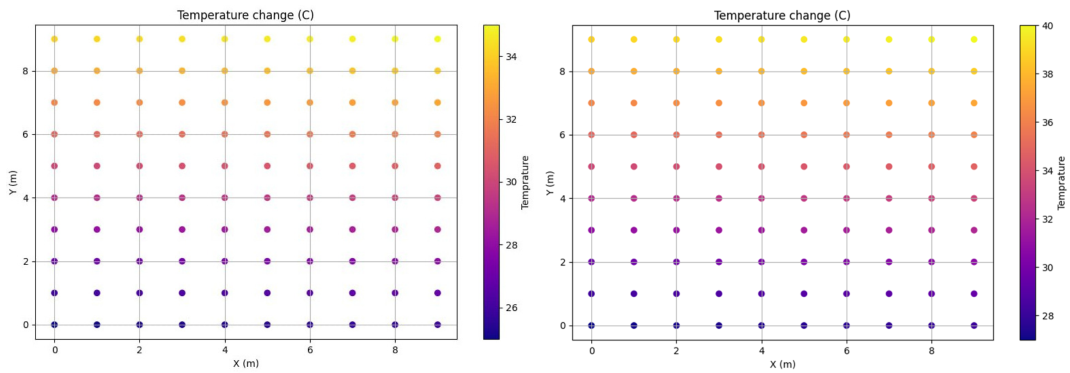
In the same way, equation (1) is approximated by *Y* with conditions (5-6), and by *Z* with conditions (7-8), and we have:

$$T_{i,j,L}^{n+1} = \frac{-\lambda_1 \overline{\overline{\alpha}}_{i,j,L-2}\, \overline{\overline{\beta}}_{i,j,L-1} - \lambda_1 \overline{\overline{\beta}}_{i,j,L-2} + 4\lambda_1 \overline{\overline{\beta}}_{i,j,L-1} - 2\Delta z \overline{\beta}_1 T_a - 2\Delta z}{3\lambda_1 - 2\Delta z \overline{\overline{\beta}}_1 + \lambda_1 \overline{\overline{\alpha}}_{i,j,L-2}\, \overline{\overline{\alpha}}_{i,j,L-1} - 4\lambda_1 \overline{\overline{\alpha}}_{i,j,L-1}}.$$

$$T_{i,j,k}^{n+1} = \overline{\overline{\alpha}}_{i,j,k}\, T_{i,j,k+1}^{n+1} + \overline{\overline{\beta}}_{i,j,k},\ \ 0 \le i \le N, 0 \le j \le M, 1 \le k \le L-1.$$

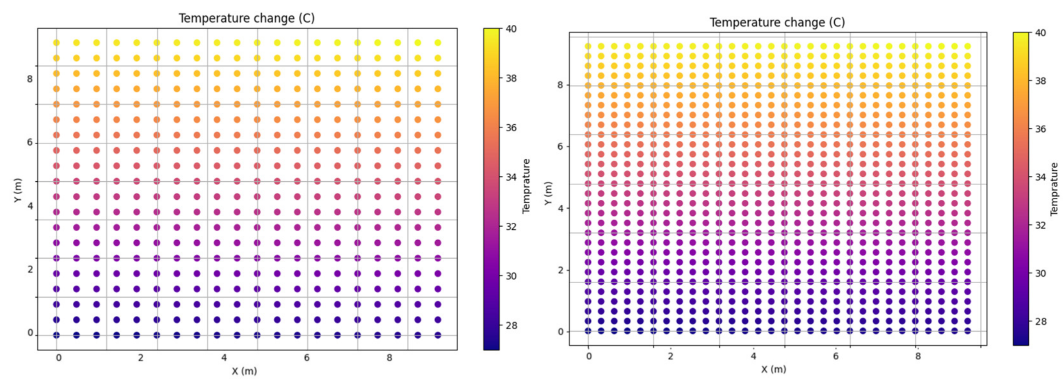
RESULTS AND DISCUSSION

Using the thermal conductivity equation, the authors developed a program for the dynamics of the mass's thermal state, and the results are illustrated in Figs. 1-2.



(a) after 10 days of storage, (b) after 20 days of storage.

FIGURE 1. Temperature changes of wheat at $T_a = 24^0 C$, $T(x, y, z, 0) = 27^0 C$.



(a) after 30 days of storage, (b) after 40 days of storage.

FIGURE 2. Temperature changes of wheat at $T_a = 32^0 C$, $T(x, y, z, 0) = 26^0 C$.

Figures 1-2 show the findings of the computational experiments on the temperature change along the O*x* and O*y* planes. In the first 10 and 20 days, the temperature changed slowly, approximately by 4°C difference (Figs. 1.a-b). After 30 days of storage, due to internal heat and moisture release, the temperature in wheat grain rose to 40°C (Fig. 2.a). When it is stored for a long time (over 40 days), wheat grain may lose quality due to increased moisture content and temperature in the interior part of the wheat pile (Fig. 2.b).

CONCLUSION

A mathematical model and numerical solution of temperature change during the storage of porous bodies under the influence of solar energy, internal heat transfer, and ambient temperatures were developed in this article. A numerical solution based on the finite-difference method with second-order accuracy in time-space variables approximates the boundary conditions with second-order accuracy, which saves absolute stability. The authors considered the main aspects of monitoring and forecasting the processes of heat transfer and moisture content of grain products during good-quality storage and their importance for the agricultural industry. The findings (Figs. 1–2) demonstrated that the role of temperature exchange with the environment and its moisture content is noticeable for small amounts of stored wheat. When wheat is stored in large quantities with high moisture content, the internal heat and moisture release becomes noticeable, leading to a widespread increase in the mass' internal temperature.

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