MODELING HEAT AND MOISTURE DISTRIBUTION IN GRAIN STORAGE USING A TWO-DIMENSIONAL MONITORING APPROACH

Adizova Zukhro Marupovna
Bukhara State University
Email: zuhroadizova096@gmail.com

Abstract. his paper presents the development of a two-dimensional mathematical model for monitoring heat and moisture transfer in grain storage systems. The model is based on the fundamental principles of heat conduction and moisture diffusion, and it takes into account the specific thermal and hygroscopic properties of stored grain. Numerical algorithms have been implemented to solve the governing partial differential equations, enabling the prediction of temperature and moisture distributions over time. The proposed model serves as a tool for assessing the stability and quality of stored grain, allowing for real-time monitoring and control of storage conditions. The results demonstrate the effectiveness of the approach in simulating the dynamic behavior of heat and moisture movement, providing valuable insights for optimizing grain storage management.

Keywords: rain storage, heat and moisture transfer, two-dimensional model, numerical simulation, monitoring, forecasting, mathematical modeling, storage conditions, quality preservation.

Аннотация. В этой статье представлено разработка двумерной математической модели для мониторинга теплопередачи и переноса влаги в системах хранения зерна. Модель основана на основных принципах теплопроводности и диффузии влаги, а также учитывает специфические тепловые и гигроскопические свойства хранимого зерна. Для решения основных частных дифференциальных уравнений были реализованы численные алгоритмы, позволяющие предсказать распределение температуры и влажности во времени. Предложенная модель служит инструментом для оценки стабильности и качества хранимого зерна, а также для мониторинга и контроля условий хранения в реальном времени. Результаты показывают эффективность подхода в симуляции динамического поведения тепла и влаги, предоставляя ценные сведения для оптимизации управления процессами хранения зерна.

Ключевые слова: хранение зерна, передача тепла и влаги, двумерная модель, численное моделирование, мониторинг, прогнозирование, математическое моделирование, условия хранения, сохранение качества.

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Annotatsiya. Ushbu maqolada don mahsulotlarini saqlash tizimlarida issiqlik va namlik uzatilishini monitoring qilish uchun ikki o'lchovli matematik model ishlab chiqilishi taqdim etiladi. Model issiqlik o'tkazilishi va namlik o'tkazish bo'yicha asosiy fizika qonunlariga asoslanadi hamda saqlanayotgan donning issiqlik va gigroskopik xususiyatlarini hisobga oladi. Boshqaruvchi xususiy tenglamalarni yechish uchun sonli algoritmlar ishlab chiqilib, ularning yordamida vaqt davomida harorat va namlik taqsimotlarini oldindan aniqlash imkoniyati yaratilgan. Taklif etilgan model saqlanayotgan don mahsulotlarining barqarorligi va sifatini baholash, shuningdek, saqlash sharoitlarini real vaqt rejimida monitoring qilish va boshqarish uchun samarali vosita bo'lib xizmat qiladi. Model issiqlik va namlik harakatining dinamikasini ishonchli simulyatsiya qilishdagi samaradorligini ko'rsatadi hamda don saqlash jarayonlarini optimallashtirish bo'yicha foydali tavsiyalar beradi.

Kalit so'zlar: don saqlash, issiqlik va namlik uzatish, ikki o'lchovli model, sonli usul, monitoring, prognozlash, matematik model, saqlash sharoitlari, sifatni saqlash.

Introduction

Food losses and spoilage worldwide have become a hot topic of research due to their significant impact on the environment, economy, and society [1]. Approximately one-third of food produced globally is wasted annually [2]. Poor grain storage management leads to rapid deterioration of its quality and nutritional value [3]. Stored grain has been studied as a critical factor in its preservation, with moisture content being one of the most important elements[4]. The moisture content is closely related to the activity of insects and microorganisms. The rate of grain spoilage is primarily determined by temperature and moisture distribution within the grain mass [5]. The interaction of these factors influences the safety of the stored grain ecosystem. Therefore, even with good storage practices, the safe storage of grain cannot be effectively ensured, and the quantity of grain may be at risk of spoilage.

In food technology, most research has focused on methods to reduce drying time, such as utilizing additional heat sources (microwaves and radiation)to accelerate the sublimation drying process and increase productivity [6]. Developing mathematical models, numerical algorithms, and software for monitoring and predicting heat and moisture transfer processes in grain storage is an important task, especially in the context of agriculture and food security. Heat and moisture transfer processes can significantly affect the quality and safety of stored grain products.

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In one study [7], the author used a hysteresis cycle model (HCM)to describe the average temperature of the grain mass and the daily fluctuations in air temperature. The forecasting model was developed using Fourier series and the least squares method. The Fourier series model was used to predict grain temperature statistics based on daily air temperature. By establishing a relationship between air temperature and grain temperature, called conversion coefficients, HCM determines the optimal model construction order using Akaike's information criteria. This method reflects the time delay in grain temperature changes in response to air temperature changes and requires only one variable (air temperature). Experiments using the model involved predicting temperature and comparing actual results obtained during grain storage over 425 days with the observed values, leading to satisfactory results.

The article [8] provides information on the environmental conditions (temperature, relative humidity)during wheat storage in corrugated galvanized iron silos, in closed rooms, and under shelters. Another study [9] highlighted the impact of relative humidity and temperature changes during storage on the moisture content of stored soybeans and corn grains.

The authors [10] developed a three-dimensional transient combined model (a model of the space, soil, and heat conduction of bulk grain) to predict grain temperature in a grain storage facility. Various grids, including linear and hybrid (linear and quadratic) elements, were used to model grain temperature. The accuracy of temperature predictions from different grids was compared. Given the slight increase in accuracy with more time spent on grid cleaning at the boundary, the same grid was preferred for predicting grain temperature in the bulk grain box with grid cleaning over the entire area.

Problem Statement.

Open-air grain storage facilities resemble a rectangle interacting with the environment. The proposed mathematical model of the heat transfer equation takes into account internal heat transfer and humidity and describes the dynamics of the thermal state of the mass:

$$\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial x}\left(a_{11}(x;z;\tau)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial z}\left(a_{11}(x;z;\tau)\frac{\partial T}{\partial z}\right)+$$

$$+\frac{\partial}{\partial x}\left(a_{12}(x;z;\tau)\frac{\partial Q}{\partial x}\right)+\frac{\partial}{\partial z}\left(a_{12}(x;z;\tau)\frac{\partial Q}{\partial z}\right)+F_1(x;z;\tau); \qquad (1)$$

$$\frac{\partial Q}{\partial \tau}=\frac{\partial}{\partial x}\left(a_{21}(x;z;\tau)\frac{\partial Q}{\partial x}\right)+\frac{\partial}{\partial z}\left(a_{21}(x;z;\tau)\frac{\partial Q}{\partial z}\right)+$$

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$$+\frac{\partial}{\partial x}\left(a_{22}(x,z,\tau)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial z}\left(a_{22}(x,z,\tau)\frac{\partial T}{\partial z}\right)+F_2(x,z,\tau) \qquad (2)$$

with initial

$$T(x,z,\tau)\big|_{\tau=0}=T_0(x,z); \qquad (3)$$

$$Q(x,z,\tau)\big|_{\tau=0}=Q_0(x,z) \qquad (4)$$

and boundary conditions

$$\mu\frac{\partial T}{\partial x}\bigg|_{x=0}=\psi_1(T-T_{tash}); \qquad (5)$$

$$\mu\frac{\partial T}{\partial x}\bigg|_{x=L_x}=\psi_1(T-T_{tash}); \qquad (6)$$

$$\mu\frac{\partial T}{\partial z}\bigg|_{z=0}=\psi_1(T-T_{tash}); \qquad (7)$$

$$\mu\frac{\partial T}{\partial z}\bigg|_{z=L_z}=\psi_1(T-T_{tash}); \qquad (8)$$

$$\omega\frac{\partial Q}{\partial x}\bigg|_{x=0}=\psi_2(Q-Q_{tash}); \qquad (9)$$

$$\omega\frac{\partial Q}{\partial x}\bigg|_{x=L_x}=\psi_2(Q-Q_{tash}); \qquad (10)$$

$$\omega\frac{\partial Q}{\partial z}\bigg|_{z=0}=\psi_2(Q-Q_{tash}); \qquad (11)$$

$$\omega\frac{\partial Q}{\partial z}\bigg|_{z=L_z}=\psi_2(Q-Q_{tash}). \qquad (12)$$

Here T and Q *are* the values of temperature and moisture of a porous body; $a_{11}(x,z,\tau)$, $a_{22}(x,z,\tau)$ - thermal conductivity coefficients; $a_{12}(x,z,\tau)$, $a_{21}(x,z,\tau)$ - moisture conductivity coefficients; $F_1(x;z;\tau)=b\cdot e^{-\zeta\tau}$ – intensity of internal heat release of the mass ; $b=\frac{Q}{c_1}$ - heat release coefficient; c_1 - specific heat capacity; ζ - empirical parameter; $F_2(x;z;\tau)=\rho m_0 e^{-\xi\tau}$ – intensity of internal moisture sources ; $\rho-$ material density; ξ- drying coefficient; m_0 - maximum evaporation rate;

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β_1 - heat transfer coefficient; T_{tash} - ambient temperature; β_2 - moisture transfer coefficient; Q_{tash} - ambient humidity.

Solution method. To solve the problem (1)–(12), we use the finite-difference method, replacing the continuous solution domain with a grid one and introducing the following space-time grid:

$$\Omega_{xz\tau} = \left\{ \left(x_i = i\Delta x,\ z_j = j\Delta z,\ \tau_n = n\,\Delta\tau \right);\ i = \overline{1, N_x};\ j = \overline{1, M_y},\ n = \overline{0, N_\tau},\ \Delta\tau = 1/N_\tau \right\},$$

we replace the differential operators of equation (1)with difference operators Ox :

$$\frac{1}{2}\frac{T_{i,j}^{n+1/2} - T_{i,j}^{n}}{\Delta\tau/2} + \frac{1}{2}\frac{T_{i+1,j}^{n+1/2} - T_{i+1,j}^{n}}{\Delta\tau/2} =$$

$$= \frac{1}{\Delta x^2}\left(a_{11,i+0,5,j} T_{i+1,j}^{n+1/2} - \left(a_{11,i+0,5,j} + a_{11,i-0,5,j} \right) T_{i,j}^{n+1/2} + a_{11,i-0,5,j} T_{i-1,j}^{n+1/2} \right) +$$

$$+ \frac{1}{\Delta z^2}\left(a_{11,i,j+0,5} T_{i,j+1}^{n} - \left(a_{11,i,j+0,5} + a_{11,i,j-0,5} \right) T_{i,j}^{n} + a_{11,i,j-0,5} T_{i,j-1}^{n} \right) +$$

$$+ \frac{1}{\Delta x^2}\left(a_{12,i+0,5,j} Q_{i+1,j}^{n} - \left(a_{12,i+0,5,j} + a_{12,i-0,5,j} \right) Q_{i,j}^{n} + a_{12,i-0,5,j} Q_{i-1,j}^{n} \right) +$$

$$+ \frac{1}{\Delta z^2}\left(a_{12,i,j+0,5} Q_{i,j+1}^{n} - \left(a_{12,i,j+0,5} + a_{12,i,j-0,5} \right) Q_{i,j}^{n} + a_{12,i,j-0,5} Q_{i,j-1}^{n} \right) + \frac{1}{2} F_{1,i,j};$$

and grouping similar members, we get:

$$\frac{a_{11,i-0,5,j}}{\Delta x^2} T_{i-1,j}^{n+1/2} - \left(\frac{a_{11,i+0,5,j} + a_{11,i-0,5,j}}{\Delta x^2} + \frac{1}{\Delta\tau} \right) T_{i,j}^{n+1/2} + \left(\frac{a_{11,i+0,5,j}}{\Delta x^2} - \frac{1}{\Delta\tau} \right) T_{i+1,j}^{n+1/2} =$$

$$-\left(\left(\frac{1}{\Delta\tau} - \frac{a_{11,i,j+0,5} + a_{11,i,j-0,5}}{\Delta z^2} \right) T_{i,j}^{n} + \frac{1}{\Delta\tau} T_{i+1,j}^{n} + \frac{a_{11,i,j+0,5}}{\Delta z^2} T_{i,j+1}^{n} + \frac{a_{11,i,j-0,5}}{\Delta z^2} T_{i,j-1}^{n} - \right.$$

$$- \left(\frac{a_{12,i+0,5,j} + a_{12,i-0,5,j}}{\Delta x^2} + \frac{a_{12,i,j+0,5} + a_{12,i,j-0,5}}{\Delta z^2} \right) Q_{i,j}^{n} + \frac{a_{12,i+0,5,j}}{\Delta x^2} Q_{i+1,j}^{n} +$$

$$\left. + \frac{a_{12,i-0,5,j}}{\Delta x^2} Q_{i-1,j}^{n} + \frac{a_{12,i,j+0,5}}{\Delta z^2} Q_{i,j+1}^{n} + \frac{a_{12,i,j-0,5}}{\Delta z^2} Q_{i,j-1}^{n} + \frac{1}{2} F_{1,i,j} \right).$$

Let's introduce the following notations:

$$a_{i,j} = \frac{a_{11,i-0,5,j}}{\Delta x^2};\ \ b_{i,j} = \frac{a_{11,i+0,5,j} + a_{11,i-0,5,j}}{\Delta x^2} + \frac{1}{\Delta\tau};\ c_{i,j} = \frac{a_{11,i+0,5,j}}{\Delta x^2} - \frac{1}{\Delta\tau};$$

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$$d_{i,j}=\left(\frac{1}{\Delta\tau}-\frac{a_{11,i,j+0,5}+a_{11,i,j-0,5}}{\Delta z^2}\right)T_{i,j}^n+\frac{1}{\Delta\tau}T_{i+1,j}^n+\frac{a_{11,i,j+0,5}}{\Delta z^2}T_{i,j+1}^n+\frac{a_{11,i,j-0,5}}{\Delta z^2}T_{i,j-1}^n-$$

$$-\left(\frac{a_{12,i+0,5,j}+a_{12,i-0,5,j}}{\Delta x^2}+\frac{a_{12,i,j+0,5}+a_{12,i,j-0,5}}{\Delta z^2}\right)Q_{i,j}^n+\frac{a_{12,i+0,5,j}}{\Delta x^2}Q_{i+1,j}^n+$$

$$+\frac{a_{12,i-0,5,j}}{\Delta x^2}Q_{i-1,j}^n+\frac{a_{12,i,j+0,5}}{\Delta z^2}Q_{i,j+1}^n+\frac{a_{12,i,j-0,5}}{\Delta z^2}Q_{i,j-1}^n+\frac{1}{2}F_{1,i,j}$$

we obtain a system of three diagonal algebraic equations:

$$a_{i,j}T_{i-1,j}^{n+1/2}-b_{i,j}T_{i,j}^{n+1/2}+c_{i,j}T_{i+1,j}^{n+1/2}=-d_{i,j}. \quad (13)$$

Next, we approximate the boundary condition (5) by and Ox obtain:

$$\mu\frac{-3T_{0,j}^{n+1/2}+4T_{1,j}^{n+1/2}-T_{2,j}^{n+1/2}}{2\Delta x}=-\psi_1\left(T_{tash}-T_{0,j}^{n+1/2}\right). \quad (14)$$

From the system of equations (13) when i =1, we obtain:

$$a_{1,j}T_{0,j}^{n+1/2}-b_{1,j}T_{1,j}^{n+1/2}+c_{1,j}T_{2,j}^{n+1/2}=-d_{1,j}. \quad (15)$$

By putting $T_{2,j}^{n+\frac{1}{3}}$ from (15)into (14), we find $T_{0,j}^{n+\frac{1}{3}}$:

$$T_{0,j}^{n+1/2}=\frac{4\mu c_{1,j}-b_{1,j}\mu}{3\mu c_{1,j}-a_{1,j}\mu+2\Delta x\psi_1 c_{1,j}}T_{1,j}^{n+1/2}+\frac{d_{1,j}\mu+2\Delta x\psi_1 c_{1,j}T_{tash}}{3\mu c_{1,j}-a_{1,j}\mu+2\Delta x\psi_1 c_{1,j}}$$

where the running coefficients $\alpha_{0,j},\beta_{0,j}$ are calculated using:

$$\alpha_{0,j}=\frac{4\mu c_{1,j}-b_{1,j}\mu}{3\mu c_{1,j}-a_{1,j}\mu+2\Delta x\psi_1 c_{1,j}} \text{ And } \beta_{0,j}=\frac{d_{1,j}\mu+2\Delta x\psi_1 c_{1,j}T_{tash}}{3\mu c_{1,j}-a_{1,j}\mu+2\Delta x\psi_1 c_{1,j}}.$$

Similarly, approximating the boundary condition (6)by Ox, we obtain:

$$\mu\frac{T_{N-2,j}^{n+1/2}-4T_{N-1,j}^{n+1/2}+3T_{N,j}^{n+1/2}}{2\Delta x}=-\psi_1\left(T_{tash}-T_{N,j}^{n+1/2}\right). \quad (16)$$

Applying the sweep method to the sequence for *N, N -1* and *N -2* we find $T_{N-1,j}^{n+1/2}$ and $T_{N-2,j}^{n+1/2}$:

$$T_{N-1,j}^{n+1/2}=\alpha_{N-1,j}T_{N,j}^{n+1/2}+\beta_{N-1,j};, \quad (17)$$

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$$T_{N-2,j}^{n+1/2} = \alpha_{N-2,j}\alpha_{N-1,j}T_{N,j}^{n+1/2} + \alpha_{N-2,j}\beta_{N-1,j} + \beta_{N-2,j}\,. \qquad (18)$$

By putting $T_{N-1,j}^{n+1/2}$ from (17) and $T_{N-2,j}^{n+1/2}$ from (18)into (16), we find $T_{N,j}^{n+1/2}$:

$$T_{N,j}^{n+1/2} = \frac{\left(4\beta_{N-1,j} - \alpha_{N-2,j}\beta_{N-1,j} - \beta_{N-2,j}\right)\mu - 2\psi_1\Delta x T_{tash}}{\alpha_{N-2,j}\alpha_{N-1,j}\mu - 4\alpha_{N-1,j}\mu + 3\mu - 2\psi_1\Delta x}. \qquad (19)$$

temperature $T_{N-1,j}^{n+1/2}$ sequence, $T_{N-2,j}^{n+1/2}$,..., $T_{1,j}^{n+1/2}$ are determined by the method of backward sweeping by decreasing the value of i sequences:

$$T_{i,j}^{n+1/2} = \alpha_{i,j}T_{i+1,j}^{n+1/2} + \beta_{i,j}, \quad i = \overline{N-1,1},\ j = \overline{0,M}\,. \qquad (20)$$

Similarly, equation (2) is approximated by Ox finite-difference relations:

$$d_{i,j} = \left(\frac{1}{\Delta\tau} - \frac{a_{21,i,j+0,5} + a_{21,i,j-0,5}}{\Delta z^2}\right)Q_{i,j}^n + \frac{1}{\Delta\tau}Q_{i+1,j}^n + \frac{a_{21,i,j+0,5}}{\Delta z^2}Q_{i,j+1}^n + \frac{a_{21,i,j-0,5}}{\Delta z^2}Q_{i,j-1}^n -$$
$$-\left(\frac{a_{22,i+0,5,j} + a_{22,i-0,5,j}}{\Delta x^2} + \frac{a_{22,i,j+0,5} + a_{22,i,j-0,5}}{\Delta z^2}\right)T_{i,j}^n + \frac{a_{22,i+0,5,j}}{\Delta x^2}T_{i+1,j}^n +$$
$$+\frac{a_{22,i-0,5,j}}{\Delta x^2}T_{i-1,j}^n + \frac{a_{22,i,j+0,5}}{\Delta z^2}T_{i,j+1}^n + \frac{a_{22,i,j-0,5}}{\Delta z^2}T_{i,j-1}^n + \frac{1}{2}F_{2,i,j}.$$

and we obtain a system of tridiagonal algebraic equations with respect to the desired variables:

$$\overline{a_{i,j}}Q_{i-1,j}^{n+1/2} - \overline{b_{i,j}}Q_{i,j}^{n+1/2} + \overline{c_{i,j}}Q_{i+1,j}^{n+1/2} = -\overline{d_{i,j}}\,. \qquad (21)$$

Next, we approximate the boundary condition (9)with the second order of accuracy by Ox and obtain:

$$\omega\frac{-3Q_{0,j}^{n+1/2} + 4Q_{1,j}^{n+1/2} - Q_{2,j}^{n+1/2}}{2\Delta x} = -\psi_2\left(Q_{tash} - Q_{0,j}^{n+1/2}\right). \qquad (22)$$

From the system of equations (21)for i =1, we obtain:

$$\overline{a_{1,j}}Q_{0,j}^{n+1/2} - \overline{b_{1,j}}Q_{1,j}^{n+1/2} + \overline{c_{1,j}}Q_{2,j}^{n+1/2} = -\overline{d_{1,j}}\,. \qquad (23)$$

By putting $Q_{2,j}^{n+1/2}$ from (23)into (22), we find the value $Q_{0,j}^{n+1/2}$:

$$Q_{0,j}^{n+1/2} = \frac{4\omega\overline{c_{1,j}} - \overline{b_{1,j}}\omega}{3\omega\overline{c_{1,j}} - \overline{a_{1,j}}\omega + 2\Delta x\psi_2\overline{c_{1,j}}}Q_{1,j}^{n+1/2} + \frac{\overline{d_{1,j}}\omega + 2\Delta x\psi_2\overline{c_{1,j}}Q_{tash}}{3\omega\overline{c_{1,j}} - \overline{a_{1,j}}\omega + 2\Delta x\psi_2\overline{c_{1,j}}}. \qquad (24)$$

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where from relation (24)the running coefficients are determined using:

$$\overline{\alpha_{0,j}} = \frac{4\omega\overline{c_{1,j}} - \overline{b_{1,j}}\omega}{3\omega\overline{c_{1,j}} - \overline{a_{1,j}}\omega + 2\Delta x\psi_2\overline{c_{1,j}}};\ \overline{\beta_{0,j}} = \frac{d_{1,j}\omega + 2\Delta x\psi_2 c_{1,j}Q_{tash}}{3\omega c_{1,j} - a_{1,j}\omega + 2\Delta x\psi_2 c_{1,j}}.$$

Similarly, approximating the boundary condition (10)by Ox and we get:

$$\omega\frac{Q_{N-2,j}^{n+1/2} - 4Q_{N-1,j}^{n+1/2} + 3Q_{N,j}^{n+1/2}}{2\Delta x} = -\psi_2\left(T_{tash} - T_{N,j}^{n+1/2}\right). \quad (25)$$

Using the sweep method for the sequence *N, N -1* and *N -2* we find the value of $Q_{N-1,j}^{n+1/2}$ and $Q_{N-2,j}^{n+1/2}$:

$$Q_{N-1,j}^{n+1/2} = \overline{\alpha_{N-1,j}}Q_{N,j}^{n+1/2} + \overline{\beta_{N-1,j}}; \quad (26)$$

$$Q_{N-2,j}^{n+1/2} = \overline{\alpha_{N-2,j}}Q_{N-1,j}^{n+1/2} + \overline{\beta_{N-2,j}} = \overline{\alpha_{N-2,j}}\left(\overline{\alpha_{N-1,j}}Q_{N,j}^{n+1/2} + \overline{\beta_{N-1,j}}\right) + \overline{\beta_{N-2,j}} =$$

$$= \overline{\alpha_{N-2,j}}\,\overline{\alpha_{N-1,j}}Q_{N,j}^{n+1/2} + \overline{\alpha_{N-2,j}}\,\overline{\beta_{N-1,j}} + \overline{\beta_{N-2,j}}. \quad (27)$$

By putting $Q_{N-1,j}^{n+1/2}$ from (26)and $Q_{N-2,j}^{n+1/2}$ from (27)into (25), we find $Q_{N,j}^{n+1/2}$:

$$Q_{N,j}^{n+1/2} = \frac{\left(4\overline{\beta_{N-1,j}} - \overline{\alpha_{N-2,j}}\,\overline{\beta_{N-1,j}} - \overline{\beta_{N-2,j}}\right)\omega - 2\psi_2\Delta x Q_{tash}}{\overline{\alpha_{N-2,j}}\,\overline{\alpha_{N-1,j}}\omega - 4\overline{\alpha_{N-1,j}}\omega + 3\omega - 2\psi_2\Delta x}. \quad (28)$$

Moisture Sequence Values $Q_{N-1,j}^{n+1/2}$, $Q_{N-2,j}^{n+1/2}$,..., $Q_{1,j}^{n+1/2}$ is determined by the method of backward sweep by decreasing *i* sequences:

$$Q_{i,j}^{n+1/2} = \overline{\alpha_{i,j}}Q_{i+1,j}^{n+1/2} + \overline{\beta_{i,j}},\ \text{where } i = \overline{N-1,1},\ j = \overline{0,M}. \quad (29)$$

Similarly, equation (1)is approximated by Oz finite-difference relations

$$\overline{\overline{a}}_{i,j} = \frac{a_{11,i,j-0,5}}{\Delta z^2};\ \overline{\overline{b}}_{i,j} = \frac{a_{11,i,j+0,5} + a_{11,i,j-0,5}}{\Delta z^2} + \frac{1}{\Delta\tau};\ \overline{\overline{c}}_{i,j} = \frac{a_{11,i,j+0,5}}{\Delta z^2} - \frac{1}{\Delta\tau};$$

$$\overline{\overline{d}}_{i,j} = \left(\frac{1}{\Delta\tau} - \frac{a_{11,i+0,5,j} + a_{11,i-0,5,j}}{\Delta x^2}\right)T_{i,j}^{n+1/2} + \frac{1}{\Delta\tau}T_{i,j+1}^{n+1/2} + \frac{a_{11,i+0,5,j}}{\Delta x^2}T_{i+1,j}^{n+1/2} +$$

$$+\frac{a_{11,i-0,5,j}}{\Delta x^2}T_{i-1,j}^{n+1/2} - \left(\frac{a_{12,i+0,5,j} + a_{12,i-0,5,j}}{\Delta x^2} + \frac{a_{12,i,j+0,5} + a_{12,i,j-0,5}}{\Delta z^2}\right)Q_{i,j}^{n+1/2} +$$

$$+\frac{a_{12,i+0,5,j}}{\Delta x^2}Q_{i+1,j}^{n+1/2} + \frac{a_{12,i-0,5,j}}{\Delta x^2}Q_{i-1,j}^{n+1/2} + \frac{a_{12,i,j+0,5}}{\Delta z^2}Q_{i,j+1}^{n+1/2} + \frac{a_{12,i,j-0,5}}{\Delta z^2}Q_{i,j-1}^{n+1/2} + \frac{1}{2}F_{1,i,j}.$$

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and we obtain a system of tridiagonal algebraic equations with respect to the desired variables:

$$\overline{\overline{a}}_{i,j}T_{i,j-1}^{n+1}-\overline{\overline{b}}_{i,j}T_{i,j}^{n+1}+\overline{\overline{c}}_{i,j}T_{i,j+1}^{n+1}=-\overline{\overline{d}}_{i,j}. \quad (30)$$

Next, we approximate the boundary condition (7)by and Oy obtain:

$$\mu\frac{-3T_{i,0}^{n+1}+4T_{i,1}^{n+1}-T_{i,2}^{n+1}}{2\Delta z}=-\psi_1\left(T_{tash}-T_{i,0}^{n+1}\right). \quad (31)$$

From the system of equations (30)when $j=1$ we obtain :

$$\overline{\overline{a}}_{i,1}T_{i,0}^{n+1}-\overline{\overline{b}}_{i,1}T_{i,1}^{n+1}+\overline{\overline{c}}_{i,1}T_{i,2}^{n+1}=-\overline{\overline{d}}_{i,1}. \quad (32)$$

By putting $T_{i,2}^{n+1}$ from (32)into (31), we find the value $T_{i,0}^{n+1}$:

$$T_{i,0}^{n+1}=\frac{4\mu\overline{\overline{c}}_{i,1}-\overline{\overline{b}}_{i,1}\mu}{3\mu\overline{\overline{c}}_{i,1}-\overline{\overline{a}}_{i,1}\mu+2\Delta z\psi_1\overline{\overline{c}}_{i,1}}T_{i,1}^{n+1}+\frac{\overline{\overline{d}}_{i,1}\mu+2\Delta z\psi_1\overline{\overline{c}}_{i,1}T_{tash}}{3\mu\overline{\overline{c}}_{i,1}-\overline{\overline{a}}_{i,1}\mu+2\Delta z\psi_1\overline{\overline{c}}_{i,1}}, \quad (33)$$

where the running coefficients are determined using the following relationships:

$$\overline{\overline{\alpha}}_{i,0}=\frac{4\mu\overline{\overline{c}}_{i,1}-\overline{\overline{b}}_{i,1}\mu}{3\mu\overline{\overline{c}}_{i,1}-\overline{\overline{a}}_{i,1}\mu+2\Delta z\psi_1\overline{\overline{c}}_{i,1}} \text{ And } \overline{\overline{\beta}}_{i,0}=\frac{\overline{\overline{d}}_{i,1}\mu+2\Delta z\psi_1\overline{\overline{c}}_{i,1}T_{tash}}{3\mu\overline{\overline{c}}_{i,1}-\overline{\overline{a}}_{i,1}\mu+2\Delta z\psi_1\overline{\overline{c}}_{i,1}}.$$

Next, approximating the boundary condition (8)we Oy obtain:

$$\mu\frac{T_{i,M-2}^{n+1}-4T_{i,M-1}^{n+1}+3T_{i,M}^{n+1}}{2\Delta z}=-\psi_1\left(T_{tash}-T_{i,M}^{n+1}\right). \quad (34)$$

Using the sweep method for the sequence *M, M -1* and *M -2* we find the value of $T_{i,M-1}^{n+1}$ and $T_{i,M-2}^{n+1}$:

$$T_{i,M-1}^{n+1}=\overline{\overline{\alpha}}_{i,M-1}T_{i,M}^{n+1}+\overline{\overline{\beta}}_{i,M-1}; \quad (35)$$

$$T_{i,M-2}^{n+1}=\overline{\overline{\alpha}}_{i,M-2}T_{i,M-1}^{n+1}+\overline{\overline{\beta}}_{i,M-2}=\overline{\overline{\alpha}}_{i,M-2}\left(\overline{\overline{\alpha}}_{i,M-1}T_{i,M}^{n+1}+\overline{\overline{\beta}}_{i,M-1}\right)+\overline{\overline{\beta}}_{i,M-2}=$$

$$=\overline{\overline{\alpha}}_{i,M-2}\overline{\overline{\alpha}}_{i,M-1}T_{i,M}^{n+1}+\overline{\overline{\alpha}}_{i,M-2}\overline{\overline{\beta}}_{i,M-1}+\overline{\overline{\beta}}_{i,M-2}. \quad (36)$$

By putting $T_{i,M-1}^{n+1}$ from (35)and $T_{i,M-2}^{n+1}$ from (36)into (34), we find $T_{i,M}^{n+1}$:

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$$T_{i,M}^{n+1}=\frac{\left(4\overline{\overline{\beta}}_{i,M-1}-\overline{\overline{\alpha}}_{i,M-2}\overline{\overline{\beta}}_{i,M-1}-\overline{\overline{\beta}}_{i,M-2}\right)\mu-2\psi_1\Delta zT_{tash}}{\overline{\overline{\alpha}}_{i,M-2}\overline{\overline{\alpha}}_{i,M-1}\mu-4\overline{\overline{\alpha}}_{i,M-1}\mu+3\mu-2\psi_1\Delta z}. \quad (37)$$

The values of temperature $T_{i,M-1}^{n+1}$, $T_{i,M-2}^{n+1}$,..., $T_{i,1}^{n+1}$ are determined by the method of backward sweeping by decreasing the sequence of values j :

$$T_{i,j}^{n+1}=\overline{\overline{\alpha}}_{i,j}T_{i,j+1}^{n+1}+\overline{\overline{\beta}}_{i,j} \text{ where } i=\overline{0,N},\ j=\overline{M-1,1}. \quad (38)$$

Similarly, equation (2)is approximated by Oz finite-difference relations

$$\overline{\overline{a}}_{i,j}=\frac{a_{21,i,j-0,5}}{\Delta z^2};\ \overline{\overline{b}}_{i,j}=\frac{a_{21,i,j+0,5}+a_{21,i,j-0,5}}{\Delta z^2}+\frac{1}{\Delta\tau};\ \overline{\overline{c}}_{i,j}=\frac{a_{21,i,j+0,5}}{\Delta z^2}-\frac{1}{\Delta\tau};$$

$$\overline{\overline{d}}_{i,j}=\left(\frac{1}{\Delta\tau}-\frac{a_{21,i+0,5,j}+a_{21,i-0,5,j}}{\Delta x^2}\right)Q_{i,j}^{n+1/2}+\frac{1}{\Delta\tau}Q_{i,j+1}^{n+1/2}+\frac{a_{21,i+0,5,j}}{\Delta x^2}Q_{i+1,j}^{n+1/2}+$$

$$+\frac{a_{21,i-0,5,j}}{\Delta x^2}Q_{i-1,j}^{n+1/2}-\left(\frac{a_{22,i+0,5,j}+a_{22,i-0,5,j}}{\Delta x^2}+\frac{a_{22,i,j+0,5}+a_{22,i,j-0,5}}{\Delta z^2}\right)T_{i,j}^{n+1/2}+$$

$$+\frac{a_{22,i+0,5,j}}{\Delta x^2}T_{i+1,j}^{n+1/2}+\frac{a_{22,i-0,5,j}}{\Delta x^2}T_{i-1,j}^{n+1/2}+\frac{a_{22,i,j+0,5}}{\Delta z^2}T_{i,j+1}^{n+1/2}+\frac{a_{22,i,j-0,5}}{\Delta z^2}T_{i,j-1}^{n+1/2}+\frac{1}{2}F_{2,i,j}.$$

and we obtain a system of tridigonal algebraic equations with respect to the sought variables:

$$\overline{\overline{a}}_{i,j}Q_{i,j-1}^{n+1}-\overline{\overline{b}}_{i,j}Q_{i,j}^{n+1}+\overline{\overline{c}}_{i,j}Q_{i,j+1}^{n+1}=-\overline{\overline{d}}_{i,j}. \quad (39)$$

Next, we approximate the boundary condition (1 1)by Oz, and obtain:

$$\omega\frac{-3Q_{i,0}^{n+1}+4Q_{i,1}^{n+1}-Q_{i,2}^{n+1}}{2\Delta z}=-\psi_2\left(Q_{tash}-Q_{i,0}^{n+1}\right). \quad (40)$$

From the system of equations (39)when j =1, we obtain :

$$\overline{\overline{a}}_{i,1}Q_{i,0}^{n+1}-\overline{\overline{b}}_{i,1}Q_{i,1}^{n+1}+\overline{\overline{c}}_{i,1}Q_{i,2}^{n+1}=-\overline{\overline{d}}_{i,1}. \quad (41)$$

By putting $Q_{i,2}^{n+1}$ from (41)into (40), we find the value $Q_{i,0}^{n+1}$:

$$Q_{i,0}^{n+1}=\frac{4\omega\overline{\overline{c}}_{i,1}-\overline{\overline{b}}_{i,1}\omega}{3\omega\overline{\overline{c}}_{i,1}-\overline{\overline{a}}_{i,1}\omega+2\Delta z\psi_2\overline{\overline{c}}_{i,1}}u_{i,1}^{n+1}+\frac{\overline{\overline{d}}_{i,1}\omega+2\Delta z\psi_2\overline{\overline{c}}_{i,1}Q_{tash}}{3\omega\overline{\overline{c}}_{i,1}-\overline{\overline{a}}_{i,1}\omega+2\Delta z\psi_2\overline{\overline{c}}_{i,1}}. \quad (42)$$

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From relation (42)it follows that the running coefficients $\overline{\overline{\alpha}}_{i,0}$ and $\overline{\overline{\beta}}_{i,0}$ is calculated using:

$$\overline{\overline{\alpha}}_{i,0}=\frac{4\omega\overline{\overline{c}}_{i,1}-\overline{\overline{b}}_{i,1}\omega}{3\omega\overline{\overline{c}}_{i,1}-\overline{\overline{a}}_{i,1}\omega+2\Delta z\psi_2\overline{\overline{c}}_{i,1}} \text{ And } \overline{\overline{\beta}}_{i,0}=\frac{\overline{\overline{d}}_{i,1}\omega+2\Delta z\psi_2\overline{\overline{c}}_{i,1}Q_{tash}}{3\omega\overline{\overline{c}}_{i,1}-\overline{\overline{a}}_{i,1}\omega+2\Delta z\psi_2\overline{\overline{c}}_{i,1}}.$$

Similarly, approximating the boundary condition (1 2)by $Oz,$ with the second order of accuracy we get :

$$\omega\frac{Q_{i,M-2}^{n+1}-4Q_{i,M-1}^{n+1}+3Q_{i,M}^{n+1}}{2\Delta z}=-\psi_2\left(Q_{tash}-Q_{i,M}^{n+1}\right). \quad (43)$$

Using the sweep method for the sequence of indices *M, M -1* and *M -2,* we find the value $Q_{i,M-1}^{n+1}$ and $Q_{i,M-2}^{n+1}$ as follows :

$$Q_{i,M-1}^{n+1}=\overline{\overline{\alpha}}_{i,M-1}Q_{i,M}^{n+1}+\overline{\overline{\beta}}_{i,M-1}; \quad (44)$$

$$Q_{i,M-2}^{n+1}=\overline{\overline{\alpha}}_{i,M-2}Q_{i,M-1}^{n+1}+\overline{\overline{\beta}}_{i,M-2}=\overline{\overline{\alpha}}_{i,M-2}\left(\overline{\overline{\alpha}}_{i,M-1}Q_{i,M}^{n+1}+\overline{\overline{\beta}}_{i,M-1}\right)+\overline{\overline{\beta}}_{i,M-2}=$$

$$=\overline{\overline{\alpha}}_{i,M-2}\overline{\overline{\alpha}}_{i,M-1}Q_{i,M}^{n+1}+\overline{\overline{\alpha}}_{i,M-2}\overline{\overline{\beta}}_{i,M-1}+\overline{\overline{\beta}}_{i,M-2}. \quad (45)$$

By putting $Q_{i,M-1}^{n+1}$ from (44)and $Q_{i,M-2}^{n+1}$ from (45)into (43), we find $Q_{i,M}^{n+1}$:

$$Q_{i,M}^{n+1}=\frac{\left(4\overline{\overline{\beta}}_{i,M-1}-\overline{\overline{\alpha}}_{i,M-2}\overline{\overline{\beta}}_{i,M-1}-\overline{\overline{\beta}}_{i,M-2}\right)\omega-2\psi_2\Delta zQ_{tash}}{\overline{\overline{\alpha}}_{i,M-2}\overline{\overline{\alpha}}_{i,M-1}\omega-4\overline{\overline{\alpha}}_{i,M-1}\omega+3\omega-2\psi_2\Delta z}. \quad (46)$$

Moisture values $Q_{i,M-1}^{n+1}, Q_{i,M-2}^{n+1},\dots,Q_{i,1}^{n+1}$ are determined sequentially by the method of backward sweeping to decrease the value of the index j :

$$Q_{i,j}^{n+1}=\overline{\overline{\alpha}}_{i,j}Q_{i+1,j}^{n+1}+\overline{\overline{\beta}}_{i,j}, \text{ Where } i=\overline{0,N},\ j=\overline{M-1,1}. \quad (47)$$

Conclusion

This article develops a mathematical model for the temperature change during grain storage under the influence of solar radiation, internal heat exchange, and ambient temperature, along with its numerical solution. The numerical solution, based on the finite difference method, approximates the boundary conditions for spatiotemporal variables with double precision, ensuring absolute stability. The authors examine the main aspects of monitoring (analysis)and forecasting heat conduction and moisture processes in grain during proper storage and their

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significance for the grain industry. The results showed that temperature exchange with the environment and humidity within it play a crucial role when storing small quantities of wheat. When wheat is stored in an environment with high humidity, internal heat and moisture will be felt, leading to changes in internal heat and moisture levels.

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