



PHASES OF MATHEMATICAL MODELING AND PUT IT INTO PRACTICE

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Abstract

Mathematical modeling is an ideal scientific symbolic formal modeling, in which the description of the object is carried out in the language of mathematics and the study of the model is carried out using certain mathematical methods. The basic principles of mathematical modeling, types of mathematical modeling problems, creating a high-quality model, mathematical modeling creation (statement of the mathematical problem), mathematical justification of the model. Examining the internal consistency of the model. Justification of the accuracy of the differential model. Proving theorems about the existence, uniqueness and stability of the solution. Qualitative study of the model. Clarification of the behavior of the model in extreme and limiting situations. Numerical study of the model. Direct and inverse problems of mathematical modeling. Universality of mathematical models. Problems such as the application of mathematical modeling in systems are covered by the principle of analogy.

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Currently, mathematical modeling is one of the most effective and widely used methods of scientific research. Almost all modern departments of physics are devoted to the construction and study of mathematical models of various physical objects and phenomena. Usually, the latest advances in biology and chemistry are related to the development and study of mathematical models of biological systems and chemical processes. Active work is being done to create mathematical models in the sciences of ecology, economics and sociology. The use of mathematical models in medicine and industry cannot be overemphasized. It became possible to approach many environmental and medical problems on a scientific, that is, logical basis: implantation and replacement of various organs, prediction of the development of epidemics, justification of plans to eliminate the consequences of major accidents and disasters. Ko'pincha matematik modellashtirish usullari yagona bo'lishi mumkin. Thus, nuclear physicists conduct serious research using mathematical models before conducting experiments. At the same time, the methodology for full-scale experiments is developed and improved on the basis of theoretical modeling, it is determined what

effects to expect, where and when, when and what to register. Such an approach can significantly reduce the cost of conducting experiments and increase efficiency.

Full-scale modeling has the following advantages compared to mathematical modeling:

- 1) Efficiency (in particular, saving real system resources);
- 2) The ability to simulate the hypothetical, that is, impossible to implement, the nature of objects (primarily at various stages of design);
- 3) The ability to implement dangerous or difficult-to-repeat modes in real life (critical mode of a nuclear reactor, operation of a missile defense system);
- 4) The ability to change time scales; Analysis of multifaceted simplicity;
- 5) Great predictive power due to the ability to identify common patterns;
- 6) Universality of the technical and software support of the work being performed (computers, programming systems and general-purpose application software packages).

There are many mathematical modeling methods that are unique. Let's give two examples. Complex mathematical modeling and computer "playing" of various options made it possible to start rational planning and implementation of the plan to eliminate the consequences of the Chernobyl disaster in the shortest possible time. Unique results were obtained from the Gaia project, which involved mathematical modeling of the consequences of nuclear war. It has been shown that intense atmospheric dusting could lead to significant global cooling ("nuclear winter") and virtually the extinction of all living things. Elements of mathematical modeling have been used since the beginning of the exact sciences: the word "algorithm" comes from the name of the medieval Arab scientist Al-Khwarazmi (al-Khwarazmi Abu Abdala Muhammad ben Musa al Majusi, 787 - app. 850). The second birth of mathematical modeling took place in the late 1940s - early 1950s and was mainly due to two reasons.

The first of them is the appearance of the first computers. The second is the social order - the implementation of the national programs of the USSR and the USA to create a nuclear missile shield, which cannot be implemented by conventional methods. Mathematical modeling has done this task perfectly: nuclear explosions and the flight of rockets and satellites were previously carried out using mathematical models in the depths of the computer, and only then were they put into practice. Currently, mathematical modeling is entering the third principally important stage of its development, being integrated into the structure of the information society. "Raw data" usually provides very little result for analysis and forecasting, decision-making and monitoring of their implementation. We need reliable ways to transform information raw materials into finished products - concrete knowledge. The history and methodology of mathematical modeling shows that it can and should become the intellectual core of information technology, the entire process of informationalization of society. The method of mathematical modeling, which represents the quantitative description of the phenomena studied in the language of mathematics, is widely used to study all types of natural phenomena and social life.

This "third way of knowledge" combines the advantages of both theory and experiment. On the one hand, working not with the object itself, but with its model, we can study its properties and behavior in all possible situations relatively quickly and without large costs (advantages of the theory). On the other hand, computational experiments with object models, relying on the power of modern computing methods and computer technologies, allow to study objects in detail and in-depth, which is not available only for theoretical studies (experimental advantages).

1. Creating a high-quality model.

The nature of laws and connections operating in the system is defined. Depending on the nature of the model, these laws can be physical, chemical, biological or economic. The task of modeling is to determine the main, characteristic features of an event or process, its defining features. In connection with the study of

physical phenomena, creating a qualitative model is the formation of physical laws of a phenomenon or process based on experience.

2. Creating a mathematical model (statement of a mathematical problem)

If the model is described by certain equations, it is called deterministic. Initial-boundary problems of mathematical physics are examples of deterministic differential models. If the model is described by probability laws, it is called stochastic.

- 1) Determination of important factors. The main principle: if several factors of the same order act in the system, then all of them should be taken into account or eliminated.
- 2) Defining additional conditions (initial, boundary, mating conditions, etc.).

3. Learning a mathematical model

- 1) Mathematical justification of the model. Examining the internal consistency of the model. Justification of the accuracy of the differential model. Proving theorems about the existence, uniqueness and stability of the solution.
- 2) Qualitative study of the model. Clarification of the behavior of the model in extreme and limiting situations.
- 3) Numerical study of the model. a) Algorithm development. b) numerical methods of studying the development model. The developed methods should be sufficiently general, algorithmic and capable of parallelization. c) Creating and implementing the program. Computer experience.

Laboratory experience

Sample

Physical device

Calibration calculations

Data analysis

Computer experience

Mathematical model

Program

Testing the program

Data analysis

Compared to laboratory (natural) experiments, computer experiments are cheaper, safer, and can be performed in situations where a full-scale experiment would be fundamentally impossible.

4. Obtaining and interpreting results.

Comparison of the obtained data with the results of qualitative analysis, natural experiment and data obtained using other digital algorithms. Improvement and modification of the model and its research methods.

5. Using the results obtained.

Prediction of new phenomena and patterns. Paul Dirac's prediction of the discovery of antiparticles based on the study of the quantum field theory model.

DIRECT AND INVERSE PROBLEMS OF MATHEMATICAL MODELING

1. Direct task: all the parameters of the studied problem are known and the behavior of the model under different conditions is studied.

2. Inverse problems:

a) Recognition task: determining model parameters by comparing observed data and simulation results. Based on the results of observations, they try to determine what processes control the movement of the object and find the defining parameters of the model. In the inverse recognition problem, it is necessary to determine the value of the model parameters based on the known behavior of the entire system. Examples of recognition tasks: -Electrical prospecting task: detection of underground structures using surface

measurements. -Magnetic defect detection: detection of a defect in a part placed between the poles of a magnet by disrupting the magnetic field on the surface of the object.

b) Synthesis problem (mathematical design problem): building mathematical models of systems and devices that must have specified technical characteristics. Unlike recognition problems, synthesis problems do not require a single solution ("fan of solutions"). The absence of a unique solution makes it possible to choose the technologically optimal result. Examples of synthesis problems: -synthesis of the antenna radiation scheme: determination of the distribution of currents creating the given antenna radiation scheme - Gradient fiber synthesis: determination of the profile of the dielectric conductivity function of the fiber with the specified properties.

3. Management system design tasks: a special field of mathematical modeling related to automated information systems and automated control systems.

TYPICAL EXAMPLES OF REVERSE RECOGNITION PROBLEMS

1. Electrical search function. Electrical methods are widely used to study the inhomogeneity of the earth's crust for the purpose of searching for minerals. The main scheme of direct current electric search is as follows. With the help of ground electrodes, the current flows from the supply battery to the ground. The voltage of the direct current field created in this way is measured at the earth's surface. An underground structure is determined using surface measurements. Methods of identifying underground structures (interpretation of observations) are based on the mathematical solution of relevant problems.

2. Electrical search function. To determine the defect (the presence of gaps), a metal part is placed between the poles of the magnet and the magnetic field on its surface is measured. Based on the magnetic field distortions, it is necessary to determine the presence of the defect, as well as its size, depth, etc.

UNIVERSALITY OF MATHEMATICAL MODELS. PRINCIPLE OF ANALOGY

The universality of mathematical models is a reflection of the principle of material unity of the world. A mathematical model should describe not only specific individual phenomena or objects, but also a very wide range of heterogeneous phenomena and objects. One of the effective methods of modeling complex objects is to use analogies with already studied phenomena. Example: vibration processes in objects of different nature.

1. An oscillating electrical circuit consisting of a capacitor and an inductor. We consider the resistance of the wires to be zero, $q(t)$ is the charge on the capacitor plates; $u(t)$ - voltage on capacitor plates, C- capacitor capacity; L- coil inductance, E - self-induction, i- current.

$$q(t) = Cu(t), E = -L \frac{di}{dt}, i = -\frac{dq}{dt}, u(t) = -E(t) \rightarrow L \frac{d^2q}{dt^2} = -\frac{1}{C}q \rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

2. Small fluctuations in the interaction of two biological populations.

N(t)- population size of herbivores 1.

M(t)- carnivorous population 2.

$$\begin{cases} \frac{dN}{dt} = (a_1 + b_1M)N, a_1 > 0, b_1 > 0, \\ \frac{dM}{dt} = (-a_2 + b_2N)M, a_2 > 0, b_2 > 0, \end{cases}$$

The system is in equilibrium if: $\frac{dN}{dt} = \frac{dM}{dt} = 0$

The linear system has the following form:

$$\begin{cases} \frac{dn}{dt} = -b_1 N_0 m \\ \frac{dm}{dt} = b_2 M_0 n \end{cases} \rightarrow \frac{d^2 n}{dt^2} + a_1 a_2 n = 0, n = N - N_0, m = M - M_0,$$

$$M_0 = \frac{a_1}{b_1}, N_0 = \frac{a_2}{b_2}$$

Where

$$N = N_0 + n, M = M_0 + m \Rightarrow \frac{d(N_0 + n)}{dt} = (a_1 - b_1(M_0 + m))(N_0 + n) \Rightarrow$$

$$\frac{dN_0}{dt} + \frac{dn}{dt} = a_1 N_0 + a_1 n - b_1 M_0 N_0 - b_1 N_0 m - b_1 M_0 n - b_1 m n$$

$$N_0 = \frac{a_2}{b_2}; M_0 = \frac{a_1}{b_1}; \frac{dN_0}{dt} = \frac{d}{dt} \left(\frac{a_2}{b_2} \right); b_1 M_0 N_0 = \frac{a_1 a_2}{b_2}; b_1 N_0 m = \frac{a_1 b_1}{b_2} m; b_1 M_0 n = \frac{a_1 b_1}{b_1} n$$

$$m \ll 1 \Rightarrow \frac{dn}{dt} = a_1 N_0 + a_1 n - \frac{a_1 a_2}{b_2} - \frac{a_2 b_1}{b_2} m - a_1 n = \frac{a_1 a_2}{b_2} + a_1 n - \frac{a_1 a_2}{b_2} - b_1 N_0 m - a_1 n = - \Rightarrow$$

$$\frac{dn}{dt} = -b_1 N_0 m$$

Likewise, we can take:

$$\frac{d(M_0 + m)}{dt} = (-a_2 + b_2(N_0 + n))(M_0 + m) = -a_2 M_0 - a_2 m + b_2 M_0 N_0 + b_2 N_0 m + b_2 M_0 n + b_2 m n;$$

$$m \ll 1 \Rightarrow N_0 = \frac{a_2}{b_2}; M_0 = \frac{a_1}{b_1}; \frac{dM_0}{dt} = 0 \Rightarrow \frac{dm}{dt} = -\frac{a_1 a_2}{b_1} - a_2 m + b_2 M_0 n = b_2 M_0 n \Rightarrow \frac{dm}{dt} = b_2 M_0 n$$

CONCLUSION

The models created in these paragraphs are based in some cases on well-known laws (problem 1 about the oscillation period), in others - on observed facts (problem 2 on two populations), and thirdly - on the nature of the object about reliable ideas. Although the nature of the considered phenomena and the approaches to obtaining models describing them are completely different, the constructed models turned out to be similar to each other. This shows the most important feature of mathematical models - their universality.

The universal feature of mathematical models is widely used in the study of objects of the most diverse nature.

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