

40
son



USTOZLAR UCHUN

ilmiy, badiiy va ommabop jurnal



DISCOURSE AND ITS
PRIVILEGE IN TEXT
FORMATION



BOSHLANG'ICH SINIF
O'QUVCHILARINING NUTQ
MADANIYATINI
SHAKLLANTIRISH
METODIKASI



KIMYO FANINI
O'QITILISHIDA DIDAKTIK
O'YINLARDAN
FOYDALANISHNING
AHAMIYATI

pedagoglar.uz

2-TO'PLAM

yanvar, 2023



IN HIGHER MATHEMATICS, THE DISCOVERIES OF GREAT MATHEMATICIANS ABOUT THE INTERDEPENDENCE OF NUMERICAL SERIES AND THE SUM OF SERIES AND THE LAWS OF ORIGIN OF THEIR FORMULAS

Bukhara State University Associate Professor, Faculty of Information

Technologies, Ph.D. Zaripova Gulbakhor Kamilovna,

E-mail: dersuzala1972@gmail.com,

Narzullayeva Feruza Sodighovna,

teacher of the Faculty of Information Technologies of Bukhara State University,

E-mail: narzullayevafe@gmail.com,

The summary: *Series theory is one of the main branches of "Higher Mathematics" and has a wide practical application. In this article, we will see the basic concepts of series theory and some practical applications of numerical and power series. Infinite series are one of the important parts of mathematical analysis. They are widely used in solving various practical problems related to approximate calculations of function values and integral values. In this article, we will look at the basic concepts related to infinite series.*

Keywords: *numerical series, convergence signs of numerical series, series with variable sign, absolute and conditional approximation, series with alternating sign, functional series, series by degrees of series of degrees, integration of differential equations using series of degrees.*

$a_1, a_2, \dots, a_n, \dots$ ie $\{a_n\}$ let there be a sequence of infinite numbers.

This $a_1 + a_2 + a_n + \dots$ (1)

an expression is called a numeric string. In this $a_1, a_2, \dots, a_n, \dots$ number series terms, a_n and is called the general term of the number series.

The sum of the first n terms of the series

$$S_n = a_1 + a_2 + \dots + a_n \quad (2)$$

(1) is called the partial sum of the series.

The partial sum of the given series is a numerical sequence because

$$S_1 = a_1, S_2 = a_1 + a_2, \dots, S_n = a_1 + a_2 + \dots + a_n$$

$\{a_n\}$ since it is a numerical sequence, we can talk about its limit.

If $\lim_{n \rightarrow \infty} S_n = S$ if there is a finite limit, it is called the sum of the series (1) and the series is called convergent.

If $\lim_{n \rightarrow \infty} S_n$ if does not exist or is equal to infinity, the series is said to be divergent.

If the series is converging, the terms in expression (1) mean that the value of the sum of infinitely many numbers is finite, if the series is receding, its sum is infinitely large or its sum does not exist (will not exist).

An example. This

$$a + aq + aq^2 + \dots + aq^{n-1} + \dots \quad (3)$$

let's look at the line.

(3) the series is a geometric progression with first term and denominator q .

If $q \neq 1$ then (3) is the row sum of the row

$$S_n = \frac{a(1 - q^n)}{1 - q} = \frac{a}{1 - q} - a \frac{q^n}{1 - q} \text{ will be.}$$

If $q < 1$ only if $\lim_{n \rightarrow \infty} q^n = 0$ and $q > 1$ $\lim_{n \rightarrow \infty} q^n = \infty$ will be.

If $q = 1$ only if (3) from

$$a + a + \dots + a + \dots \text{ being, } S_n = n \cdot a \text{ and } \lim_{n \rightarrow \infty} S_n = \infty \text{ from being, } q \leq 1 \text{ if the}$$

series is convergent and $q \geq 1$ then it follows that it is moving away.

If $q = -1$ only if

$$a - a + a - a + \dots + \dots \text{ being, } \lim_{n \rightarrow \infty} S_n - \text{ does not exist because } n \text{ is even}$$

$S_n = 0$ and $n -$ if it is odd $S_n = a$ will be, that is $\lim_{n \rightarrow \infty} a_n$ The existence of depends on the way n tends to infinity. Even in this case $\{S_n\}$ will not have a limit.

Thus, when (3) is a numerical series formed from the limits of an infinite decreasing geometric progression, and converges $|q| \geq 1$ becomes distant when.

$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \left(\frac{a}{1-q} - \frac{aq^n}{1-q} \right) = \frac{a}{1-q}$ is convergent when (3) is a series and is its sum $\frac{a}{1-q}$ will be equal to.

Explanation. (1) from dropping or adding the first n terms of a numerical sequence, its approach or distance does not change, but the sum does.

Signs of convergence of a number line: $\sum_{n=1}^{\infty} a_n$ all terms of the series $a_k \geq 0$ such a series is called a positive series. If all the terms of the series are negative, the negative sign is removed from the parentheses and the positive term is brought to the series.

For this reason, we present the signs of convergence of the numerical series for the positive case series and note that it is also appropriate for the negative series.

In solving many practical problems with the help of series theory, it is enough to know whether it converges or diverges without finding its sum.

The fact that a series is convergent means that its sum is finite.

Now we present several symptoms of the convergence of a number series.

1⁰. a necessary sign of series convergence.

Theorem. (1) if the numerical series approaches, its n term tends to zero when n increases infinitely, i.e. $\lim_{n \rightarrow \infty} a_n = 0$.

$$M: \frac{1}{2} + \frac{2}{4} + \frac{3}{4} + \dots + \frac{n}{n-1} + 1 \neq 0$$

Now $\lim_{n \rightarrow \infty} a_n = 0$ is, and we give an example that the number series (1) is decreasing:

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots$$

Let's look at the number line. This series is called a harmonic series.

Ravshanki, for the harmonic series $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{n} = 0$, that is, the necessary condition of series convergence is fulfilled.

We prove that the harmonic series is degenerate.

Harmonic series $n = 2^k (k = 1, 2, \dots)$ Let's look at the partial sum of numbers:

$$S_{2^1} = S_2 = 1 + \frac{1}{2} > 2 \cdot \frac{1}{2} = 1, \quad S_{2^2} = S_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = S_{2^1} + \left(\frac{1}{3} + \frac{1}{4}\right) > 1 + 2 \cdot \frac{1}{4} = \frac{3}{2}$$

$$S_{2^3} = S_8 = S_{2^2} + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) > 3 \cdot \frac{1}{2} + 4 \cdot \frac{1}{8} = 4 \cdot \frac{1}{2} = 2$$

If we continue this process:

$$S_{2^k} = S_{2^{k-1}} + \left(\frac{1}{2^{k-1}} + \frac{1}{2^{k-1}+1} + \dots + \frac{1}{2^k}\right) > \frac{k}{2} + \frac{2^{k-1}}{2^k} = \left(k + 1\right)^{\frac{1}{2}}.$$

It seems that $\lim_{n \rightarrow \infty} S_{2^k} = \infty$, ie $\{S_n\}$ the sequence does not have a finite limit, so it can be seen that the harmonic series is decreasing.

2⁰. Affirmative expression is a comparison of rows.

Let the affirmative be given two lines.

$$a_1 + a_2 + \dots + a_n + \dots \quad (1)$$

$$b_1 + b_2 + \dots + b_n + \dots \quad (4)$$

Theorem. If the terms of the first line (1) are not greater than the terms of the fourth line (4), i.e:

$$a_k \leq b_k (k = 1, 2, \dots) \quad (5)$$

in this case:

- 1). (4) as the series converges, the series (1) also converges;
- 2). (1) moves away, row (4) also moves away.

An example. $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} + \dots$ check if the line is getting closer or farther apart.

Solving. (4) as a line:

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots + \dots \quad (4')$$

We look at the harmonic series and check that condition (5) is fulfilled:

$$a_n = \frac{1}{\sqrt{n}}, \quad b_n = \frac{1}{n}, \quad \text{and } a_n \geq b_n, \quad \text{because } n \geq \sqrt{n} \text{ and } \frac{1}{\sqrt{n}} \geq \frac{1}{n}$$

(4') is a harmonic series, which is decreasing, and therefore the given number series is also decreasing.

3⁰. D'alambert symptom. (D'alambert Jean 1717 – 1783 French scientist)

Theorem. If the positive expression (1) of the series, $(n+1)$ come on n - the ratio of the limit $n \rightarrow \infty$ at L If has a (finite) limit, ie:

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = L \quad (6)$$

if so, at that time:

1. $L < 1$ when (1) the number series converges.
2. $L > 1$ when (1) the numbered line goes away.
3. $L = 1$ this theorem cannot answer the convergence or divergence of the series. In this case (1) other line convergence symbols are used to determine whether the line is converging or diverging.

4⁰. **Cauchy's sign** (*Augustine Louis Cauchy. 1789 – 1875, a famous French mathematician.*)

Theorem. If statement (1) is for a series $\sqrt[n]{a_n}$ amount $n \rightarrow \infty$ at L has a finite limit, ie:

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = L \quad (7)$$

1. $L < 1$ when (1) the series converges.
2. $L > 1$ when (1) the row is moved away.
3. $L = 1$ this theorem also cannot answer the convergence or divergence of the series.

An example. $\frac{1}{3} + \left(\frac{2'}{5}\right)^2 + \left(\frac{3'}{7}\right)^2 + \dots + \left(\frac{n}{2n+1}\right)_2 + \dots$ we check the convergence of the series.

Solving. Both Dalamber's sign of series convergence and Cauchy's sign can be applied to this series approximation.

Indeed:

$$1) a_n = \left(\frac{n}{2n+1}\right)^n, \quad a_{n+1} = \left(\frac{n+1}{2n+1}\right)^n, \quad a_{n+1} = \left(\frac{n+1}{2n+3}\right)^{n+1}, \text{ from being;}$$

$$L = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{n+1}{2n+3}\right)^n \left(\frac{n+1}{2n+3}\right)}{\left(\frac{n}{2n+1}\right)^n} = \lim_{n \rightarrow \infty} \left(\frac{n+1}{2n+3} \cdot \frac{2n+1}{n}\right)^n \lim_{n \rightarrow 0} \frac{n+1}{2n+3} = \frac{1}{2} < 1.$$

that is, the given series is convergent.

$$2) L = \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{n}{2n+1}\right)^n} = \lim_{n \rightarrow \infty} \frac{n}{2n+1} = \frac{1}{2} < 1$$

When checking a number of rows in practice $L = 1$ the case is common. For

example, for the Hormanic series $L = \lim_{n \rightarrow \infty} \frac{\frac{1}{n!+1}}{\frac{1}{n}} = 1$

In this case, the following series convergence sign, called the integral sign of series convergence, often works.

5⁰. The integral sign of series convergence.

Theorem. If the terms of the series (1) are not positive and increasing, i.e

$$a_1 \geq a_2 \geq \dots \geq a_n \geq \dots$$

and that which does not grow $f(x)$ such that there exists a continuous function

$$f(1) = a_1, \quad f(2) = a_2, \dots, f(n) = a_n, \dots$$

In this case, if:
$$\int_1^{\infty} f(x) dx \quad (8)$$

if the eigenintegral approaches, the series (1) also approaches, (8) if the

eigenintegral moves away, the number series (1) also moves away.

For example, consider the harmonic series:

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots$$

$$1 > \frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \dots > \frac{1}{n} > \dots \text{ and } f(x) = \frac{1}{x} (f'(x) = -\frac{1}{x^2})$$

from being $\int_1^{\infty} \frac{dx}{x}$ we consider the proper integral.

According to the definition of integral integral:

$$\int_1^{\infty} \frac{dx}{x} = \lim_{A \rightarrow \infty} \int_1^A \frac{dx}{x} = \lim_{A \rightarrow \infty} \ln|x| \Big|_1^A = \lim_{A \rightarrow \infty} \ln A = \infty,$$

ie, $\int_1^{\infty} \frac{dx}{x}$ the characteristic integral is diverging and therefore the harmonic

series is also convergent.

Strings with variable sign. Absolute and conditional approximation. If a series has both positive and negative terms between its terms, the series is called a variable-signed series.

Theorem. A variable-signed string

$$u_1 + u_2 + \dots + u_n + \dots \quad (9)$$

formed from the absolute values of the terms

$$|u_1| + |u_2| + \dots + |u_n| + \dots \quad (10)$$

as the series converges, so does the series with the given variable sign.

If the series (10) is approximated by the terms of the variable series whose sign (9) is convergent, the given series is called an absolute convergent.

If the variable sign (9) converges, but (10) recedes, the given variable sign series (9) is called a conditionally or non-absolutely convergent series.

The pointer is alternating rows. In solving many practical problems, for example, in calculating the value of an exact integral using series, in solving differential equations, alternating series, whose sign is a special case of a variable series, is widely used.

$$u_1 + u_2 + \dots + u_n + \dots \quad (9)$$

A line pointer is called an alternating line if U_n and U_{n+1} terms are any n has a different sign for, that is, if the terms of the series alternately change their sign, that is,

$$u_1 - u_2 + u_3 - u_4 + \dots + (-1)^{n-1} u_n + \dots \quad (11)$$

Approaching or moving away of a series whose sign is alternating is determined using the following theorem.

Theorem (Leibnitz's sign). *(Leibnitz Gottfried Friedrich 1646 – 1716 was a great German mathematician).*

If (11) is the term of the alternating series:

1) $u_1 \geq u_2 \geq \dots u_n \geq \dots$; 2) $\lim_{n \rightarrow \infty} u_n = 0$ if (11) the series converges.

An example. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + (-1)^n \frac{1}{n} + \dots$ check for line convergence.

Solving. We check the fulfillment of the conditions of Leibniz's sign:

$$1) \quad 1 > \frac{1}{2} > \frac{1}{3} > \dots > \frac{1}{n} > \dots \quad 2) \quad \lim_{n \rightarrow \infty} U_n = \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

Since both conditions of Leibniz signs are satisfied, the given series is convergent. The given sign is formed from the absolute sum of the terms of the alternating series

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots$$

and the series is moving away from the harmonic series. So, the alternating series with the given sign is a conditional (non-absolute) converging series.

Explanation. The sign is an alternating line $S - S_n = r_n$ residual verb

$|S - S_n| = |r_n| \leq U_{n+1}$ it is not difficult to prove that, privately $S_1 \leq U_1$ originates.

$|r_n| \leq U_{n+1}$ inequality is widely used to estimate error in approximate calculations.

Functional lines. If $u_1 + u_2 + u_3 + \dots + u_n + \dots$ the terms of the series

x are functions of, this series is called a functional series.

This: $f_1(x) + f_2(x) + \dots + f_n(x) + \dots$ (1)

Let's look at the functional line.

x is clear to $x = x_0$ by giving a value, we create different string strings:

$$f_1(x_0) + f_2(x_0) + \dots + f_n(x_0) + \dots \quad (2)$$

(2) is an array of numbers, which can be zoomed in or zoomed out.

x The set of values to which the functional series converges is called the domain of convergence of this series.

Clearly, the sum in the convergence domain of the series is a function of and $S(x)$ is determined in appearance.

We mainly consider a special case of the functional series, ie $f'_n(x) = a_n x^n$ we study a functional series called a power series formed when.

Graded rows. This $a_0 + a_1 x + a_1 x^2 + \dots + a_n x^n + \dots$ (3)

A functional array of the form is called a graded array, where $a_0, a_1, \dots, a_n, \dots$ are fixed numbers and are called the coefficients of the power series. The domain of approximation of a power series can be an interval, a segment, a half-segment, or even a point.

The domain of approximation of a power series is determined using the following theorem.

Theorem (Abel's theorem) Abel Niel's Henrik (1802 - 1829) Norwegian mathematician.

1) If the power series converges to a value other than zero x of

$$|x| < |x_0|$$

converges absolutely in all values satisfying the inequality;

2) If (3) any of the rank series x'_0 distance in value, x of

$$|x| < |x'_0|$$

goes away at every value that satisfies the inequality.

Abel's theorem allows finding the interval of approximation of a power

series. Of a truly rank series x_0 approaches at the point, based on Abel's theorem $(-|x_0|, |x_0|)$ converges absolutely in the interval and x'_0 if the point is receding $(-|x'_0|, |x'_0|)$ is regressive outside the interval.

Therefore, there is such a number that, $|x| < R$ The absolute value of the power series is approximated when and $|x| > R$ will move away R is called the radius of convergence of the power series.

$(-R, R)$ at both ends of the interval $(x = -R, x = R)$ the convergence of a given series is checked as a separate numerical series. $x = -R, x = R$ the area of convergence of the series, depending on whether the series converges or diverges at points $(-R, R), (-R; R], [-R, R), [-R, R]$ can be.

Convergence radius of the graded
row: $R = \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right|$ (4); $R = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$ (5) is found by one of the formulas.

(3) An important property of the power series is that if (1) $(-R, R)$ converges in the interval, its sum is continuous in this interval $(\alpha, \beta) \in (-R, R)$ can be differentiated and integrated in the interval, i.e: if

$$S(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \dots$$

$$S'(x) = a_1 + 2a_2x + 3a_3x^2 + \dots + na_nx^{n-1} + \dots;$$

$$\int_{\alpha}^{\beta} S(x)dx = a_1 \int_{\alpha}^{\beta} dx + 2a_2 \int_{\alpha}^{\beta} xdx + 3a_3 \int_{\alpha}^{\beta} x^2dx + \dots + na_n \int_{\alpha}^{\beta} x^{n-1}dx + \dots \text{ will be.}$$

$X - a$ Graded rows by degrees of.

$$a_0 + a_1(x - a) + a_2(x - a)^2 + \dots + a_n(x - a)^n + \dots \quad (6)$$

A series of the form is also called a graded series and $a = 0$ when is the power series we learned (3). (6) To determine the domain of convergence of a power series $x - a = t$ if we replace, $a_0 + a_1t + a_2t^2 + \dots + a_nt^n + \dots$ (7)

a graded row is created. If (7) is the interval of convergence of the power

series $-R < t < R$ (7) is the interval of convergence of the power series $a - R < x < a + R$ will be.

Taylor and Maclaren lines. (Brooke Taylor. 1685 – 1731) (McLauren Colin 1698 – 1748 English mathematician.)

$f(x)$ in the function $x=a$ around the point $(n+1)$ - be a function with all derivatives of order $f(x)$ Taylor formula for functions:

$$f(x) = f(a) + \frac{x-a}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots + \frac{(x-a)^n}{n!} f^{(n)}(a) + R_n(x) \quad (7)$$

It was mentioned that the formula is appropriate here

$$R_n(x) = \frac{(x-a)^{n+1}}{(n+1)!} f^{(n+1)}[a + \theta(x-a)], \quad 0 < \theta < 1 \quad (8)$$

If $f(x)$ function $x=a$ has all order derivatives around a point (if the derivatives are bounded) $n \rightarrow \infty$ residual term $R_n(x)$ tends to zero, i.e

$$\lim_{n \rightarrow \infty} R_n(x) = 0 \quad (9)$$

In this case (7). $n \rightarrow \infty$ if we go to the limit:

$$f(x) = f(a) + \frac{x-a}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots + \frac{(x-a)^n}{n!} f^{(n)}(a) + \dots \quad (10)$$

(10) is called a Taylor series.

Privately $a=0$ if,

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^n}{n!} f^{(n)}(0) + \dots$$

This is the case for all elementary functions known to us a point and R is available, $(a-R, a+R)$ in the interval these functions are Taylor or ($a=0$ if) We emphasize that it will spread to the McLaren line.

Now we present the expansion of some elementary functions into Taylor and Maclaren series without proof:

$$\begin{aligned}
 1. e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} \dots \quad -\infty < x < +\infty \\
 2. \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots \quad -\infty < x < +\infty \\
 3. \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots \quad -\infty < x < \infty \\
 4. \operatorname{sh} x &= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \\
 5. \operatorname{ch} x &= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \\
 6. \operatorname{arctg} x &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{2n-1} + \dots \\
 7. \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n} + \dots \quad -1 < x \leq 1 \\
 8. \operatorname{tg} x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \dots \\
 9. \frac{x}{1+x^2} &= xx + x^3 + x^5 + x^7 + \dots \\
 10. \operatorname{arc} \sin x &= x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \\
 11. \ln(x + \sqrt{x^2 + 1}) &= x - \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{x^5}{5} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \\
 12. \frac{1}{2} \ln \frac{1+x}{1-x} &= x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots
 \end{aligned}$$

Binomial series. We will study the so-called binomial series, which has an important place in practice in the theory of series.

$f(x) = (1+x)^m$, (m – an arbitrary real constant) if we expand the function in the Maclaren series:

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \dots \quad (1)$$

(1) A graded series is called a binomial series and $|x| < 1$ approaches when.

Privately:

1) $m = 1$ when:

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 \dots \quad (2)$$

2) $m = \frac{1}{2}$ when:

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8}x^4 + \dots \quad (3)$$

3) $m = -\frac{1}{2}$ when:

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}x^4 - \dots \quad (4)$$

When approximating the value of a binomial series function at a given point, when expanding some functions into a power series, e.g., $\arcsin x$, $\arccos x$, $\arctg x$, $\operatorname{arcctg} x$, $\ln(1+x)$ widely used.

In our article entitled "In higher mathematics, the discoveries of great mathematicians about the interdependence of numerical series and the sum of series and the laws of origin of their formulas", Jean D'alambert French Mathematician, Cauchy Augustin Louis Famous French Mathematician, Leibniz Gottfried Friedrich Great German Mathematician and Abel Niels Henrik Norway At the same time, we have analyzed the work of English mathematician MacLaurin Colin on the interdependence of numerical series and sum of series.

List of used literature:

1. V.E. Barbaumov, V.I. Ermakov, N.N. Kriventsova, A.S. Lebedev, V.I. Matveev, B.M. Rudyk, E.A. Silaeva, O.K. Smagina. Tutorial. - 2nd ed., revised. and additional - M.: Higher School, 1997. - 384 p. — ISBN 5-060030-74-1.
2. V. T. Lisichkin and I. L. Soloveichuk. Maths. M. 1991.
3. B. M. Rudyk, Bobrik G. I., Grintsevichyus R. K., Sagitov R. V., Shershnev V. G., Matveev V. I., Gladkikh I. M. Course of higher mathematics for economists. M. 2016. 647 str. ISBN: 978-5-16-011091-2
4. V.I. Belousova, G.M. Ermakova, M.M. Mikhaleva, Yu.V. Shapar, I.A. Shestakova. Higher mathematics: textbook. / - Yekaterinburg: Ural Publishing House. un-ta, 2016.- Ch.I.- 296 p. ISBN 978-5-7996-1779-0.

5. Smirnov V. I. Course of higher mathematics. Volume I / Prev. L. D. Faddeeva, before. and approx. E. A. Grinina: 24th ed. - St. Petersburg: BHV-Petersburg, 2008. - 624 p.: ill. - (Educational literature for universities) ISBN 978-5-94157-909-9.
6. M. Ya. Vygodsky. Handbook of higher mathematics. M. 1977.
7. Prihodko E.M., Pasko T.P., Chebova D.L. Maths. Arithmetic, algebra, trigonometry. For students - foreigners. - Moscow: Higher School. 1971 304 p.
8. Demidovich B.P., Kudryavtsev V.A. A short course in higher mathematics. M.: LLC "Publishing house Astrel"; LLC "AST Publishing House", 2001. - 656 p.
9. V. A. Kudryavtsev, Demidovich B.P. A short course in higher mathematics. M.1989.
10. Bronshtein I. N., Semendyaev K. A. A Handbook of Mathematics for Engineers and student of higher education institutions: a teaching aid. - Publishing house "Lan", 2010 - 608 pp.: ill. - (Textbooks for universities. Special literature). ISBN 978-5-8114-0906-8