ALGEBRAIC EQUATIONS. THEORY, METHODS, ALGORITHMS SOLUTIONS

T. R. Suyarov

Bukhara State University, Bukhara, Uzbekistan,

Email: t.r.suyarov@buxdu.uz.

Abstracts. *In the proposed work, practically all questions related to the theory of algebraic and irrational equations and methods of their analytical solution are presented from a unified position. The main attention is paid to algorithmic methods of solving the presented problems, so that it is possible to perform the necessary calculations in practice. Each chapter ends with typical tasks that take place in the entrance tests, GIA, USE and olympiads of various levels. Each chapter ends with typical problems occurring at entrance examinations, GIA, USE and Olympiads of various levels. By These tasks can be conditionally divided into three levels.*

Keywords: *algebraic equation, polynomial, Bezu's theorem, solving equations*

1. RATIONAL EQUATIONS

It is common to write an integer algebraic equation in the form:

$$P(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n \tag{1.1}$$

where $a_0, a_1, \ldots, a_n$ given numbers, x is the unknown (variable), n is the degree of the of the algebraic equation (the greatest degree of the equation). An expression $P(x)$ is called a polynomial or polynomial of degree n if the coefficient at the highest degree of the unknown is not zero $a_0 \neq 0$. If in equation (1.2) $a_0 = 1$, the whole algebraic equation is called reduced [1].

Equations containing polynomials and algebraic fractions of the form $\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ - polynomials, are called fractional algebraic equations or fractional-rational equations.

When solving a linear equation $a \cdot x = b$ there are three possible cases:

1. $a \neq 0$, then $x = \frac{b}{a}$ - the only root of the equation;

2. $a = 0, b = 0$, then the equation takes the form $0 \cdot x = 0$, which is true for any x, i.e. the answer is $x \in R$

3. $a = 0, b \neq 0$, then the equation takes the form $0 \cdot x = b$, it has no roots.

An equation of the form $ax^2 + bx + c = 0, (a \neq 0)$ is called a quadratic equation with one variable. Its roots are calculated using the formulas

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Expression $D = b^2 - 4ac$ is called the discriminant of the quadratic equation. Thus, a quadratic equation has real roots only in the case of $D \geq 0$.

[2] If $b = 2k, k \in Z$, i.e. b - is an even number, then the quadratic equation can be written as $ax^2 + 2kx + c = 0$. Then the formula for the roots of the quadratic equation can be simplified and used in the form $x_{1,2} = \frac{-k \pm \sqrt{k^2 - ac}}{a}$.

Finally, if by the same token $a = 1$, then the formulas for determining the roots of the equation $x^2 + 2kx + c = 0$ are further simplified and take the form

$$x_{1,2} = -k \pm \sqrt{k^2 - c}$$

2.METHOD OF TRANSFORMATION OF ALGEBRAIC EXPRESSIONS

[3] The solution of very many rational equations is based on the successful grouping and then reducing the grouped summands to a common denominator. to a common denominator. In simpler cases, grouping is not necessary. By getting rid of

the common denominator, both parts of the equation are essentially multiplied by the function containing the unknown quantity. This raises the danger of obtaining extraneous solutions, which is eliminated either by checking or by finding the region of The following table summarises the following The following table provides a summary of the relevant constraints and further verification of their fulfilment.

Example 2.1. Solve the equation

$$\frac{x+1}{2x-1}+\frac{x}{x-3}=\frac{3x}{2x-10}.$$

Solution. The UCL is easy to find, so let us write out the constraints on the unknown at which the denominators of the fractions do not go to zero

$$\begin{cases} x \in R \\ 2x-1 \neq 0 \\ x-3 \neq 0 \\ 2x-10 \neq 0 \end{cases} \Rightarrow \begin{cases} x \in R \\ x \neq \frac{1}{2} \\ x \neq 3 \\ x \neq 5 \end{cases}$$

Now we don't have to keep the common denominator when performing transformations. Transferring all summands to the left-hand side and multiplying both parts of the equation by $(2x-1)(x-3)(2x-10) \neq 0$, we get

$$(x+1)(x-3)(2x-10)+x(2x-1)(2x-10)-3x(2x-1)(x-3)=0$$

$$2(x^2-2x-3)(x-5)+2(2x^2-x)(x-5)-3(2x^2-x)(x-3)=0$$

$$2(x-5)(x^2-2x-3+2x^2-x)-3(2x^3-7x^2+3x)=0$$

$$6(x-5)(x^2-x-1)-3(2x^3-7x^2+3x)=0$$

$$3\big(2(x^3-6x^2+4x+5)-(2x^3-7x^2+3x)\big)$$

$$-5x^2+5x+10=0 \;\Rightarrow\; x^2-x-2=0 \Rightarrow x_1=-1; x_2=2$$

Both solutions belong to the ODZ.

3.METHOD OF DECOMPOSITION INTO FACTORS

This method is to use grouping of summations grouping of summands, as well as the formulae of reduced multiplication, to bring the original equation to the form where the product of the terms is written on the left and the zero is written on the right. zero on the right. Then each of the factors is equated to zero, and by solving simple equations to find the roots of the original equation. of the original equation.

Bezu's theorem finds application not only in solving equations, but also in problems related to the divisibility of polynomials (finding the residue when dividing polynomials, determining multiplicity of polynomials, etc.), with the decomposition of polynomials into multipliers, with determining multiplicity of roots. decomposition of polynomials into multipliers, determination of multiplicity of roots and many others.

Theorem 1 (Bezu's theorem) [4]. The residue from dividing the polynomial $P_n(x)$ by the bipartite $(x-a)$ is equal to the value of this polynomial at $x=a$.

Corollary 1: The residue from dividing the polynomial $P_n(x)$ by the bipartite $ax+b$ is equal to the value of this polynomial at x =-b/a, i.e. $R = P_n(-b/a)$.

Corollary 2: If the number a is a root of the polynomial $P_n(x)$, then this polynomial is divisible by $(x-a)$ without remainder.

Corollary 3: If a polynomial $P(x)$ has pairwise different roots $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ then it is divisible by the product of the two terms $(x-\alpha_1) \ldots (x-\alpha_n)$ without remainder.

Corollary 4: A polynomial of degree n has at most n distinct roots .

Corollary 5. For any polynomial $P(x)$ and a number α, the difference ($P(x)$ - $P(\alpha)$) is divisible without remainder by the two-term $(x-\alpha)$.

Corollary 6: The number a is a root of a polynomial $P(x)$ of degree at least one if and only if $P(x)$ is divisible by $(x-\alpha)$ without remainder.

Corollary 7: A polynomial with no real roots does not contain linear multipliers in the expansion into multipliers.

Corollary 8: If the coefficients of the reduced integer algebraic equation

$$x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0$$

are integers, then the integer roots should be found among the divisors of the free term a_n.

Example 1.2. Find the remainder from division of polynomial $x^3 - 3x^2 + 6x - 5$ by a two-term$x - 2$.

Solution. By Bezu's theorem $R = P_3(2) = 2^3 - 3 \cdot 2^2 + 6 \cdot 2 - 5 = 3$.

Example 1.3. Find the remainder of division of a polynomial of degree four $32x^4 - 64x^3 + 8x^2 + 36x + 4$ by a two-term $2x - 1$.

Solution. According to Corollary 1 of Bezu's theorem

$$R = P_4\left(\frac{1}{2}\right) = 32\left(\frac{1}{2}\right)^4 - 64\left(\frac{1}{2}\right)^3 + 8\left(\frac{1}{2}\right)^2 + 36\left(\frac{1}{2}\right) + 4 = 18.$$

The main attention at consideration of the following problems we will consider the solution of integer algebraic equations by means of Bezu's theorem. Corollaries 2 and 3 of this theorem will be used. The sequence of actions in this case is formalised and contains the following steps.

1.Find the integer root of x_1 by looking at the divisors of the free term and substituting them into the equation.

2. Divide the polynomial from the left side of the equation by the bipartition $x - x_1$ According to Corollary 6, after this division, the remainder will be zero. This fact can serve as a control of the reliability of the performed transformations. If the remainder is not equal to zero, the error should be sought either at the stage of root selection or in the procedure of division of a polynomial by a polynomial. the root selection stage, or in the procedure of division of a polynomial by a polynomial

3.The quotient of the division in the previous paragraph is a polynomial of degree $(\mathrm{n} - 1)$. We return to point 1 and similarly look for its root.

Reference

[1] Bashmakov M.I., Bekkre B.M., Golhovoy V.M. Tasks in Mathematics. Algebra and analysis / Edited by D.K. Fadeev. - Moscow: Nauka, Main Editorial Office of Physical and Mathematical Literature, 1982. - 192c.

[2] Bronstein I. N. Reference book on mathematics for engineers and students of higher technical schools / I. N. Bronstein, K. A. Semendyaev. Students of vocational schools / I. N. Bronstein, K. A. Semendyaev. - Moscow: Nauka, 1986. - 544 c.

[3] Foreign Mathematical Olympiads / S.V. Konyagin [et al] - M.: Nauka, Main Editorial Office of Physical and Mathematical Literature, 1987. -416c.

[4] Korn G. Handbook of mathematics for scientists and engineers / G. Korn, T. Korn. Engineers / G. Korn, T. Korn. - Moscow : Nauka, 1977. - 832 c.