MINISTRY OF HIGHER EDUCATION, SCIENCE AND INNOVATIONS OF THE REPUBLIC OF UZBEKISTAN

V.I.ROMANOVSKIY INSTITUTE OF MATHEMATICS

UNIVERSITY OF EXACT AND SOCIAL SCIENCES

OXUS UNIVERSITY

INTERNATIONAL SCIENTIFIC CONFERENCE

MATHEMATICAL ANALYSIS AND DYNAMICAL SYSTEMS

CONFERENCE PROGRAMME

MAY 20 - 21, 2025

TASHKENT, UZBEKISTAN

V.I.Romanovskiy Institute of Mathematics

University of Exact and Social Sciences

OXUS University

ABSTRACTS

OF THE INTERNATIONAL CONFERENCE

MATHEMATICAL ANALYSIS AND DYNAMICAL SYSTEMS

May 20–21, 2025, Tashkent, Uzbekistan

TASHKENT – 2025

The problem of determining the unknown coefficient in the two-dimensional wave equation under initial and non-local boundary conditions

Suyarov T.R.
Bukhara State University, 11 M. Ikbol St., Bukhara 200100, Uzbekistan
tsuyarov007@gmail.com

Let $\Omega = D \times (0,T)$, where $D = \{(x,y) : 0 < x, y < 1\}$. We consider the initial boundary value problem for wave equation

$$u_{tt}(x,y,t) - \Delta u + q(t)u(x,y,t) = f(x,y,t), \tag{1}$$

with initial conditions

$$u(x,y,t)|_{t=0} = \varphi_1(x,y), \ (x,y) \in \bar{D}, \tag{2}$$

$$u_t(x,y,t)|_{t=0} = \varphi_2(x,y), \ (x,y) \in \bar{D}, \tag{3}$$

and non-local boundary conditions

$$u_x(0,y,t) = u_x(1,y,t), \quad u(0,y,t) = 0, \quad (y,t) \in [0,1] \times [0,T], \tag{4}$$

$$u(x,0,t) = u(x,1,t), \quad u_y(x,1,t) = 0, \quad (x,t) \in [0,1] \times [0,T], \tag{5}$$

where $\bar{D} = \{(x,y) : 0 \leqslant x, y \leqslant 1\}, \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}; \varphi_1(x,y), \varphi_2(x,y)$ and $f(x,y,t)$ are given functions.

In the **direct problem**, it is required to define a function $u(x,y,t) \in C^2(\bar{\Omega})$ satisfying the equalities (1)-(5), for given sufficiently smooth functions $q(t), f(x,y,t), \varphi_i(x,y), i = 1,2$, where $\bar{\Omega} = \{(x,y,t) : 0 \leqslant x, y \leqslant 1, 0 \leqslant t \leqslant T\}$.

The **inverse problem** is to find function $q(t) \in C[0,T]$, if with respect to the solution of the direct problem (1)-(5) the overdetermination condition is known

$$\int_0^1 \int_0^1 w(x,y)u(x,y,t)dxdy = h(t), \tag{6}$$

Lemma 1. *Let $\varphi_1(x,y) \in C^4(\bar{D}), \varphi_2(x,y) \in C^3(\bar{D})$ and $f(x,y,t) \in C^{2,0}_{xy,t}(\bar{\Omega})$. Besides, the following equalities hold*

$$\begin{aligned}
&\varphi_1(0,y) = \varphi_{1xx}(0,y) = \varphi_1(1,y) = \varphi_{1xx}(1,y), \quad 0 \leqslant y \leqslant 1;\\
&\varphi_1(x,0) = \varphi_{1yy}(x,0) = \varphi_1(x,1) = \varphi_{1yy}(x,1), \quad 0 \leqslant x \leqslant 1;\\
&\varphi_2(0,y) = \varphi_{2xx}(0,y) = \varphi_2(1,y) = \varphi_{2xx}(1,y), \quad 0 \leqslant y \leqslant 1;\\
&\varphi_2(x,0) = \varphi_{2yy}(x,0) = \varphi_2(x,1) = \varphi_{2yy}(x,1), \quad 0 \leqslant x \leqslant 1;\\
&f(0,y,t) = f(1,y,t), \quad (y,t) \in [0,1] \times [0,T];\\
&f(x,0,t) = f(x,1,t), \quad (x,t) \in [0,1] \times [0,T].
\end{aligned}$$

Then the numerical series:

$$2\Psi_{0,0}(T) + \sum_{k=1}^{\infty} \Psi_{0,2k-1}(T) + \sum_{k=1}^{\infty} \Psi_{0,2k}(T) + 2\sum_{m=1}^{\infty} \Psi_{2m-1,0}(T) + \sum_{m,k=1}^{\infty} \Psi_{2m-1,2k-1}(T)$$

$$+\sum_{m,k=1}^{\infty}\Psi_{2m-1,2k}(T)+2\sum_{m=1}^{\infty}\Psi_{2m,0}(T)+\sum_{m,k=1}^{\infty}\Psi_{2m,2k-1}(T)+\sum_{m,k=1}^{\infty}\Psi_{2m,2k}(T), \qquad (7)$$

$$4\sum_{m=1}^{\infty}\sqrt{\lambda_m}\Psi_{2m-1,0}(T)+2\sum_{m=1}^{\infty}\lambda_m\Psi_{2m-1,0}(T)+2\sum_{m,k=1}^{\infty}\sqrt{\lambda_m}\Psi_{2m-1,2k-1}(T)$$

$$+\sum_{m,k=1}^{\infty}\lambda_m\Psi_{2m-1,2k-1}(T)+2\sum_{m,k=1}^{\infty}\sqrt{\lambda_m}\Psi_{2m-1,2k}(T)+2\sum_{m=1}^{\infty}\lambda_m\Psi_{2m,0}(T)$$

$$+\sum_{m,k=1}^{\infty}\lambda_m\Psi_{2m-1,2k}(T)+\sum_{m,k=1}^{\infty}\lambda_m\Psi_{2m,2k-1}(T)+\sum_{m,k=1}^{\infty}\lambda_m\Psi_{2m,2k}(t), \qquad (8)$$

$$2\sum_{k=1}^{\infty}\sqrt{\gamma_k}\Psi_{0,2k-1}(T)+\sum_{k=1}^{\infty}\gamma_k\Psi_{0,2k-1}(T)+\sum_{k=1}^{\infty}\gamma_k\Psi_{0,2k}(T)$$

$$+2\sum_{m,k=1}^{\infty}\sqrt{\gamma_k}\Psi_{2m-1,2k-1}(T)+\sum_{m,k=1}^{\infty}\gamma_k\Psi_{2m-1,2k-1}(T)+\sum_{m,k=1}^{\infty}\gamma_k\Psi_{2m-1,2k}(t)$$

$$+2\sum_{m,k=1}^{\infty}\sqrt{\gamma_k}\Psi_{2m,2k-1}(T)+\sum_{m,k=1}^{\infty}\gamma_k\Psi_{2m,2k-1}(T)+\sum_{m,k=1}^{\infty}\gamma_k\Psi_{2m,2k}(T), \qquad (9)$$

in (7)-(9) are converge.

Thus, we have proved the following theorem.

Theorem 1. *Let $q(t) \in C[0,T]$ and if the functions $\varphi_1(x,y), \varphi_2(x,y)$ and $f(x,y,t)$ satisfy the conditions of the Lemma 1, then there exists a unique solution to the problem (1)-(5).*

The following assertion is main result in this poper:

Theorem 2. *Let the conditions of Lemma 1 and $h(t) \in C^2[0,T], |h(t)| \geqslant h_0 > 0$, be satisfied and*

$$w(1,y)=w(0,y)=0, \quad w_x(1,y)=0, \quad y\in[0,1],$$
$$w(x,0)=0, \quad w_y(x,1)=w_y(x,0)=0, \quad x\in[0,1].$$

Then the exists $T^* \in (0,T)$ so that the inverse problem (1)-(6) has unique solution $q(t) \in C\,[0,T^*]$.

Theorem 3. *Let $(\varphi_1,\varphi_2,h,f) \in D_{\nu_0}, \left(\widetilde{\varphi}_1,\widetilde{\varphi}_2,\widetilde{h},\widetilde{f}\right) \in D_{\nu_0}$ and $q,\widetilde{q} \in G_{\nu_1}$ Then, for solution to the inverse problem (1)-(6) the following stability estimate holds:*

$$\|q-\widetilde{q}\|_{C[0,T]} \leqslant r\left[\|\varphi_1-\widetilde{\varphi}_1\|_{C^4[0,1]}+\|\varphi_2-\widetilde{\varphi}_2\|_{C^3[0,1]}+\|h-\widetilde{h}\|_{C^2[0,T]}+\|f-\widetilde{f}\|_{C^2(\bar{\Omega})}\right],$$

where the constant r depends only on ν_0,ν_1,T.

References

1. Romanov, V. G. Inverse Problems of Mathematical Physics, Utrecht, VNU Science Press, 1987.

2. Hasanov A., Romanov, V. G. Introduction to Inverse Problems for Differential Equations, Springer International Publishing, 2017.

3. Kabanikhin, S. I. Inverse and Ill-Posed Problems: Theory and Applications, Berlin, De Gruyter, 2011.