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Direct problem for an incompressible viscoelastic polymer fluid at rest

Turdiev Kh. Kh. [1] **Suyarov T. R.** [2]

Tinch holatdagi siqilmaydigan elastik polimer suyuqliklari uchun to'g'ri masala

Ushbu maqolada tinch holatdagi siqilmaydigan elastik qayishqoq polimer suyuqliklar uchun, tenglamalar sistemasi chiziqli holatga keltirilgan. Chiziqli tenglamalar sistemasi, x o'zgaruvchiga nisbatan Furye almashtirishi qo'llanilib, kanonik ko'rinishga keltirilgan. Kanonik ko'rinishdagi, tinch holatdagi siqilmaydigan elastik qayishqoq polimer suyuqliklar sistemasi uchun to'g'ri masala qo'yilgan. Ushbu masala yopiq ikkinchi tur Volterra tipidagi integral tenglamalar sistemasiga keltirilgan. Yadrosi va ozod hadi uzluksiz bo'lgan Volterra tipidagi integral tenglamalar biror yopiq sohada uzluksiz yechimga ega. Shunday qilib maqolada quyilgan to'g'ri masala yechimi mavjudligi va yagonaligi haqidagi teorema isbotlangan.

Kalit so'zlar: Giperbolik sistema; siqilmaydigan elastik qayishqoq polimer suyuqliklari; integral tenglama; integro-differensial tenglama; Vinogradov-Pokrovskiyning reologik modeli.

Прямой задачи для несжимаемой вязкоупругой полимерной жидкости в состоянии покоя

В этой статье, приведение линейной системы уравнений для несжимаемой вязкоупругой полимерной жидкости в состоянии покоя. Система линейных уравнений приводится к каноническому виду, применения преобразований Фурье по переменной x. В кононическом виде, ставится прямая задачи для системы несжимаемой вязкоупругой полимерной жидкости в состоянии покоя. Задачи заменяются замкнутой системой интегральных уравнений второго рода Вольтерровского типа.Интегральные уравнения типа Вольтерра с непрерывным ядром и свободным членом имеют непрерывное решение в замкнутой области.Таким образом, доказываются теорема существования и единственности решений поставленных задач.

Ключевые слова: Гиперболическая система; несжимаемой вязкоупругой полимерной жидкость; интегральное уравнение; интегро-дифференциальное уравнение; реологическая модель Виноградова-Покровского.

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Keywords: Hyperbolic system; incompressible viscoelastic polymer fluid; integral equation; integro-differential equations; rheological model of Vinogradov-Pokrovsky.

[1]Bukhara State University, Bukhara, Uzbekistan. E-mail: hturdiev@mail.ru
[2]Bukhara State University, Bukhara, Uzbekistan. E-mail: tsuyarov996@gmail.com

Introduction

The question of the linear stability of stationary solutions of the equations of an incompressible viscoelastic polymer fluid to be extremely important. The Pokrovsky-Vinogradov rheological model [1, 2] is used as a model for the hydrodynamics of a polymer fluid. Polymer fluids are fluid medium consisting of long macromolecules entangled with each other. In flows with nonzero velocity gradients, such molecules interact in a complex way, resting against each other, catching and releasing with time. This character of the molecular structure of a fluid leads to a number of properties, such as the strain memory effect, pseudoclassical (change in fluid viscosity with shear rate), and spatial anisotropy.The mathematical description of such a complex behavior of the medium is a difficult task, in the process of solving which one has to make a large number of assumptions and assumptions, often not obvious and controversial, not always well proved from a physical point of view. Probably, it is not worth counting on the emergence of a certain universal model of the dynamics of polymeric materials, since it is hardly possible to take into account all the various properties of the behavior of these medium with in the framework of one model. The result of this is a large number of different rheological models for the dynamics of liquid polymers, differing in approaches and, as a result, in the relationships and properties obtained.In addition, even geometrically simple flows within the framework of such models have unusual features that are often unique for individual models and require careful analysis. By themselves, these models are quite complex mathematically, and the properties of problem solutions for them are often poorly understood.One way or another, any rheological model of liquid polymers is based on a constitutive relation connecting the stress tensor of the medium with the velocity gradient tensor. The form of this relation depends on the generalizing assumptions made to obtain it, and is different from model to model. In general, there are two main approaches, or, if you like, two main ideas, to get this ratio.The first approach is focused on the analysis of experimental measurements of fluid properties obtained in the study of viscometric flows of real polymers.Using experimental data, within the framework of this approach, one can make a number of general assumptions regarding conservation laws and obtain constitutive relations by selecting the values of one or more of the introduced parameters, achieving correspondence solutions of equations with empirical data.The second approach focuses on modeling the dynamics of the medium macromolecules themselves and their interaction with each other. Since the movement of molecules itself is random, to model their dynamics one has to involve stochastic equations, which in one way or another take into account the Brownian component of the dynamics of microscopic particles. Accordingly, to obtain macroscopic relationships, the averaging of the liquid characteristics over the statistical ensemble is used. Models that mainly adhere to the first approach are called phenomenological [3, 4], and the second statistical [5, 6]. Models that combine these approaches in one way or another are usually called mesoscopic. The latter include and the Pokrovsky-Vinogradov model used in this work. The system of differential equations from [2] was studied in detail in [7], where the stationary solutions. The linear stability of such solutions was considered in [8; 9]. In this paper, we discuss the question of the stability of the direct problem of a simpler stationary flow - a state of rest (a state of mechanical equilibrium).

Preliminaries

Following [2], we formulate a generalized rheological model of Vinogradov - Pokrovsky, describing the flow of an incompressible viscoelastic polymer fluid. Consider a flat channel with perforated horizontal walls of height l.Let $\boldsymbol{u}=(u,v)$ be the dimensionless fluid velocity vector referred to the characteristic velocity $u_H, (x,y)-$ space vector referred to l, and t - dimensionless time referred to l/u_H. Since the flow of an incompressible fluid is considered, the model continuity equation in these notations has a standard form:

$$\operatorname{div}\mathbf{u} = u_x + v_y = 0, \tag{1}$$

Let us discuss the formulation of the momentum conservation law for the model used.As in the case of a viscous Newtonian fluid, the momentum flux must contain terms describing the irreversible momentum transfer between zones having different velocities.The complexity of viscoelastic fluids lies in the non-local nature of this transport. In other words, the internal stresses of a viscoelastic system depend not only on the velocity field at a given moment of time, but also on the flow history. The dependence between the fluid velocity field at the time t_0 and the stresses of the medium at the time $t_1 > t_0$ decreases with an increase in the elapsed time between these moments and for simple flows with constant shear is characterized by the value $\exp\left(-\left(t_1-t_0\right)/\tau\right)$, where $\tau-$ is the relaxation time. Accordingly, the relationship between the stress tensor and the velocity gradient a tensor cannot be represented as a local algebraic equation similar to that used in the Navier-Stokes equation and requires the use of additional differential equations. We present the equation for the law of conservation of momentum in general form:

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} + \nabla p = \frac{1}{\mathrm{Re}}\operatorname{div}\Pi \tag{2}$$

Here and further

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + (\boldsymbol{u},\nabla) = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$$

The right side of the equation contains the symmetric dimensionless tensor of the second rank $\Pi = \{a_{ij}\}, i,j = 1.2$, which characterizes the additional viscoelastic stresses of the system. In the model we use, this tensor is also called the anisotropy tensor, and its physical meaning is the deviation of the deformed viscoelastic system from the equilibrium position. In the approximation of small deformations, it is assumed that the internal friction coefficients included in the defining relation of the model are determined through the anisotropy tensor and the coefficients k and $\beta(0<\beta<1)$ are phenomenological parameters characterizing the dimensions and orientation molecular tangles in a polymer. Then the constitutive relation for the anisotropy tensor consists of three differential equations:

$$\left.\begin{array}{l} \frac{da_{11}}{dt}-2A_1u_x-2a_{12}u_y+K_Ia_{11}+\beta\left\|\boldsymbol{\sigma}_1\right\|^2=0 \\ \frac{da_{12}}{dt}-A_1v_x-A_2u_y+K_Ia_{12}+\beta\left(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2\right)=0 \\ \frac{da_{22}}{dt}-2a_{12}v_x-2A_2v_y+K_Ia_{22}+\beta\left\|\boldsymbol{\sigma}_2\right\|^2=0 \end{array}\right\}. \tag{3}$$

Here: $p-$ pressure; $\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2-$ symmetric matrix columns $\Pi=(a_{ij})=(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)$;

$$\begin{aligned} &\left\|\boldsymbol{\sigma}_i\right\|^2=\left(\boldsymbol{\sigma}_i,\boldsymbol{\sigma}_i\right), i=1,2, \\ &\operatorname{div}\Pi=\left(\operatorname{div}\boldsymbol{\sigma}_1,\operatorname{div}\boldsymbol{\sigma}_2\right)^T, \\ &K_I=W^{-1}+\frac{\bar{k}}{3}I, I=a_{11}+a_{22}, \bar{k}=k-\beta, \end{aligned}$$

$\mathrm{Re}=\rho u_H l/\eta_0-$ Reynolds number; $\rho(=\text{const})$ - medium density; $W=\tau_0u_H/l-$ Weissenberg number; η_0 - initial values of shear viscosity; l - characteristic length (see. picture);

$$\begin{aligned} &A_i=W^{-1}+a_{ii}, \quad i=1,2 \\ &\frac{d}{dt}=\frac{\partial}{\partial t}+(\mathbf{u},\nabla). \end{aligned}$$

In the system (1)-(3) the time t, coordinates x,y, velocity vector components u,v, pressure p attributed to $l/u_H, l, u_H, \rho u_H^2$. In the case of considering the flows of a polymer fluid in a channel, we must set the following boundary conditions on the channel walls:

$$u=v=0, \quad p_y=\frac{1}{\mathrm{Re}}\left(\left(a_{12}\right)_x+\left(a_{22}\right)_y\right) \quad \text{at } y=0,1. \tag{4}$$

As we have already noted, stationary solutions of the mathematical model (1)-(4) were e studied in detail in [7].Stationary solutions were constructed there, similar to the solutions of Poiseuille and Couette for the Navier - Stokes system of equations.Questions related to the linear stability of such solutions were considered in [8; 9].In this work, we take the state of rest (mechanical equilibrium) as the initial stationary flow:

$$u=v=a_{11}=a_{12}=a_{22}=0, p=const. \tag{5}$$

The eigenvalue problem

In [7], a linear system was constructed, obtained by linearizing the equations of an incompressible viscoelastic polymer fluid (1)-(3). Linearization was carried out with respect to stationary solutions, similar to the Poiseuille solutions for the system of Navier - Stokes equations.If we take the state of rest (5) in the channel as a stationary solution, then the linear system will have the following form:

$$\begin{gathered} \mathbf{U}_t+A_1\mathbf{U}_y+A_2\mathbf{U}_x+A_3\mathbf{U}+\mathbf{F}=0 \\ t>0, x\in\mathbb{R}^1, \quad 0<y<1, \end{gathered} \tag{6}$$

$$\mathbf{U}=\begin{pmatrix} u \\ v \\ \alpha_{12} \\ \alpha_{22} \end{pmatrix}, A_1=\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -\kappa_0^2 & 0 & 0 & 0 \\ 0 & -2\kappa_0^2 & 0 & 0 \end{pmatrix}, A_2=\begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & -1 & 0 \\ 0 & -\kappa_0^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$A_3=W^{-1}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}, \quad \mathbf{F}=\begin{pmatrix} \Omega_x \\ \Omega y \\ 0 \\ 0 \end{pmatrix}; \tag{7}$$

u,v - small perturbations of the velocity vector components; a_{11},a_{12},a_{22} - small perturbations of the components of the symmetric anisotropy tensor; $\alpha_{ij}=a_{ij}/\mathrm{Re}, i,j=1,2$; $\Omega=p-\alpha_{22}, p-$ small pressure disturbant; $\kappa_0^2=\frac{1}{W\,\mathrm{Re}}$.
Now we apply the Fourier transform with respect to the variable x and write down the symmetric hyperbolic system

$$\tilde{U}_t+A_1\tilde{U}_y+B_1\tilde{U}=\int_0^t\Psi(t-\tau)\tilde{U}(y,t)d\tau-\tilde{F}_0(y,t), \quad 0<y<1, \tag{8}$$

where $\tilde{U} = (\tilde{u}, \tilde{v}, \tilde{\alpha}_{12}, \tilde{\alpha}_{22})$-column vector, $\Psi(t) = \text{diag}\,(\psi_1, \psi_2, \psi_3, \psi_4)$ $\tilde{\Omega} = \tilde{p} - \tilde{\alpha_{22}}$

$$A_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -\kappa_0^2 & 0 & 0 & 0 \\ 0 & -2\kappa_0^2 & 0 & 0 \end{pmatrix}, \tag{9}$$

$$B_1 = (i\xi A_2 + A_3) = \begin{pmatrix} 0 & 0 & 0 & i\xi \\ 0 & 0 & -i\xi & 0 \\ 0 & -i\xi\kappa_0^2 & \frac{1}{W} & \frac{1}{W} \\ 0 & 0 & \frac{1}{W} & \frac{1}{W} \end{pmatrix}, \tilde{F}_0 = \begin{pmatrix} i\xi\tilde{p} \\ \tilde{p}_y \\ 0 \\ 0 \end{pmatrix}.$$

System (8) leads to canonical species.In the obscure case, there is such a nondegenerate matrix T that $T^{-1}A_1T = \Lambda$, where $\Lambda-$ is a diagonal matrix, on whose diagonal are the eigenvalues of the matrix $\tilde{A}_1$.

$$T = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ -\kappa_0 & \kappa_0 & 0 & 0 \\ 0 & 0 & -\sqrt{2}\kappa_0 & \sqrt{2}\kappa_0 \end{pmatrix}. \tag{10}$$

The inverse matrix to T is defined by the following formula

$$T^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2\kappa_0} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2\kappa_0} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2\sqrt{2}\kappa_0} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2\sqrt{2}\kappa_0} \end{pmatrix}. \tag{11}$$

Now we replace the in equation (8) a new function with the aid of equality

$$\tilde{U} = TV$$

and multiply this equation on the left by the matrix T^{-1}. Then we get following equation for the function V

$$\left(I_4\frac{\partial}{\partial t} + \Lambda\frac{\partial}{\partial y} + C\right)V = \int_0^t \bar{\Psi}(y, t-\tau)V(y,\tau)d\tau + J \tag{12}$$

where I_4 - unit matrix of order $4, \Lambda = \text{diag}\left(\kappa_0, -\kappa_0, \sqrt{2}\kappa_0, -\sqrt{2}\kappa_0\right)$,
$C = T^{-1}B_1T$, $\bar{\Psi}(y,t) = T^{-1}\Psi(y,t)T, J = -T^{-1}\tilde{F}_0(y,t)$.

Formulation of the problem

In a direct task with specified matrices $\bar{\Psi}, C$ and a J vector function must be defined in the area $D = \{(y,t) : 0 < y < 1, t > 0\}$ vector function $V(y,t)$, satisfying equation (12) under the following initial and boundary conditions:

$$V_i(y,t)|_{t=0} = \varphi_i(y), i = \overline{1,4} \tag{13}$$

$$V_i(y,t)|_{y=0} = g_i(t), i = 1,3;\ V_i(y,t)|_{y=1} = g_i(t), i = 2,4 \tag{14}$$

where $\varphi(y) = (\varphi_1, \varphi_2, \varphi_3, \varphi_4)(y), g(t) = (g_1, g_2, g_3, g_4)(t)$ given functions.

$$\begin{gathered} \frac{\partial V_i}{\partial t} + \lambda_i\frac{\partial V_i}{\partial y} = -\sum_{j=1}^{4} c_{ij}V_i(y,t) + \\ + \int_0^t \psi_i V_i(y, t-\tau)d\tau + J_i(y,t), i = \overline{1,4}. \end{gathered} \tag{15}$$

To date, the problems of determining kernels from one second-order integro-differential equation [10]-[42] have been widely studied. The numerical solution of direct and inverse problems for such equations was studied in [24]-[38]. As a rule, second order equations are derived from systems of first order partial differential equations under some additional assumptions.
The inverse problem of determining the kernels of the integral terms from a system of general first-order integro-differential equations with two independent variables was studied in [43]. A theorem of local existence and global uniqueness is obtained.

Investigation of direct problem

Let $\Pi = \{(y,t) : 0 < y < 1, t > 0\}$ be the projection of the domain D to plane of the variables y, t. Consider an arbitrary point $(y,t) \in \Pi$ on the plane of the variables μ, τ and draw a characteristic of the i - equation of system (12) through till the intersection with the boundary of Π in the domain $\tau < t$. The equation is taking as

$$\mu = y + \lambda_i(\tau - t). \tag{16}$$

For $\lambda_i > 0$ $(i = 1, 3\ )$ this point lies either on the segment $[0,1]$ of the axis t = 0, or on the straight line $y = 0$, and for $\lambda_i < 0$ $(i = 2, 4\ )$ either on the segment $[0,1]$ or on the straight line $y = 1$.
Integrating equations (15) over characteristic (18) from $\left(y_0^i, t_0^i\right)$ to (y, t), we find

$$\begin{aligned} V_i^1(y,t) = V_i^1\left(y_0^i, t_0^i\right) + \int_0^t \left[J_i(\mu,\tau) - \sum_{j=1}^4 c_{ij} V_j^1(\mu,\tau) \right] \Bigg|_{\mu = y + \lambda_i(\tau - t)} d\tau + \\ + \int_{t_0^i}^t \int_0^\tau \psi_i V_j^1(\mu, \tau - \eta) d\eta \Bigg|_{\mu = y + \lambda_i(\tau - t)} d\tau, i = \overline{1,4} \end{aligned} \tag{17}$$

Let we find t_0^i in (17) and consequence it depends on the coordinates of (y,t).It is easy to observe that $t_0^i(y,t)$ has the form

$$t_0^i(y,t) = \begin{cases} t - \frac{y}{\lambda_i}, t \geq \frac{y}{\lambda_i}, \\ 0, 0 < t < \frac{y}{\lambda_i}, \end{cases} \quad i = 1, 3;$$

$$t_0^i(y,t) = \begin{cases} t + \frac{1-y}{\lambda_i}, t \geq \frac{y-1}{\lambda_i}, \\ 0, 0 < t < \frac{y-1}{\lambda_i}, \end{cases} \quad i = 2, 4.$$

Then the condition that the pair (y_0^i, t_0^i) satisfy to the equation (16) implies

$$y_0^i(y,t) = \begin{cases} 1, t \geq \frac{y}{\lambda_i}, \\ y - \lambda_i t, 0 < t < \frac{y}{\lambda_i}, \end{cases} \quad i = 1, 3;$$

$$y_0^i(y,t) = \begin{cases} 1, t \geq \frac{y-1}{\lambda_i}, \\ y - \lambda_i t, 0 < t < \frac{y-1}{\lambda_i}, \end{cases} \quad i = 2, 4. \tag{18}$$

The free terms of the equations(16) are defined through the initial and boundary conditions (13) and (14) as follows:

$$V_i\left(z_0^i, t_0^i\right) = \begin{cases} g_i\left(t - \frac{y}{\lambda_i}\right), t \geq \frac{y}{\lambda_i}, \\ \varphi_i\left(y - \lambda_i t\right), 0 \leq t < \frac{y}{\lambda_i}, \end{cases} \quad i = 1, 3.$$

$$V_i\left(y_0^i, t_0^i\right) = \begin{cases} g_i\left(t + \frac{y-1}{\lambda_i}\right), t \geq \frac{y-1}{\lambda_i}, \\ \varphi_i\left(y - \lambda_i t\right), 0 \leq t < \frac{y-1}{\lambda_i} \end{cases}, \quad i = 2, 4. \tag{19}$$

It is required that $V_i\left(z_0^i, t_0^i\right)$ be continuous in Π. Note that, for these conditions to be fulfilled, the given functions φ_i and g_i must satisfy the metting conditions at the angular points of Π :

$$g_i(0) = \varphi_i(1),\ i = 1, 3;\ \ g_i(0) = \varphi_i(0),\ i = 2, 4. \tag{20}$$

Here and below, the values of g_i at $t = 0$ and φ_i at $y = 0, 1$ are understood as the limit values at these points as the argument tends from the point where these functions are defined.
Suppose that all given functions in (17) are continuous functions of their arguments in Π.Then we have a closed system of Volterra-type integral equations with continuous kernels and free terms. As usual, such a system has a unique solution in the bounded sub domain

$$\Pi_T = \{(y,t) : 0 \leq y \leq 1, 0 \leq t \leq T\},$$

$T > 0$- some fixed number.
Theorem 1. *Suppose that $\varphi(y) \in C^1[0,1], g(t) \in C^1[0,T], \Psi(t) \in C^1[0,T], J \in C^1(\Pi_T)$ and conditions (13) and (19) are fulfilled. Then there is a unique solution. Then in the domain Π_T there is a unique classical solution to problem (17).*

Proof. Stop first of all on the properties of the functions $V_i(y,t)$ defined by equations (17). Show that in this case equations (17) fuse the only solution the continuous functions class that can be obtained by consecutive approximations. Let

$$\varphi_0 := \max_{1 \leq i \leq 4} \left\{ \|\varphi_i\|_{C^2[0,1]} \right\}, g_0 := \max_{1 \leq i \leq 4} \left\{ \|g_i\|_{C^2[0,T]} \right\},$$

$$J_0 := \max_{1 \leq i, \leq 4} \left\{ \|J_i\|_{C^2[\bar{\Pi}_T]} \right\}, c_0 := \max_{1 \leq i \leq 4} \left\{ \|c_{ij}\|_{C[0,1]} \right\}, q_0 = \max \left\{ \varphi_0, g_0, J_0 \right\},.$$

$$\psi_0 := \max_{1 \leq j \leq 4} \left\{ \|\psi_j\|_{C[0,T]} \right\}, \quad m_0 = \max \left\{ 4c_0, \psi_0 \right\}$$

Let us construct a method of successive approximations for equation (17) according to the following scheme [44],[45]:

$$
\begin{gathered}
V_i^0(y,t)=V_i\left(y_0^i,t_0^i\right)+\int_{t_0^i}^t J_i(y,\tau)_{\mu=y+\lambda_i(\tau-t)}d\tau,\\
V_i^1(y,t)=V_i^0(y,t)-\int_{t_0^i}^t\sum_{j=1}^4 c_{ij}(\mu)V_j^0(\mu,\tau)_{\mu=y+\lambda_i(\tau-t)}d\tau+\\
+\int_{t_0^i}^t\int_0^\tau \psi_i(\eta)V_i^0(\mu,\tau-\eta)_{\mu=y+\lambda_i(\tau-t)}d\eta d\tau,\quad i=\overline{1,4}\\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
V_i^l(y,t)=V_i^0(y,t)-\int_{t_0^i}^t\sum_{j=1}^4 c_{ij}(\mu)V_j^{l-1}(\mu,\tau)_{\mu=x+\lambda_i(\tau-t)}d\tau+\\
+\int_{t_0^i}^t\int_0^\tau \psi_i(\eta)V_j^{l-1}(\mu,\tau-\eta)_{\mu=y+\lambda_i(\tau-t)}d\eta d\tau,\quad i=\overline{1,4}\\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{gathered}
\tag{21}
$$

It is obvious that each of the functions $V_i^l(y,t)$ in the area Π_T is continuous.
There are evaluations in this area

$$
\begin{gathered}
\left|V_i^0(y,t)\right|=\left|V_i\left(y_0^i,t_0^i\right)+\int_{t_0^i}^t J_i(\mu,\tau)_{\mu=+\lambda_i(\tau-t)}d\tau\right|\le\\
\le q_0(1+t)\\
\left|V_i^1(y,t)\right|=\mid V_i^0(y,t)-\int_{t_0^i}^t\sum_{j=1}^n c_{ij}(\mu)V_j^0(\mu,\tau)_{\mu=y+\lambda_i(\tau-t)}d\tau+\\
+\int_{t_0^i}^t\int_0^\tau \psi_i(\eta)V_i^0(\mu,\tau-\eta)_{\mu=y+\lambda_i(\tau-t)}d\eta d\tau\mid\le\\
\le q_0(1+t)+m_0q_0\left[t+\frac{t^2}{2!}\right]+m_0q_0\left[\frac{t^2}{2!}+\frac{t^3}{3!}\right]\\
\left|V_i^2(y,t)\right|=\mid V_i^0(y,t)-\int_{t_0^i}^t\sum_{j=1}^n c_{ij}(\mu)V_j^1(\mu,\tau)_{\mu=y+\lambda_i(\tau-t)}d\tau+\\
+\int_{t_0^i}^t\int_0^\tau \psi_i(\eta)V_i^1(\mu,\tau-\eta)_{\mu=y+\lambda_i(\tau-t)}d\eta d\tau\mid\le\\
\le q_0(1+t)+m_0q_0\left[t+\frac{t^2}{2!}\right]+m_0q_0\left[\frac{t^2}{2!}+\frac{t^3}{3!}\right]+\\
+m_0^2q_0\left[\frac{t^2}{2!}+\frac{t^3}{3!}\right]+m_0^2q_0\left[\frac{t^3}{3!}+\frac{t^3}{3!}\right]+m_0^2q_0\left[\frac{t^4}{4!}+\frac{t^5}{5!}\right]
\end{gathered}
$$

$$
\left|V_i^l(y,t)\right|\le\sum_{d=0}^l\sum_{j=0}^{d+1}m_0^dq_0\left[\frac{t^{d+j-1}}{(d+j-1)!}+\frac{t^{d+j}}{(d+j)!}\right]\tag{22}
$$

Let's show that the Neumann's row

$$
V_i^0(y,t)+\sum_{l=1}^\infty\left(V_i^l(y,t)-V_i^{l-1}(y,t)\right).
$$

Its partial sum coincides with the function $V_i^l(y,t)$ and, therefore, this series is majorize by the series

$$
\sum_{l=0}^\infty\sum_{j=0}^{l+1}m_0^lq_0\left[\frac{t^{l+j-1}}{(l+j-1)!}+\frac{t^{l+j}}{(l+j)!}\right],
$$

which, in turn, for all $(y,t)\in\Pi_T$ can be mojorize by the a converging numerical row

$$
\sum_{l=0}^\infty\sum_{j=0}^{l+1}m_0^lq_0\left[\frac{T^{l+j-1}}{(l+j-1)!}+\frac{T^{l+j}}{(l+j)!}\right].
$$

So the Neumann's row

$$
V_i^0(y,t)+\sum_{l=1}^\infty\left(V_i^l(y,t)-V_i^{l-1}(y,t)\right),
$$

is absolutely and evenly converges, and therefore its sum is a continuous in the Π_T function. As usual, it is not difficult to prove that the sum of a series is a solution of the integral equation (20). Now let's you the uniqueness of this solution. Suppose there are two different solutions to the equation (20) $V_i^1(y,t)$ and $V_i^2(y,t)$:

$$V_i^1(y,t) = V_i\left(y_0^i, t_0^i\right) + \int_{t_0^i}^{t} J_i(\mu,\tau)_{\mu=y+\lambda_i(\tau-t)}d\tau -$$

$$-\int_{t_0^i}^{t}\sum_{j=1}^{n} c_{ij}(\mu)V_j^1(\mu,\tau)_{\mu=y+\lambda_i(\tau-t)}d\tau +$$

$$+\int_{t_0^i}^{t}\int_0^{\tau}\psi_i(\eta)V_i^1(\mu,\tau-\eta)_{\mu=y+\lambda_i(\tau-t)}d\eta d\tau, \quad i=\overline{1,4},$$

$$V_i^2(y,t) = V_i\left(y_0^i, t_0^i\right) + \int_{t_0^i}^{t} J_i(\mu,\tau)_{\mu=y+\lambda_i(\tau-t)}d\tau -$$

$$-\int_{t_0^i}^{t}\sum_{j=1}^{n} c_{ij}(\mu)V_j^2(\mu,\tau)_{\mu=y+\lambda_i(\tau-t)}d\tau +$$

$$+\int_{t_0^i}^{t}\int_0^{\tau}\psi_i(\eta)V_i^2(\mu,\tau-\eta)_{\mu=y+\lambda_i(\tau-t)}d\eta d\tau, \quad i=\overline{1,4}.$$

Then their difference

$$X_i(y,t) = V_i^1(y,t) - V_i^2(y,t), \quad i=\overline{1,4}$$

satisfies the integral equation

$$X_i(y,t) = -\int_{t_0^i}^{t}\sum_{j=1}^{n} c_{ij}(\mu)X_j(\mu,\tau)_{\mu=y+\lambda_i(\tau-t)}d\tau +$$
$$+\int_{t_0^i}^{t}\int_0^{\tau}\psi_i(\eta)X_i(\mu,\tau-\eta)_{\mu=y+\lambda_i(\tau-t)}d\eta d\tau, \quad i=\overline{1,4}. \tag{23}$$

By $\widetilde{m}_i(t)$ we denote the maximum modulus of the function $m_i(y,t)$ for each fixed $t\in[0,T]$ and $y\in[0,1]$

$$\bar{X}_i(t) = \max_{y\in[0,1]}|X_i(y,t)|.$$

Let us show that integral equations (23) have only trivial solutions.The proof of this fact can be carried out using the following statement.
Consider (23), whence we obtain the estimate

$$X_i(t) = | -\int_{t_0^i}^{t}\sum_{j=1}^{n} c_{ij}(\mu)X_j(\mu,\tau)_{\mu=y+\lambda_i(\tau-t)}d\tau +$$

$$+\int_{t_0^i}^{t}\int_0^{\tau}\psi_i(\eta)X_i(\mu,\tau-\eta)_{\mu=y+\lambda_i(\tau-t)}d\eta d\tau \; |$$

$$\leq m_0(1+T)\int_{t_0^i}^{t}\tilde{X}_i(\tau)d\tau.$$

The last expressions are that the initial-boundary value problem (12)-(14) has a unique solution if the conditions of Theorem 1 are satisfied. □

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