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THE INVERSE PROBLEM FOR SYSTEMS OF INCOMPRESSIBLE VISCOELASTIC POLYMERIC FLUID AT REST

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Abstract:

Introduction. *Many physical processes are described by a system of equations of the hyperbolic type of the first order. For example, systems of an incompressible viscoelastic polymeric fluid, etc. It is well known that second-order equations are derived from them with the help of a number of additional restrictions. The solution of inverse problems leads directly to the solution of these systems. Systematic research in this area was carried out in the 1970s by L.P. Nizhnik, S.P. Belinsky, V.G. Romanov and L.I. Slinyuchev began in the work of scientists.*

Methods. *In this work we will use the differential equations, functional analysis, algebraic methods and also the principle of contraction mappings.*

Results. *The article checks the inverse problem for hyperbolic systems of 4 first-order integro-differential equations with an integral member of the convolution type. The direct problem is the initial-boundary value problem for this system on the finite interval [0, 1]. When certain data matching conditions are fully met, the inverse problem is reduced to solving a system of Volterra-type integral equations with respect to the unknowns. In addition, a theorem on the local unique solvability of the problem for sufficiently small.*

Discussions. *The study of direct and inverse problems posed to a mixed type equation is one of the advanced critical and rapidly emerging areas of world science. Their numerical implementation provides an applied application for the study of these problems. In this paper, we numerically study the boundary value problem posed to a model equation of mixed type. To do this, you need to know the concept of approximation and stability. The stability of the difference scheme has been proven. The order of approximation is calculated in the work. Further, when the stability and approximation are proved, it is possible to show the approximation of the numerical solution to the exact solution.*

Conclusion. *To sum up, we look the kernel is* 4×4 *dimensional diagonal matrix that depends on time. To define that function, we put initial-boundary conditions on characteristic lines. Proved the theorem of unique solvability. We get the following results, firstly consider the inverse problem of the determination kernel in hyperbolic system of n number first-order integro–differential equations, which is*

of the form of 4×4 *matrices depending on variable* t*, next obtain the theorem of exists unique solution, finally proved local theorem in a small interval.*
Keywords: *Hyperbolic system, diagonal and inverse quadratic matrices, vector functions, convolution kernel, integral equations of Volterra type, principle of contraction mappings.*

Introduction. First-order hyperbolic systems of equations describe many physical processes associated with the motion of a polymer fluid in a flat channel. As an example, we can point out systems of equations for an incompressible viscoelastic polymer fluid.

The Pokrovsky–Vinogradov rheological model is used as a model for the hydrodynamics of a polymeric fluid [1, 2].

Polymeric liquids are fluid media consisting of long macromolecules entangled witheach other. In flows with nonzero velocity gradients, such molecules interact in a complex way, resting against each other, catching and releasing with time. This feature of the molecular structure of a fluid leads to a number of features like the strain memory effect, pseudoplasticity (change in fluid viscosity with shear rate) and spatial anisotropy.

The mathematical description of such a complex behavior of the medium is a difficult task, in the process of solving which one has to make a large number of assumptions and assumptions, often not obvious and controversial, not always well argued from a physical point of view. Apparently, it is not worth counting on the appearance of a certain universal model of the dynamics of polymeric materials, since it is hardly possible to take into account all the various features of the behavior of these media within the framework of one model. The result of this is a large number of different rheological models for the dynamics of liquid polymers, differing in approaches and, as a result, in the relationships and properties obtained. In addition, even geometrically simple flows within the framework of such models have unusual features that are often unique for individual models and require careful analysis. By themselves, these models are quite complex mathematically, and the properties of problem solutions for them areoften poorly understood.

One way or another, any rheological model of liquid polymers is based on a constitutiverelation connecting the stress tensor of the medium with the velocity gradient tensor. The form of this ratio depends from the generalizing assumptions made to obtain it and differ from model to model. In general, we can single out two main approaches, or if you like, two main ideas that make it possible to obtain this ratio. The first approach is focused on the analysis of experimental measurements of fluid properties obtained in the study of viscometric flows of real polymers. Using experimental data, within the framework of this approach, one can make a number of general assumptions regarding conservation laws and obtain constitutive relations by selecting the values of one or more of the introduced parameters, achieving correspondence solutions of equations with empirical data.

The second approach focuses on modeling the dynamics of the medium macromolecules themselves and their interaction with each other. Since the motion of molecules in itself is random, to model their dynamics one has to involve stochastic equations, which in one way or another take into account the Brownian component of the dynamics of microscopic particles. Accordingly, to obtain macroscopic relationships, the liquid characteristics are averaged over the statistical ensemble. Models that mainly adhere to the first approach are called phenomenological [3, 4], and the second – statistical [6, 5]. Models that somehow combine these approaches are commonly referred to as mesoscopic. The latter include and the Pokrovsky–Vinogradov model used in this work.

One of the classical fluid flows is a stationary flow in a straight cylindrical or flat channel. Its implementation for a viscous fluid in the stationary case is the well- known Poiseuille flow, which is one of the examples of exact analytical solutions of the Navier–Stokes equations. A natural desire is to analyze a flow of a viscoelastic fluid similar in geometry. In addition to its relative simplicity, this type of flow can be considered one of the most interesting from a practical point of view, since the study of the flow of polymer melts through pipes is important for the production of polymer materials, additive technologies, and related industries. Similar flows, among other models, have also been studied for the Pokrovsky–Vinogradov model, but, unlike the Poiseuille flow for the Navier–Stokes equations, cases that allow one to find analytical solutions have not yet been considered.

Analysis of dynamic equations describing such processes, show that Volterrod operators of the convolution type of some function are added to the right-hand side of the systems of hyperbolic equations, depending on time and the elliptic part of the corresponding hyperbolic operators on the left side.

Following [1], we formulate the generalized rheological model of Vinogradov – Pokrovsky, which describes the flows of an incompressible viscoelastic polymeric fluid (for example,

in a flat channel). In dimensionless form (the process of nondimensionalization is described in detail in [2]), this mathematical model has the following in

$$
\begin{gathered}
u_x + v_y = 0, \\
\frac{d\mathbf{u}}{dt} + \nabla p = \frac{1}{Re} div\Pi, \\
\frac{da_{11}}{dt} - 2A_1 u_x - 2a_{12} u_y + K_I a_{11} + \beta \|\sigma_1\|^2 = 0, \\
\frac{da_{12}}{dt} - A_1 v_x - A_2 u_y + K_I a_{12} + \beta(\sigma_1, \sigma_2) = 0, \\
\frac{da_{22}}{dt} - 2a_{12} v_x - 2A_2 v_y + K_I a_{22} + \beta \|\sigma_2\|^2 = 0,
\end{gathered} \tag{1}
$$

here: t - time; $u, v -$ components of the velocity vector $\mathbf{u}$ in Cartesian coordinates $x, y; p -$ pressure; $a_{ij}, i, j = 1,2$ - components of the symmetric anisotropy tensor Π of the second rank; $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2 -$ columns of the symmetric matrix $\Pi = (a_{ij}) = (\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)$;

$$\|\sigma_i\|^2 = (\sigma_i, \sigma_i), i = 1,2;$$

$$div\Pi = (div\sigma_1, div\sigma_2)^T,$$

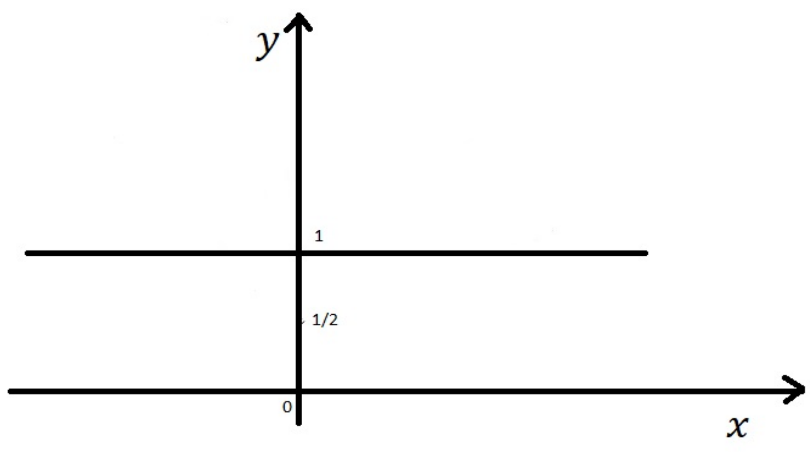
$K_I = W^{-1} + \frac{\bar{k}}{3} I, I = a_{11} + a_{22}, \bar{k} = k - \beta$ the constants $k, \beta (0 < \beta < 1) -$ phenomenological parameters do not depend on the molecular weight of the polymer and its concentration, characterizing, respectively, the dimensions and orientation of the molecular coils of the polymer associated with anisotropy; $Re = \frac{\rho u_H l}{\eta_0} -$ Reynolds number; $\rho (= \text{const})$ - medium density; $W = \frac{\tau_0 u_H}{l} -$ Weisenberg number; η_0, τ_0 - initial values of shear viscosity and relaxation time; l - characteristic length (see figure); u_H - characteristic speed;

$$A_i = W^{-1} + a_{ii}, \quad i = 1,2;$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{u}, \nabla).$$

In the (1) frame, time t, coordinates x, y, velocity vector components u, v, pressure p are related to $\frac{l}{u_H}, l, u_H, \rho u_H^2$. As we have already noted, the stationary solutions of the mathematical model (1) were studied in detail in [2]. Stationary solutions were constructed there, similar to the Poiseuille and Couette solutions for the Navier–Stokes system of equations. Questions related with linear stability of such solutions were considered in [3; 4]. In this work as the initial stationary flow, we take the state of rest (mechanical equilibrium):

$$u = v = a_{11} = a_{12} = a_{22} = 0, p = const. \qquad (2)$$



1. Flat channel.

In [2], a linear system was constructed, obtained by linearizing equations of an incompressible viscoelastic polymeric fluid (1). Linearization was carried out with respect to stationary solutions similar to the Poiseuille solutions for the system of Navier-Stokes equations. If we take the state of rest (2) in the channel (see figure) as a stationary solution, then the linear system will have the following view:

$$\mathrm{U}_t + A_1\mathrm{U}_y + A_2\mathrm{U}_x + A_3\mathrm{U} + \mathrm{F}_0 = 0, \tag{3}$$

$$t > 0, x \in \mathbb{R}^1, \quad 0 < y < 1,$$

$$\mathbf{U} = \begin{pmatrix} u \\ v \\ \alpha_{12} \\ \alpha_{22} \end{pmatrix}, A_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -\kappa_0^2 & 0 & 0 & 0 \\ 0 & -2\kappa_0^2 & 0 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & -1 & 0 \\ 0 & -\kappa_0^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$A_3 = W^{-1}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}, \quad \mathbf{F_0} = \begin{pmatrix} \Omega_x \\ \Omega y \\ 0 \\ 0 \end{pmatrix}; \tag{4}$$

u, v are small perturbations of the velocity vector components; $a_{11}(= -a_{22}), a_{12}, a_{22}$ - small perturbations of the components of the symmetric anisotropy tensor; $\alpha_{ij} = \frac{a_{ij}}{Re}, i, j = 1,2$; $\Omega = p - \alpha_{22}, p -$ small pressure perturbation; $\kappa_0^2 = \frac{1}{WRe}$.

Equation (3) will be considered with integral terms of the convolution type on the right side:

$$\mathbf{U}_t + A_1\mathbf{U}_y + A_2\mathbf{U}_x + A_3\mathbf{U} + \mathbf{F_0} = \int_0^t \Psi(\tau)\mathbf{U}(y, t-\tau)d\tau,$$

$$t > 0, x \in \mathbb{R}^1, \quad 0 < y < 1,$$

where $\Psi(t) = diag(\psi_1, \psi_2, \psi_3, \psi_4)(t)$-diagonal matrix, characterizing the viscous properties of the medium.

Now, applying the Fourier transform with respect to the variable x, we rewrite this system in the form:

$$\widetilde{U}_t + A_1\widetilde{U}_y + B_1\widetilde{U} = \int_0^t \Psi(t-\tau)\widetilde{U}(y,\tau)d\tau - \tilde{F}_0(y,t), \quad 0 < y < 1, \tag{5}$$

where $\widetilde{U} = (\tilde{u}, \tilde{v}, \tilde{\alpha}_{12}, \tilde{\alpha}_{22})$-column vector, $\widetilde{\Omega} = \tilde{p} - \tilde{\alpha}_{22}$,

$$A_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -\kappa_0^2 & 0 & 0 & 0 \\ 0 & -2\kappa_0^2 & 0 & 0 \end{pmatrix}, \tag{6}$$

$$B_1 = (i\xi A_2 + A_3) = \begin{pmatrix} 0 & 0 & 0 & i\xi \\ 0 & 0 & -i\xi & 0 \\ 0 & -i\xi\kappa_0^2 & \frac{1}{W} & \frac{1}{W} \\ 0 & 0 & \frac{1}{W} & \frac{1}{W} \end{pmatrix}, \tilde{F}_0 = \begin{pmatrix} i\xi\tilde{p} \\ \tilde{p}_y \\ 0 \\ 0 \end{pmatrix}.$$

Let us reduce system (5) to the canonical form.

In the case under consideration, there exists a nonsingular matrix T such that $T^{-1}A_1T = \Lambda$, where $\Lambda -$ is a diagonal matrix with eigenvalues of the matrix A_1,

$$T = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ -\kappa_0 & \kappa_0 & 0 & 0 \\ 0 & 0 & -\sqrt{2}\kappa_0 & \sqrt{2}\kappa_0 \end{pmatrix}. \quad (7)$$

The inverse matrix to T is defined by the following formula

$$T^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2\kappa_0} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2\kappa_0} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2\sqrt{2}\kappa_0} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2\sqrt{2}\kappa_0} \end{pmatrix}. \quad (8)$$

Now, in equation (5), we introduce a new function using the equality

$$\widetilde{U} = TV \quad (9)$$

and multiply this equation on the left by the matrix T^{-1}. Then for the function V we obtain the equation

$$\left(I_4\frac{\partial}{\partial t} + \Lambda\frac{\partial}{\partial y} + C\right)V = \int_0^t R(t-\tau)V(y,\tau)d\tau + F \quad (10)$$

where I_4 - means the identity matrix of order 4, $\Lambda = diag\left(\kappa_0, -\kappa_0, \sqrt{2}\kappa_0, -\sqrt{2}\kappa_0\right), C = T^{-1}B_1T$, $R(t) = T^{-1}\Psi(t)T, F = -T^{-1}\tilde{F}_0(y,t)$.

Formulation of the problem

In the direct problem, given matrices R, C and vector functions F, it is required to determine in the domain $\Pi = \{(y,t): 0 < y < 1, t > 0\}$ the vector function $V(y,t)$, matching equation (10) under the following initial and boundary conditions:

$$V_i(y,t)|_{t=0} = \varphi_i(y), i = \overline{1,4}; \quad (11)$$

$$V_i(y,t)|_{y=0} = g_i(t), i = 1,3; \quad V_i(y,t)|_{y=1} = g_i(t), i = 2,4; \quad (12)$$

here $\varphi(y) = (\varphi_1, \varphi_2, \varphi_3, \varphi_4)(y), g(t) = (g_1, g_2, g_3, g_4)(t)$ given functions.

In the inverse problem, the matrix function $R(t) > 0, t > 0$ is assumed to be unknown it is required to find it if, with respect to the solution of problem (10) (12), additional conditions are known that are specified on the side boundaries of the domain Π,

$$\begin{aligned} V_i(t)|_{y=1} &= h_i(t), i = 1,3; \\ V_i(t)|_{y=0} &= h_i(t), i = 2,4; \end{aligned} \qquad (13)$$

in this case, $R(0)$ are assumed to be given.

To date, the problems of determining kernels from one second-order integro-differential equation [6]-[23] have been widely studied. The numerical solution of direct and inverse problems for such equations was studied in [24]-[38]. As a rule, second-order equations are derived from systems of first-order partial differential equations under some additional assumptions.

The inverse problem of determining the kernels of the integral terms from a system of general first-order integro-differential equations with two independent variables was studied in [39]. A theorem of local existence and global uniqueness is obtained.

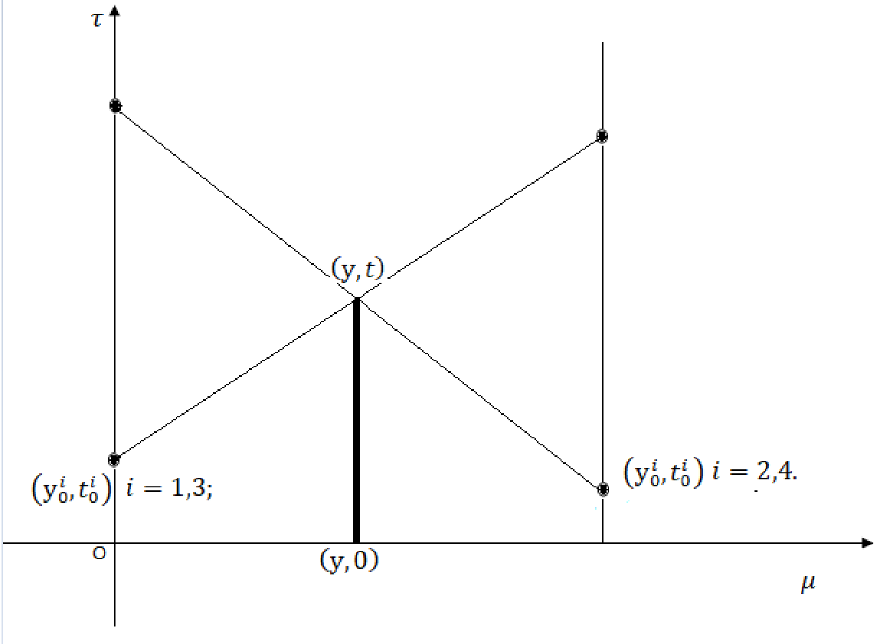
It seems quite natural to study inverse problems of determining the kernels of the integral terms of a system of integro-differential equations directly in terms of the system itself. This article is a natural continuation of this circle of problems and to a certain extent generalizes the results of [39] to the case of a system of equations for an incompressible viscoelastic polymer fluid (1).

Study of the direct problem

Consider an arbitrary point $(y, t) \in \Pi$ on the plane of variables μ, τ and characterize the equation of system (10) through it until it intersects the boundary Π in the region $\tau < t$. Its equation has the form

$$\mu = y + \lambda_i(\tau - t), \qquad (14)$$

at $\lambda_i > 0$ (that is $i = 1,3$) this point lies either on the segment $[0,1]$ of the axis $t = 0$, or on the line $y = 0$, and for $\lambda_i < 0$, (that is $i = 2,4$) either on the segment $[0,1]$ or on the line $y = 1$ (Rice 2).



2. Characteristic lines.

Integrating the i-th component of equality (10) with respect to characteristic (14) from the point (y_0^i, t_0^i) to the point (y, t), we find

$$V_i^1(y,t) = V_i^1(y_0^i, t_0^i) + \int_0^t \left[F_i(\mu,\tau) - \sum_{j=1}^{4} c_{ij} V_j^1(\mu,\tau) \right] \Bigg|_{\mu = y + \lambda_i(\tau - t)} d\tau +$$

$$+ \int_{t_0^i}^{t} \int_0^{\tau} \sum_{j=1}^{4} R(\tau)_{ij} V_j^1(\mu, \tau - \eta) d\eta \Bigg|_{\mu = y + \lambda_i(\tau - t)} d\tau, i = \overline{1,4}. \quad (15)$$

Let us first define in (15) t_0^i It depends on the coordinates of the point (y,t). It is easy to see that $t_0^i(y,t)$ has view,

$$t_0^i(y,t) = \begin{cases} \begin{cases} t - \frac{y}{\lambda_i}, t \geq \frac{y}{\lambda_i}, \\ 0, \quad 0 < t < \frac{y}{\lambda_i}, \end{cases} & i = 1,3; \\ \begin{cases} t + \frac{1-y}{\lambda_i}, t \geq \frac{y-1}{\lambda_i}, \\ 0{,}0 < t < \frac{y-1}{\lambda_i} \end{cases} & i = 2,4. \end{cases} \quad (16)$$

Then, from the condition that the pair (y_0^i, t_0^i) satisfies equation (15) it follows

$$y_0^i(y,t)=\begin{cases}\begin{cases}0, & t\geq \dfrac{y}{\lambda_i},\\ y-\lambda_i t, & 0<t<\dfrac{y}{\lambda_i},\end{cases} & i=1,3;\\ \begin{cases}1, & t\geq \dfrac{y-1}{\lambda_i},\\ y-\lambda_i t, 0<t<\dfrac{y-1}{\lambda_i}\end{cases} & i=2,4.\end{cases}$$

The free terms of the integral equations (15) are determined through the initial and boundary conditions (11) and (12) as follows:

$$\mathrm{V}_i\left(y_0^i,t_0^i\right)=\begin{cases}\begin{cases}\mathrm{g}_i\left(t-\frac{y}{\lambda_i}\right), & t\geq \frac{y}{\lambda_i},\\ \varphi_i(y-\lambda_i t), & 0\leq t<\frac{y}{\lambda_i},\end{cases} & i=1,3;\\ \begin{cases}\mathrm{g}_i\left(t+\frac{1-y}{\lambda_i}\right), & t\geq \frac{y-1}{\lambda_i},\\ \varphi_i(y-\lambda_i t), & 0\leq t<\frac{y-1}{\lambda_i},\end{cases} & i=2,4.\end{cases} \tag{17}$$

We require that the functions $V_i\left(y_0^i,t_0^i\right)$ be continuous in the domain Π. Note that in order to fulfill these conditions, the given functions φ_i and g_i must satisfy the matching conditions at the corner points of the domain Π:

$$\varphi_i(0)=\mathrm{g}_i(0),\quad i=1,3;\quad \varphi_i(1)=\mathrm{g}_i(0),\quad i=2,4. \tag{18}$$

Here the values of functions g_i at t = 0 and functions φ_i at $y=0,1$; are understood as the limit at these points as the argument tends from the side of the point where these functions are defined.

Let us assume that all given functions included in (15) are continuous functions of their arguments in Π. Then this system of equations is a closed system of Voltaire-type integral equations of the second kind with continuous kernels and free terms. As usual, such a system has a unique solution in the bounded subdomain $\Pi_T=\{(y,t): 0\leq y\leq 1, 0\leq t\leq T\}, T>0$ is some fixed number, area Π.

Let us introduce the vector function $w(y,t)=\frac{\partial}{\partial t}V(y,t)$ To obtain a problem for a function $w(y,t)$ similar to (10)-(12), we differentiate equations (10) n boundary conditions (12) with respect to the variable t, and conventionally for $t=0$ we find usingthe equation (10) and initial conditions (11). In doing so, we get

$$\frac{\partial w_i}{\partial t}+\gamma_i\frac{\partial w_i}{\partial y}=-\sum_{j=1}^{4}\mathrm{c}_{ij}w_j(y,t)+\sum_{j=1}^{4}R_{ij}(t)\varphi_j(y)+$$

$$+\int_0^t \sum_{j=1}^4 R_{ij}(t)w_j(y,t-\tau)d\tau + \frac{\partial}{\partial t}F_i(y,t), i=\overline{1,4}; \quad (19)$$

$$w_i(y,t)|_{t=0} = \Phi_i(y), i=\overline{1,4}; \quad (20)$$

$$\begin{gathered} w_i(y,t)|_{y=0} = \tfrac{d}{dt}g_i(t), i=1,3; \\ w_i(y,t)|_{y=1} = \tfrac{d}{dt}g_i(t), i=2,4; \end{gathered} \quad (21)$$

where

$$\Phi_i(y) = F_i(y,0) - \lambda_i\frac{\partial}{\partial y}\varphi_i(y) - \sum_{j=1}^4 c_{ij}\varphi_i(y), i=\overline{1,4}. \quad (22)$$

Again, integration along the corresponding characteristics will lead problem (19)-(21) to the integral equations

$$\begin{gathered} w_i(y,t) = w_i(y_0^i,t_0^i) + \int_{t_0^i}^t \sum_{j=1}^4 R_{ij}(\tau)\varphi_j(\mu)\Big|_{\mu=y+\lambda_i(\tau-t)} d\tau + \\ + \int_{t_0^i}^t \left[\frac{\partial}{\partial t}F_i(\mu,\tau) - \sum_{j=1}^4 c_{ij}w_j(\mu,\tau)\right]_{\mu=y+\lambda_i(\tau-t)} d\tau + \quad (23) \\ + \int_{t_0^i}^t \int_0^\tau \sum_{j=1}^4 R_{ij}(\eta)w_j(\mu,\tau-\eta)d\eta\Big|_{\mu=y+\lambda_i(\tau-t)} d\tau. \quad i=\overline{1,4} \end{gathered}$$

For functions w_i additional conditions (13) conditions look like

$$w_i(0,t) = \tfrac{d}{dt}h_i(t), i=2,4; \quad w_i(1,t) = \tfrac{d}{dt}h_i(t), i=1,3. \quad (24)$$

In equations (23) $w_i(y_0^i,t_0^i)$ are defined as follows:

$$w_i(y_0^i,t_0^i) = \begin{cases} \dfrac{d}{dt}g_i\left(t-\dfrac{y}{\lambda_i}\right), t \ge \dfrac{y}{\lambda_i} \\ \Phi_i(y-\lambda_i t), \quad 0 \le t < \dfrac{y}{\lambda_i}, \quad i=1,3; \end{cases}$$

$$w_i(y_0^i,t_0^i) = \begin{cases} \dfrac{d}{dt}g_i\left(t+\dfrac{1-y}{\lambda_i}\right), \quad t \ge \dfrac{y-1}{\lambda_i} \\ \Phi_i(y-\lambda_i t), \quad 0 \le t < \dfrac{y-1}{\lambda_i}, \quad i=2,4; \end{cases}$$

Let the conditions

$$F_i(0,0)-\gamma_i\left[\frac{\partial}{\partial y}\varphi_i(y)\right]_{y=0}-\sum_{j=1}^{4}c_{ij}\varphi_j(0)=\left[\frac{d}{dt}g_i(t)\right]_{t=0}, i=1,3; \qquad (25)$$

$$F_i(1,0)-\gamma_i\left[\frac{\partial}{\partial y}\varphi_i(y)\right]_{y=1}-\sum_{j=1}^{4}c_{ij}\varphi_j(1)=\left[\frac{d}{dt}g_i(t)\right]_{t=0}, i=2,4; \qquad (26)$$

It is not difficult to see that the conditions for matching the initial (20) and boundary (21) data at the corner points of the domain Π coincide with relations (25) and (26). From here it is clear that if the same equalities (25) and (26) hold, equations (19) will have unique continuous solutions $w_i(y,t)$ or the same $(\partial/\partial t)\mathrm{V}_i(y,t)$. So, we have proved the following statement:

Theorem 1. Let be $\varphi(y)\epsilon C^1[0,1]$, $g(t)\epsilon C^1[0,1], \Psi(t)\epsilon C[0,T]$,

$\varphi(y)\epsilon C^1[0,1]$ $F(y,t)$ and $F_t(y,t)\epsilon C(\Pi_T)$ the conditions are met (18), (25) и (26). Then there is a unique continuo- us solution to problem (19)-(21) in the domain Π.

Study of the inverse problem. Derivation of an equivalent system of integral equations

Consider an arbitrary point $(y,0)\in\Pi$ and draw characteristic (14) through it until it intersects with the side boundaries of the region Π. Integrating the i -th component of equation (19), using the data (22), we find

$$w_i(y,0)=\frac{d}{dt}h_i(t_i(y))-\int_0^{t_i(y)}\left[\sum_{j=1}^{4}R_{ij}(\tau)\varphi_j(\mu)-\sum_{j=1}^{4}c_{ij}w_j(\mu,\tau)\right]\Bigg|_{\mu=y+\lambda_i\tau}d\tau-$$

$$-\int_0^{t_i(y)}\frac{\partial}{\partial t}F_i(\mu,\tau)\Bigg|_{\mu=y+\lambda_i\tau}d\tau-\int_0^{t_i(y)}\int_0^{\tau}\sum_{j=1}^{4}R_{ij}(\alpha)w_j(\mu,\tau-\alpha)\mathrm{d}\alpha\Bigg|_{\mu=y+\lambda_i\tau}d\tau, \qquad (27)$$

where

$$t_i(y)=\frac{1}{\lambda_i}\begin{cases}-y, & i=1,3,\\ 1-y, & i=2,4.\end{cases}$$

Taking into account the initial conditions (21), we rewrite equations (27) in the form

$$\int_0^{t_i(y)} \sum_{j=1}^{4} c_{ij} \mathrm{w}_j(y+\lambda_i\tau,\tau)d\tau - \int_0^{t_i(y)} \int_0^{\tau} \sum_{j=1}^{4} R_{ij}(\alpha) w_j(y+\lambda_i\tau, \tau-\alpha)d\alpha d\tau =$$

$$= \Phi_i(y) - \frac{d}{dt} h_i(t_i(y)) + \int_0^{t_i(y)} \left[\sum_{j=1}^{4} R_{ij}(\tau)\varphi_j(y+\lambda_i\tau) + \frac{\partial}{\partial t} F_i(y+\lambda_i\tau,\tau)\right] d\tau, i=\overline{1,4}. \tag{28}$$

We differentiate the equations with respect to the variables y. Then we have

$$-\sum_{j=1}^{4} c_{ij} \mathrm{w}_j(y+\lambda_i t_i(y), t_i(y)) + \lambda_i \int_0^{t_i(y)} \sum_{j=1}^{4} c_{ij} \frac{\partial}{\partial y} \mathrm{w}_j(y+\lambda_i\tau,\tau)d\tau +$$

$$+\int_0^{t_i(y)} \sum_{j=1}^{4} R_{ij}(\tau) w_j(y+\lambda_i t_i(y), t_i(y)-\tau)d\tau -$$

$$-\lambda_i \int_0^{t_i(y)} \int_0^{\tau} \sum_{j=1}^{4} R_{ij}(\alpha) \frac{\partial}{\partial y} w_i(y+\lambda_i\tau, \tau-\alpha)d\alpha d\tau$$

$$= \lambda_i \frac{d}{dy}\Phi_i(y) + \frac{\partial^2}{\partial t^2}\ \mathrm{h}_i(t_i(y)) -$$

$$-\left[\sum_{j=1}^{4} R_{ij}(t_i(y))\varphi_j(y+\lambda_i t_i(y)) + \frac{\partial}{\partial t} F_i(y+\lambda_i t_i(y), t_i(y))\right] +$$

$$+\lambda_i \int_0^{t_i(y)} \left[\sum_{j=1}^{4} R_{ij}(\tau) \frac{\partial}{\partial y}\varphi_j(y+\lambda_i\tau) + \frac{\partial^2}{\partial t \partial y} F_i(y+\lambda_i\tau,\tau)\right] d\tau, i=\overline{1,4}. \tag{29}$$

Next, in equations (29), we replace $t_j(y)$ by t, and obtain the following equalities

$$-\sum_{j=1}^{4} c_{ij} \mathrm{w}_j(0,\mathrm{t}) + \lambda_i \int_0^{\mathrm{t}} \sum_{j=1}^{4} c_{ij} \frac{\partial}{\partial y} \mathrm{w}_j(y+\lambda_i\tau,\tau)d\tau +$$

$$+\sum_{j=1}^{4} R_{ij}(t)\varphi_j(0) + \int_0^{\mathrm{t}} \sum_{j=1}^{4} R_{ij}(\tau) w_j(0, \mathrm{t}-\phi)d\tau -$$

$$-\lambda_i \int_0^{\mathrm{t}} \int_0^{\tau} \sum_{j=1}^{4} R_{ij}(\alpha) \frac{\partial}{\partial y} w_i(y+\lambda_i\tau, \tau-\alpha)d\alpha d\tau = \lambda_i \frac{d}{dy}\Phi_i(y) +$$

$$+\frac{\partial^2}{\partial t^2}\,\mathrm{h}_i(\mathrm{t})-\frac{\partial}{\partial t}F_i(0,t)+ \tag{30}$$

$$+\lambda_i\int_0^{\mathrm{t}}\left[\sum_{j=1}^{4}R_{ij}(\tau)\frac{\partial}{\partial y}\varphi_j(y+\lambda_i\tau)+\frac{\partial^2}{\partial t\partial y}F_i(y+\lambda_i\tau,\tau)\right]d\tau, i=1,3;$$

$$-\sum_{j=1}^{4}c_{ij}\mathrm{w}_j(1,\mathrm{t})+\lambda_i\int_0^{\mathrm{t}}\sum_{j=1}^{4}c_{ij}\frac{\partial}{\partial y}\mathrm{w}_j(y+\lambda_i\tau,\tau)d\tau+$$

$$+\sum_{j=1}^{4}R_{ij}(t)\varphi_j(1)+\int_0^{\mathrm{t}}\sum_{j=1}^{4}R_{ij}(\tau)w_j(1,\mathrm{t}-\tau)d\tau-$$

$$-\lambda_i\int_0^{\mathrm{t}}\int_0^{\tau}\sum_{j=1}^{4}R_{ij}(\alpha)\frac{\partial}{\partial y}w_i(y+\lambda_i\tau,\tau-\alpha)d\alpha d\tau=\lambda_i\frac{d}{dy}\Phi_i(y)+$$

$$+\frac{\partial^2}{\partial t^2}\,\mathrm{h}_i(\mathrm{t})-\frac{\partial}{\partial t}F_i(1,t)+ \tag{31}$$

$$+\lambda_i\int_0^{\mathrm{t}}\left[\sum_{j=1}^{4}R_{ij}(\tau)\frac{\partial}{\partial y}\varphi_j(y+\lambda_i\tau)+\frac{\partial^2}{\partial t\partial y}F_i(y+\lambda_i\tau,\tau)\right]d\tau, i=2,4;$$

We write equalities (30)-(31) in the following compact form

$$-\sum_{j=1}^{4}c_{ij}\mathrm{w}_j(\theta_i,\mathrm{t})+\lambda_i\int_0^{\mathrm{t}}\sum_{j=1}^{4}c_{ij}\frac{\partial}{\partial y}\mathrm{w}_j(y+\lambda_i\tau,\tau)d\tau+$$

$$+\sum_{j=1}^{4}R_{ij}(t)\varphi_j(\theta_i)+\int_0^{\mathrm{t}}\sum_{j=1}^{4}R_{ij}(\tau)w_j(\theta_i,\mathrm{t}-\tau)d\tau-$$

$$-\lambda_i\int_0^{\mathrm{t}}\int_0^{\tau}\sum_{j=1}^{4}R_{ij}(\alpha)\frac{\partial}{\partial y}w_i(y+\lambda_i\tau,\tau-\alpha)d\alpha d\tau=\lambda_i\frac{d}{dy}\Phi_i(y)+$$

$$+\frac{\partial^2}{\partial t^2}\,\mathrm{h}_i(\mathrm{t})-\frac{\partial}{\partial t}F_i(\theta_i,t)+ \tag{32}$$

$$+\lambda_i\int_0^{\mathrm{t}}\left[\sum_{j=1}^{4}R_{ij}(\tau)\frac{\partial}{\partial y}\varphi_j(y+\lambda_i\tau)+\frac{\partial^2}{\partial t\partial y}F_i(y+\lambda_i\tau,\tau)\right]d\tau, i=\overline{1,4};$$

here

$$\theta_i = \begin{cases} 0, & i = 1,3; \\ 1, & i = 2,4; \end{cases} \quad \lambda_i = \begin{cases} 1, \sqrt{2}, & i = 1,3; \\ -1, -\sqrt{2}, & i = 2,4. \end{cases}$$

Let us introduce the following notation

$$\Upsilon = (\theta_i; \varphi(\theta_i)) := \left(\Upsilon_{il}(\theta_i; \varphi(\theta_i))\right)_{i,l=1}^{4} =$$

$$= \frac{1}{2}\begin{pmatrix} \varphi_1(\theta_1) + \varphi_2(\theta_1) & 0 & \varphi_1(\theta_1) - \varphi_2(\theta_1) & 0 \\ \varphi_1(\theta_2) + \varphi_2(\theta_2) & 0 & -\varphi_1(\theta_2) + \varphi_2(\theta_2) & 0 \\ 0 & \varphi_3(\theta_3) + \varphi_4(\theta_3) & 0 & \varphi_3(\theta_3) - \varphi_4(\theta_3) \\ 0 & \varphi_3(\theta_4) + \varphi_4(\theta_4) & 0 & -\varphi_3(\theta_4) + \varphi_4(\theta_4) \end{pmatrix},$$

$$\Upsilon = (y; \omega(y,t)) := (\Upsilon_{il}(y; \omega(y,t)))_{i,l=1}^{4} =$$

$$= \frac{1}{2}\begin{pmatrix} \omega_1(y,t) + \omega_2(y,t) & 0 & \omega_1 - \omega_2 & 0 \\ \omega_1(y,t) + \omega_2(y,t) & 0 & -\omega_1 + \omega_2 & 0 \\ 0 & \omega_3(y,t) + \omega_4 & 0 & \omega_3 - \omega_4 \\ 0 & \omega_3(y,t) + \omega_4 & 0 & -\omega_3 + \omega_4 \end{pmatrix}. \qquad (33)$$

Taking into account (33), we write equations (23) in the following form:

$$w_i(y,t) = w_i(y_0^i, t_0^i) + \int_{t_0^i}^{t} \left[\sum_{j=1}^{4} \Upsilon_{ij}(\mu; \varphi(\mu))\psi_j(\tau) - \sum_{j=1}^{4} c_{ij} w_i(\mu,\tau)\right]_{\mu=y+\lambda_i(\tau-t)} d\tau +$$
$$+ \int_{t_0^i}^{t} \int_0^{\tau} \sum_{j=1}^{4} \Upsilon_{ij}(\mu; w_i(\mu, \tau-\eta))\psi_j(\eta) d\eta\Big|_{\mu=y+\lambda_i(\tau-t)} d\tau + \int_{t_0^i}^{t} \frac{\partial}{\partial t} F_i(\mu,\tau)\Big|_{\mu=y+\lambda_i(\tau-t)} d\tau. \qquad (34)$$

Also using (33) system (32), we rewrite in the form:

$$\sum_{j=1}^{4} \Upsilon_{ij}(\theta_i; \varphi(\theta_i))\psi_j(t) = \lambda_i \int_0^t \frac{\partial}{\partial y} \sum_{j=1}^{4} \Upsilon_{ij}\left(\tau; \frac{\partial}{\partial y}\varphi_j(y+\lambda_i\tau)\right)\psi_j(\tau)d\tau -$$

$$- \int_0^t \sum_{j=1}^{4} \Upsilon_{ij}\left(\theta_i; w_j(\theta_i, t-\tau)\right)\psi_j(\tau)d\tau + \qquad (35)$$

$$+ \lambda_i \int_0^t \int_0^{\tau} \sum_{j=1}^{4} \Upsilon_{ij}\left(y; \frac{\partial}{\partial y} w_j(y, \tau-\alpha)\right)\psi_j(\alpha)\Bigg|_{y=\theta_i+\lambda_i(\tau-t)} d\alpha d\tau -$$

$$-\lambda_i \int_0^t \sum_{j=1}^{4} c_{ij} \frac{\partial}{\partial y} \mathrm{w}_j(y+\lambda_i\tau, \tau)d\tau + \lambda_i \frac{d}{dy}\Phi_i(y) + \frac{\partial^2}{\partial t^2}\, \mathrm{h}_i(t) - - \frac{\partial}{\partial t} F_i(\theta_i, t)$$

$$+ \sum_{j=1}^{4} c_{ij} \mathrm{w}_j(\theta_i, t) + \lambda_i \int_0^t \frac{\partial^2}{\partial t \partial y} F_i(y+\lambda_i\tau, \tau)d\tau, i = \overline{1,4}.$$

In what follows, we will assume that the conditions

$$det\Upsilon\left(\theta_j; \varphi(\theta_j)\right) = D_0 \neq 0,$$

$$\varphi_1(0)\varphi_1(1) \neq \varphi_2(0)\varphi_2(1) \quad \varphi_3(0)\varphi_3(1) \neq \varphi_4(0)\varphi_4(1). \tag{36}$$

Now solving system (35) with respect to $\psi_j(t)$, we obtain

$$\psi_j(t) = \frac{1}{D_0}\sum_{k=1}^{4}\left[\lambda_k\int_0^t\sum_{l=1}^{4}\Upsilon_{kl}\left(\tau;\frac{\partial}{\partial y}\varphi_l(y+\lambda_k\tau)\psi_l(\tau)d\tau\right]\mathrm{A}_{kj}\left(\theta_j;\varphi(\theta_j)\right) -$$

$$-\frac{1}{D_0}\sum_{k=1}^{4}\left[\lambda_k\int_0^t\sum_{l=1}^{4}\Upsilon_{kj}\big(\theta_k;\omega_l(\theta_k,t-\tau)\big)\psi_l(\tau)d\tau\right]\mathrm{A}_{kj}\left(\theta_j;\varphi(\theta_j)\right) +$$

$$+\frac{1}{D_0}\sum_{k=1}^{4}\left[\lambda_k\int\limits_0^t\int\limits_0^\tau\sum_{l=1}^{4}\Upsilon_{kl}\left(y;\frac{\partial}{\partial y}\omega_l(y,\tau-\alpha)\right)\psi_l(\phi)\right|_{y=\theta_j-\lambda_k(t-\tau)} d\alpha d\tau \mathrm{A}_{kj} -$$

$$-\frac{1}{D_0}\sum_{k=1}^{4}\left[\lambda_k\int\limits_0^t\sum_{l=1}^{4}c_{kl}\frac{\partial}{\partial y}\omega_l(y+\lambda_k t,\tau)\psi_l d\tau\right]\mathrm{A}_{kj}\left(\theta_j;\varphi(\theta_j)\right) + \tag{37}$$

$$+\frac{1}{D_0}\sum_{k=1}^{4}\left[\lambda_k\frac{\partial}{\partial y}\Phi_k(y)+\frac{\partial^2}{\partial t^2}h_k(t)-\frac{\partial}{\partial y}F_k(\theta_k,t)\right]\mathrm{A}_{kj}\left(\theta_j;\varphi(\theta_j)\right) +$$

$$+\frac{1}{D_0}\sum_{k=1}^{4}\left[\sum_{l=1}^{4}c_{kl}\frac{\partial}{\partial y}\omega_l(\theta_j,t)+\lambda_k\int\limits_0^t\frac{\partial^2}{\partial t\partial y}F_k(y+\lambda_k t,\tau)d\tau\right]\mathrm{A}_{kj}\left(\theta_j;\varphi(\theta_j)\right),$$

where A_{kj} are algebraic complements of elements Υ_{ij}-matrices. Equations (37) include unknown functions $\frac{\partial}{\partial y}w_j$, $j = \overline{1,4}$. For them we find the integral equations from (34) by differentiating them with respect to the variable y. At the same time, we have

$$\frac{\partial}{\partial y}w_i(y,t) = \frac{\partial}{\partial y}w_i(y_0^i,t_0^i) - \frac{\partial}{\partial y}t_0^i\left[\frac{\partial}{\partial t}F_i(y_0^i,t_0^i) - \sum_{j=1}^{4}c_{ij}w_j(y_0^i,t_0^i) +\right.$$

$$\left.+\sum_{j=1}^{4}\Upsilon_{ij}(y_0^i;\varphi)\psi_j(t_0^i)\right] + \int_{t_0^i}^{t}\left[\frac{\partial}{\partial t\partial y}F_i(\mu,\tau) - \sum_{j=1}^{4}c_{ij}\frac{\partial}{\partial y}w_j(\mu,\tau) +\right.$$

$$+\frac{\partial}{\partial y}t_0^i\int_0^{t_0^i}\sum_{j=1}^{4}\Upsilon_{ij}\left(y_0^i;G_j(y_0^i,t_0^i-\tau)\right)\psi_j(\tau)d\tau + \tag{38}$$

$$+\int_{t_0^i}^{t}\int_0^{\tau}\sum_{j=1}^{4}\frac{\partial}{\partial y}\Upsilon_{ij}\left(\mu;w_j(\mu,\tau-\eta)\right)\psi_j(\eta)d\eta\Bigg|_{\mu=y+\lambda_i(\tau-t)} d\tau, i=\overline{1,4}$$

where

$$G_j\left(y_0^i,t_0^i-\tau\right)=\begin{cases}\frac{d}{dt}h_j\left(\frac{1-y}{\lambda_i}-\tau\right), j=2,4,\\ \frac{d}{dt}g_j\left(\frac{1-y}{\lambda_i}-\tau\right), j=1,3.\end{cases}$$

We require compliance with the conditions of agreement

$$[\frac{d}{dt}\,\mathrm{h}_i(t)]_{t=0}=F_i(0,0)-\lambda_i\frac{\partial}{\partial y}\varphi_i(y)\Big|_{y=0}-\sum_{j=1}^{4}c_{ij}\varphi_j(0), i=2,4, \qquad (39)$$

$$[\frac{d}{dt}\,\mathrm{h}_i(t)]_{t=0}=F_i(1,0)-\lambda_i\frac{\partial}{\partial y}\varphi_i(y)\Big|_{y=1}-\sum_{j=1}^{4}c_{ij}\varphi_j(1), i=1,3. \qquad (40)$$

Main result and its proof

The main result of this work is the following assertion:

Theorem 2. Let the conditions of Theorem 1 be satisfied, besides $\varphi(y)\in C^2[0,1], g(t)\in C^2[0,T]$, $h(t)\in C^2[0,T]$, and condition (36) and matching conditions (39), (40) are satisfied. Then on the interval [0,1] there is a unique solution of the inverse problem (19)-(22), from the class $\Psi(t)\in C[0,1]$, and each component $\psi_i\in C[0,1]$] is defined by specifying $h_i(t)$ for $t\in[0,1], i=\overline{1,4}$.

Proof. Consider now the square

$$\Pi_0:=\{(y,t): 0\le y\le 1, 0\le t\le 1\}.$$

Equations (34),(37) and (38), supplemented by the initial and boundary conditions from equality (19), form in Π_0 a closed system of equations with respect to the unknowns $w_i(y,t),\psi_i(\mathrm{t}),\frac{\partial}{\partial y}w_i(y,t), i=\overline{1,4}$.

Equations (34), (37) and (38) show that the values of the functions $w_i(y,t),\psi_i(\mathrm{t}),\frac{\partial}{\partial y}w_i(y,t), i=\overline{1,4}$ at $(\mathrm{y},\mathrm{t})\in\Pi_0$. are expressed in terms of integrals of some combinations of the same functions over segments lying in Π_0.

We write equations (34), (37), and (38) as a closed system of Voltaire-type integral equations of the second kind. To do this, we introduce into consideration the vector function $v(y,t)=\left(v_i^1,v_i^2,v_i^3\right), i=\overline{1,4}$ defining their components by equalities

$$v_i^1(y,t) = w_i(y,t), v_i^2(y,t) = \psi_i(t),$$

$$v_i^3(y,t) = \frac{\partial}{\partial y} w_i(y,t) + \frac{\partial}{\partial y} t_0^i \sum_{j=1}^4 \Upsilon_{ij}\left(y_0^i; \varphi\left(y_0^i\right)\right)\psi_j\left(t_0^i\right).$$

Then the system of equations (34),(37) and (38) takes the operator form

$$v = \mathrm{L}v \tag{41}$$

where operator $\mathrm{L} = \left(\mathrm{L}_i^1, \mathrm{L}_i^2, \mathrm{L}_i^3\right), i = \overline{1,4}$ in accordance with the right-hand sides of equations (34), (37) and (38) is defined by the equalities

$$L_i^1 v = v_i^{01}(y,t) + \int_{t_0^i}^{t} \left[\sum_{j=1}^4 \Upsilon_{ij}(\mu; \varphi(\mu)) v_j^2(\tau) - \sum_{j=1}^4 c_{ij} v_j^1(\mu,\tau)\right]_{\mu = y + \lambda_i(\tau - t)} d\tau +$$

$$+ \int_{t_0^i}^{t} \int_0^{\tau} \sum_{j=1}^4 \Upsilon_{ij}\left(\mu; v_j^1(\mu, \tau - \eta)\right) v_j^2(\eta) d\eta \Bigg|_{\mu = y + \lambda_i(\tau - t)} d\tau, \quad i = \overline{1,4}. \tag{42}$$

$$L_i^2 v = v_i^{02}(y,t) + \frac{1}{\Upsilon_0}\int_0^t \sum_{k=1}^4 \sum_{l=1}^4 \lambda_k \Upsilon_{kl}\left(\tau; \frac{\partial}{\partial y}\varphi_l(y + \lambda_k \tau)\right) v_l^1(\tau) d\tau \mathrm{A}_{ki}\left(\theta_i; \varphi(\theta_i)\right)$$

$$-\frac{1}{\Upsilon_0}\int_0^t \sum_{k=1}^4 \sum_{l=1}^4 \lambda_k \Upsilon_{kl}\left(\theta_k; \frac{d}{dt} h_l(\theta_k, t - \tau)\right) v_l^2(\tau) d\tau \mathrm{A}_{ki}\left(\theta_i; \varphi(\theta_i)\right) + \tag{43}$$

$$+\frac{1}{\Upsilon_0}\int_0^t \int_0^\tau \sum_{k=1}^4 \sum_{l=1}^4 \lambda_k \Upsilon_{kl}\left(y; \frac{\partial}{\partial y} v_l^1(y, \tau\right.$$

$$\left. - \alpha)\right) v_l^2(\tau)\Bigg|_{y = \theta_i - \lambda_k(t - \tau)} d\alpha d\tau \mathrm{A}_{ki}\left(\theta_i; \varphi(\theta_i)\right) -$$

$$-\frac{1}{\Upsilon_0}\sum_{k=1}^4 \left[\int_0^t \sum_{l=1}^4 \lambda_k c_{kl} \frac{\partial}{\partial y} v_l^1(y + \lambda_k t, \tau) v_l^2(\tau) d\tau\right] \mathrm{A}_{ki}\left(\theta_i; \varphi(\theta_i)\right), \quad i = \overline{1,4}.$$

$$L_i^3 v = v_i^{03}(y,t)$$

$$+ \int_{t_0^i}^t \left[\sum_{j=1}^4 \frac{\partial}{\partial y} \Upsilon_{ij}(\mu; \varphi(\mu)) v_j^2(\tau) - \sum_{j=1}^4 c_{ij} \frac{\partial}{\partial y} v_j^1(\mu,\tau)\right]\Bigg|_{\mu = y + \lambda_i(\tau - t)} d\tau$$

$$-$$

$$-\frac{\partial}{\partial y}t_0^i\int_0^{t_0^i}\sum_{j=1}^{4}\Upsilon_{ij}\left(y_0^i;G\left(y_0^i,t_0^i-\tau\right)\right)v_j^2(\tau)d\tau+ \tag{44}$$

$$+\int_{t_0^i}^{t}\int_0^{\tau}\sum_{j=1}^{4}\frac{\partial}{\partial y}\Upsilon_{ij}\left(\mu;w_j(\mu,\tau-\eta)\right)v_j^2(\eta)d\eta\Bigg|_{\mu=y+\lambda_i(\tau-t)}d\tau,\quad i=\overline{1,4}.$$

The following notations are introduced in these formulas:

$$v_i^{01}(y,t)=w_i\left(y_0^i,t_0^i\right)+\int_{t_0^i}^{t}\frac{\partial}{\partial t}F_i(\mu,\tau)\Bigg|_{\mu=y+\lambda_i(y-t)}d\tau,$$

$$v_i^{02}(y,t)=\frac{1}{\Upsilon_0}\sum_{k=1}^{4}\left[\lambda_k\frac{\partial}{\partial y}\Phi_k(y)+\frac{\partial^2}{\partial t^2}h_k(t)-\frac{\partial}{\partial y}F_k(\theta_k,t)\right]\mathrm{A}_{ki}\left(\theta_i;\text{ц}(\theta_i)\right)+$$

$$+\frac{1}{\Upsilon_0}\sum_{k=1}^{4}\left[\sum_{l=1}^{4}c_k\frac{\partial}{\partial y}h_l(\theta_i,t)+\lambda_k\int_0^t\frac{\partial^2}{\partial t\partial y}F_k(y+\lambda_k t,\tau)d\tau\right]\mathrm{A}_{ki}\left(\theta_i;\varphi(\theta_i)\right),$$

$$v_i^{03}(y,t)=\frac{\partial}{\partial y}w_i\left(y_0^i,t_0^i\right)-\frac{\partial}{\partial y}t_0^i\left[\frac{\partial}{\partial t}F_i\left(y_0^i,t_0^i\right)-\sum_{j=1}^{4}c_{ij}w_i\left(y_0^i,t_0^i\right)+\right.$$

$$\left.+\sum_{j=1}^{4}\Upsilon_{ij}\left(y_0^i;\text{ц}\left(y_0^i\right)\right)\psi_j\left(t_0^i\right)\right]+\int_{t_0^i}^{t}\frac{\partial}{\partial t\partial y}F_i(\mu,\tau)\Bigg|_{\mu=y+\lambda_i(\tau-t)}.$$

On the set of continuous functions C_σ (Π0), we define the norm

$$\| v \|_\sigma=\max_{1\le i\le 4,1\le l\le 3}\sup_{(y,t)\in\Pi_0}\left|v_i^l(y,t)e^{-\sigma t}\right|,$$

$\sigma\geq 0$ -some number to be chosen later.

Obviously, for $\sigma=0$ this space coincides with the space of continuous functions with the usual norm $\| v \|_\sigma$. Because of the inequalityc

$$e^{-\sigma}\| v \|\le\| v \|_\sigma\le\| v \|,$$

norm $\| v \|_\sigma$ and$\| v \|$ equivalent.

Next, consider the set of functions $S(v^0,r)\subset C_\sigma(\Pi_0)$, satisfying the inequality

$$\|v-v^0\|_\sigma\le r, \tag{45}$$

where is a vector function $v^0(y,t)=\left(v_i^{01}(y,t), v_i^{02}(t), v_i^{03}(y,t)\right), i=\overline{1,4}$ is determined by the free terms of the operator equation (43). It is easy to see that $v \in S(v^0,r)$ satisfies the estimate$\parallel v \parallel_\sigma \leq \|v^0\|_\sigma + r \leq \|v^0\| + r := r_0$. So r_0 –is a known number.

Let us introduce the following notation:

$$\varphi_0 := \max_{1\le i\le 4}\|\varphi_i\|_{C^2[0,1]}, g_0 := \max_{1\le i\le 4}\|g_i\|_{C^2[0,1]}, F_0 := \max_{1\le i\le 4}\|F_i\|_{C^2[\Pi_0]}, h_0 := \max_{1\le i\le 4}\|h_i\|_{C^2[0,1]},$$

$$\Upsilon_0\varphi_0 = \max_{1\le i,j\le 4}\left\|\Upsilon_{ij}(y+\lambda_i(\tau-t);\varphi)\right\|_{C^1(\Pi_0)}, A_0 := \max_{1\le i,j\le 4}\left\{\left|A_{\mathrm{ij}}\left(\theta_{\mathrm{i}};\varphi(\theta_{\mathrm{i}})\right)\right|\right\}.$$

The operator A takes the space $C_\sigma(\Pi_0)$ into itself. Let us show that, given an appropriate choice of σ, it is a contraction operator on the set $S(v^0,r)$. Let us first verify that the operator A maps the set $S(v^0,r)$ into itself, i.e. it follows from the condition $v(y,t) \in S(v^0,r)$that $Av \in S(v^0,r)$, if σ satisfies some restrictions. Indeed, for any $(y,t) \in \Pi_0$ and any $v \in S(v^0,r)$ the inequalities hold:

$$\left|\left(L_i^1 v - v_i^{01}\right)e^{-\sigma t}\right| = \left|\int_{t_0}^t \sum_{j=1}^4 \Upsilon_{ij}(\mu;\varphi(\mu))e^{-\sigma(t-\tau)}v_j^2(\tau)e^{-\sigma t}\right|_{\mu=y+\lambda_i(\tau-t)} d\tau -$$

$$-\int_{t_0^i}^t \sum_{j=1}^4 c_{ij}e^{-\sigma(t-\tau)}v_j^1(\mu,\tau)e^{-\sigma t}\Bigg|_{\mu=y+\lambda_i(\tau-t)} d\tau +$$

$$+\int_{t_0^i}^t \int_0^\tau \sum_{j=1}^4 \Upsilon_{ij}(\mu;v_i^1(\mu,\tau-\eta))e^{-\sigma(\tau-\eta)}v_j^2(\eta)e^{-\sigma\eta}d\eta\Bigg|_{\mu=y+\lambda_i(\tau-t)} d\tau\Bigg| \leq$$

$$\leq 4\left[(\Upsilon_0\varphi_0 + c_0)\parallel v \parallel_{\mathrm{y}} + \Upsilon_0 \parallel v \parallel_\sigma^2\right]\int_0^t e^{-\sigma(t-\tau)}d\tau \leq$$

$$\leq \frac{4}{\sigma}\left((\Upsilon_0\varphi_0 + c_0) + \Upsilon_0 r_0\right)r_0 := \frac{1}{\sigma}\alpha_{11},$$

$$\left|\left(L_i^2 v - v_i^{02}(y,t)\right)e^{-\sigma t}\right|$$

$$= \left|\frac{1}{D_0}\int_0^t \sum_{k=1}^4 \sum_{l=1}^4 \lambda_k\Upsilon_{kl}\left(y+\lambda_k\tau;\frac{\partial}{\partial y}\varphi_l(y+\lambda_k\tau)\right)e^{-\sigma(t-\tau)} \times\right.$$

$$\times v_l^2(\tau)e^{-\sigma\tau}d\tau \mathrm{A}_{ki}\left(\theta_i;\varphi(\theta_i)\right)$$

$$-\frac{1}{D_0}\int_0^t \sum_{k=1}^4 \sum_{l=1}^4 \lambda_k\Upsilon_{kl}\left(\theta_k;\frac{d}{dt}h_l(\theta_k,t-\tau)\right)e^{-\sigma(t-\tau)} \times$$

$$\times v_l^2(\tau)e^{-\sigma\tau}d\tau \mathrm{A}_{ki}\big(\theta_i;\varphi(\theta_i)\big)$$

$$+\frac{1}{D_0}\int_0^t\int_0^\tau\sum_{k=1}^4\sum_{l=1}^4\lambda_k \Upsilon_{kl}\left(y;\frac{\partial}{\partial y}v_l^1(y,\tau-\alpha)\right)e^{-\sigma(t-\tau)}\times$$

$$\times v_l^2(\tau)e^{-\sigma\tau}\Big|_{y=\theta_i-\lambda_k(t-\tau)}d\alpha d\tau \mathrm{A}_{ki}\Bigg(\theta_i;\varphi(\theta_i)-\frac{1}{D_0}\int_0^t\sum_{k=1}^4\sum_{l=1}^4\lambda_k c_{kl}e^{-\sigma(t-\phi)}\times$$

$$\times\left[v_l^3(y+\lambda_k t,\tau)-\frac{\partial}{\partial y}t_0^i\sum_{p=1}^4\Upsilon_{lp}\left(y_0^i;\varphi\big(y_0^i\big)\right)v_p^2\big(t_0^i\big)\right]e^{-\sigma\tau}d\tau \mathrm{A}_{ki}\big(\theta_i;\varphi(\theta_i)\big)\Bigg|\le$$

$$\le\frac{16A_0}{D_0}[\Upsilon_0(\varphi_0+h_0+\|v\|_\sigma)+c_0(1+\Upsilon_0\varphi_0)]\|v\|_\sigma\int_0^t e^{-\sigma(t-\tau)}d\tau\le$$

$$\le\frac{16A_0}{\sigma D_0}[\Upsilon_0(\varphi_0+h_0+r_0)+c_0(1+\Upsilon_0\varphi_0)]r_0:=\frac{1}{\sigma}\alpha_{12},$$

$$|\big(L_i^3v-v_i^{03}\big)e^{-\sigma t}|=\Big|\int_{t_0^i}^t\sum_{j=1}^4\frac{\partial}{\partial y}\Upsilon_{ij}(\mu;\varphi(\mu))e^{-\sigma(t-\tau)}v_j^2(\tau)e^{-\sigma t}\Big|_{\mu=y+\lambda_i(\tau-t)}d\tau-$$

$$-\int_{t_0^i}^t\sum_{j=1}^4 c_{ij}e^{-\sigma(t-\tau)}\Bigg(v_j^3$$

$$-\frac{\partial}{\partial y}t_0^i\int_0^{t_0^i}\sum_{p=1}^4\Upsilon_{jp}\left(y_0^j;\varphi\big(y_0^j\big)\right)v_p^2\big(t_0^i\big)\Bigg)e^{-\sigma\tau}\Bigg|_{\mu=y+\lambda_i(\tau-t)}$$

$$\times d\tau-\frac{\partial}{\partial y}t_0^i\int_0^{t_0^i}\sum_{j=1}^4\Upsilon_{ij}\left(y_0^i;G\big(y_0^i,t_0^i-\tau\big)\right)e^{-\sigma(t-\tau)}v_j^2(\tau)e^{-\sigma\tau}d\tau+$$

$$+\int_{t_0^i}^t\int_0^\tau\sum_{j=1}^4\frac{\partial}{\partial y}\Upsilon_{ij}\big(\mu;v_j^1(\mu,\tau-\eta)\big)e^{-\sigma(\tau-\eta)}v_j^2(\eta)e^{-\sigma\eta}d\eta\Bigg|_{\mu=y+\lambda_i(t-t)}d\tau\Bigg|\le$$

$$\le 4[\Upsilon_0(\varphi_0+h_0+\|v\|_\sigma)+c_0(1+\Upsilon_0\varphi_0)]\|v\|_\sigma\int_0^t e^{-\sigma(t-\tau)}d\tau\le$$

$$\leq \frac{4}{\sigma}[\Upsilon_0(\varphi_0 + h_0 + r_0) + c_0(1 + \Upsilon_0\varphi_0)]r_0 := \frac{1}{\sigma}\alpha_{13}.$$

From here and from formulas (41) and (42)-(44) it follows that

$$\|\mathrm{L}v - v^0\|_\sigma = \max\left\{\max_{1\leq i\leq 4}\sup_{(y,t)\in\Pi_0}\left|\left(\mathrm{L}_i^1 v - v_i^{01}\right)e^{-\sigma t}\right|, \max_{1\leq i\leq 4}\sup_{t\in[0,1]}\left|\left(\mathrm{L}_i^2 v - v_i^{02}\right)e^{-\sigma t}\right|,\right.$$

$$\left.\max_{1\leq i\leq 4}\sup_{t\in[0,1]}\left|\left(\mathrm{L}_i^3 v - v_i^{03}\right)e^{-\sigma t}\right|\right\} \leq \frac{1}{\sigma}\alpha_0,$$

where $\alpha_0 := \max(\alpha_1, \alpha_2, \alpha_3)$. Choosing $\sigma > (1/r)\alpha_0$, we obtain that the operator L takes the set $S(v^0, \rho)$ into itself.

Let us now take any functions $v, \tilde{v} \in S(v^0, r)$ and estimate the norm of the difference $Uv - U\tilde{v}$. Using the obvious inequality

$$\left|v_i^k v_i^l - \tilde{v}_i^k \tilde{v}_i^l\right|e^{-\sigma t} \leq \left|v_i^k - \tilde{v}_i^k\right|\left|v_i^l\right|e^{-\sigma t} + \left|\tilde{v}_i^k\right|\left|v_i^l - \tilde{v}_i^l\right|e^{-\sigma t} \leq 2r_0 \parallel v - \tilde{v} \parallel_\sigma,$$

and estimates for the integrals similar to those given above, we obtain

$$\left|(L_i^1 v - L_i^1\tilde{v})e^{-\sigma t}\right| =$$

$$= \left|\int_{t_0^i}^t \sum_{j=1}^4 \Upsilon_{ij}(\mu; \varphi(\mu))e^{-\sigma(t-\tau)}\left(v_j^2(\tau) - \tilde{v}_j^2(\tau)\right)e^{-\sigma t}\right|_{\mu=y+\lambda_i(\tau-t)} d\tau -$$

$$-\int_{t_0^i}^t \sum_{j=1}^4 c_{ij}e^{-\sigma(t-\tau)}\left(v_j^1(\mu,\tau) - \tilde{v}_j^1(\mu,\tau)\right)e^{-\sigma t}\Bigg|_{\mu=y+\lambda_i(\tau-t)} d\tau +$$

$$+\int_{t_0^i}^t \int_0^\tau \sum_{j=1}^4 \Upsilon_{ij}(\mu; v_i^1(\mu, \tau-\eta))e^{-\sigma(\tau-\eta)}\left(v_j^2(\eta) - \tilde{v}_j^2(\eta)\right)e^{-\sigma\eta}d\eta\Bigg|_{\mu=y+\lambda_i(\tau-t)} d\tau\Bigg| \leq$$

$$\leq 4[(\Upsilon_0\varphi_0 + c_0) \parallel v - \tilde{v} \parallel_\sigma + 2r_0\Upsilon_0 \parallel v - \tilde{v} \parallel_\sigma]\int_0^t e^{-\sigma(t-\tau)}d\tau \leq$$

$$\leq \frac{4}{\sigma}\left((\Upsilon_0\varphi_0 + c_0) + 2\Upsilon_0 r_0\right) \parallel v - \tilde{v} \parallel_\sigma := \frac{1}{\sigma}\alpha_{21} \parallel v - \tilde{v} \parallel_\sigma.$$

Similarly, we obtain the following estimates

$$\left|\left(L_i^2 v - L_i^2 v\right)e^{-\sigma t}\right| \leq \frac{16A_0}{\sigma D_0}[\Upsilon_0(\varphi_0 + h_0 + 2r_0) + c_0(1 + \Upsilon_0\varphi_0)] \parallel v - \tilde{v} \parallel_\sigma :$$

$$= \frac{1}{\sigma}\alpha_{22} \parallel v - \tilde{v} \parallel_\sigma,$$

$$\left|\left(L_i^3 v - L_i^3 v\right)e^{-\sigma t}\right| \le \frac{4}{\sigma}\left[\Upsilon_0(\varphi_0 + h_0 + 2r_0) + c_0(1 + \Upsilon_0\varphi_0)\right] \parallel v - \tilde{v} \parallel_\sigma := $$

$$= \frac{1}{\sigma}\alpha_{2_3} \parallel v - \tilde{v} \parallel_\sigma .$$

Hence we have

$$\parallel \mathrm{L}v - \mathrm{L}\tilde{v} \parallel_\sigma = \max\left\{\max_{1\le i\le 4}\sup_{(y,t)\in\Pi_0}\left|\left(\mathrm{L}_i^1 v - \mathrm{L}_i^1\tilde{v}\right)e^{-\sigma t}\right|, \max_{1\le i\le 4}\sup_{t\in[0,1]}\left|\left(\mathrm{L}_i^2 v - \mathrm{L}_i^2\tilde{v}\right)e^{-yt}\right|,\right.$$

$$\left.\max_{1\le i\le 4}\sup_{t\in[0,1]}\left|\left(\mathrm{L}_i^3 v - \mathrm{A}_i^3\tilde{v}\right)e^{-\sigma t}\right|\right\} \le \frac{1}{\sigma}\beta_0 \parallel v - \tilde{v} \parallel_\sigma ,$$

Where $\beta_0 := \max(\beta_1, \beta_2, \beta_3)$. Choosing now $\sigma > \beta_0$, we get that the operator L shrinks the distance between elements $v, \tilde{v}$ by $S(v^0, \rho)$.

As follows from the above estimates, if the number σ is chosen from the condition $\sigma > \sigma^* := \max\{\alpha_0, \beta_0\}$, then the operator L is contractive on $S(v^0, \rho)$. In this case, according to the Banach principle [40, pp. 87-97], equation (41) has a unique solution in $S(v^0, \rho)$.Theorem 2 is proved.

REFERENCES:

1. Yu. A. Altukhov, A. S. Gusev, G. V. Pyshnogray, Introduction to the Mesoscopic Theory of Fluid Polymer Systems, Nauka, AltGPA Publishing House, Barnaul, 2012.

2. A. M. Blokhin, N. V. Bambaeva, Stationary solutions of the equations of incompressible viscoelastic polymer fluid, Comput. mat. and mat. physical, 2014, vol. 54, no. 5, pp. 55-69.

3. A.I. Leonov, A.N. Prokunin, Nonlinear phenomena in flows of viscoelastic polymer fluids., Chapman and Hall, New York, 1994.475 p.

4. J.G. Oldroyd, On the formulation of rheological equations of state, Proc. R. Soc., M, 1950. V.200. No 1063. P.523-541.

5. R.B. Bird, C.F. Curtiss, R.C. Armstrong, O Hassager., ynamics of Polymeric Liquids, D Vol. 2. Wiley, New York., 1987.464 p..

6. M. Doi, S.F. Edwards, The Theory of Polymer Dynamics., Oxford, Clarendon, 1986. 391 p.

7. V. Volterra, Theory of functionals, integral and integro-differential equations, Science, Ch. ed. fiz. -mat. letters, M, 1982.

8. Mura. Toshio, Micromechanics of defects in solids, Second, Revised Edition, IL, USA, Northwestern University, Evanston, 1987.

9. L. D. Landau, E. M. Lifshitz, Electrodynamics of continuous media, Science, M, 1959.

10. S. K. Godunov, Equations of Mathematical Physics (2nd ed.), Science, M, 1979.

11. A. A. Kilbas, Integral equations: a course of lectures, Minsk, 2005.

12. F. R. Gantmakher, Matrix theory, Science, Ch. ed. Phys.-Math. lit., M, 1988.

13. V. G. Romanov, "Inverse problems for equation with a memory", Eurasian Jour. of Math. and Computer Applications, 2:4 (2014), 51-80.

14. V. G. Romanov, "Problem of determining the permittivity in the stationary system of Maxwell equations", Dokl. Math., 95:3 (2017), 230-234.

15. A. Lorenzi, "An identification problem related to a nonlinear hyperbolic integro-

differential equation", Nonlinear Anal., Theory, Methods Appl., 22:1 (1994), 21-44.

16. *D. K. Durdiev, J. D. Totieva, "Zadacha ob opredelenii odnomernogo yadra uravneniya vyazkouprugosti", Sib. jurn. industr. matem., 16:2 (2013), 72-82.*

17. *V. G. Yaxno, Obratnie zadachi dlya differentsialnix uravneniy uprugosti, Nauka, Novosibirsk, 1989.*

18. *J. Janno, L. Von Wolfersdorf, "Inverse problems for identification of memory kernels in viscoelasticity", Math. Methods Appl. Sci., 20:4 (1997), 291-314.*

19. *V. G. Romanov, "Otsenki ustoychivosti resheniya v zadache ob opredelenii yadra uravneniya vyazkouprugosti", Sib. jurn. industr. matem., 15:1 (2012), 86-98.*

20. *D. K. Durdiev, J. D. Totieva, "Zadacha ob opredelenii mnogomernogo yadra uravneniya vyazkouprugosti", Vladikavk. matem. jurn., 17:4 (2015), 18-43.*

21. *D. K. Durdiev, "Some multidimensional inverse problems of memory determination in hyperbolic equations", Jurn. matem. fiz., anal., geom., 3:4 (2007), 411-423.*

22. *D. K. Durdiev, J. Sh. Safarov, "Obratnaya zadacha ob opredelenii odnomernogo yadra uravneniya vyazkouprugosti v ogranichennoy oblasti", Matem. zametki., 97:6 (2015), 855-867.*

23. *D. K. Durdiev, Z. R. Bozorov, "Zadacha opredeleniya yadra integrodifferentsialnogo volnovogo uravneniya so slabo gorizontalnoy odnorodnostyu", Dalnevost. matem. jurn., 13:2 (2013), 209-221.*

24. *V. G. Romanov, "Zadacha ob opredeleniya yadra v uravnenii vyazkouprugosti", Dokl. AN., 446:1 (2012), 18-20.*

25. *D. K. Durdiev, A. A. Raxmonov, "Obratnaya zadacha dlya sistemi integro-differentsialnix uravneniy SH-voln v vyazkouprugoy poristoy srede: globalnaya razreshimost", TMF., 195:3 (2018), 491-506.*

26. *D. K. Durdiev, A. A. Raxmonov, "Zadacha ob opredelenii dvumernogo yadra v sisteme integro-differentsialnix uravneniy vyazkouprugoy poristoy sredi", Sib. jurn. industr. matem., 23:2 (2020), 63-80.*

27. *D. K. Durdiev, A. A. Rahmonov, "A 2D kernel determination problem in a visco-elastic porous medium with a weakly horizontally inhomogeneity", Mathematical Methods in the Applied Sciences, 43:15 (2020), 8776-8796.*

28. *D. K. Durdiev, Z. D. Totieva, "The problem of determining the one-dimensional matrix kernel of the system of viscoelasticity equations", Mathematical Methods in the Applied Sciences, 41:17 (2018), 8019-8032.*

29. *Z. D. Totieva, D. K. Durdiev, "The Problem of Finding the One-Dimensional Kernel of the Thermoviscoelasticity Equation", Mathematical Methods in the Applied Sciences, 103:1-2 (2018), 118-132.*

30. *Z. S. Safarov, D. K. Durdiev, "Inverse Problem for an Integro-Differential Equation of Acoustics", Differential Equations, 54:1 (2018), 134-142.*

31. *D. K. Durdiev, Z. D. Totieva, "The problem of determining the one-dimensional kernel of viscoelasticity equation with a source of explosive type", Journal of Inverse and Ill-Posed Problems, 28:1 (2020), 43-52.*

32. *U. D. Durdiev, "Obratnaya zadacha dlya sistemi uravneniy vyazkouprugosti v odnorodnix anizotropnix sredax", Sib. jurn. industr. matem., 22:4 (2019), 26-32.*

33. *R. V. Brizitskiy, A. S. Savenkova, "O regulyarnosti resheniya odnoy kraevoy zadachi dlya uravneniy Maksvella", Dalnevost. matem. jurn., 9:1-2 (2009), 24-28.*

34. *I. V. Proxorov, V. V. Zolotarev, I. B. Agafonov, "Zadacha akusticheskogo zondirovaniya vo fluktuiruyushem okeane", Dalnevost. matem. jurn., 11:1 (2011), 76-87.*

35. *M. K. Teshaev, I. I. Safarov, N. U. Kuldashov, M. R. Ishmamatov, T. R. Ruziev, "On the Distribution of Free Waves on the Surface of a Viscoelastic Cylindrical Cavity", Journal of Vibrational Engineering and Technologies, 8:4 (2020), 579-585..*

36. *I. Safarov, M. Teshaev, E. Toshmatov, Z. Boltaev, F. Homidov, "Torsional vibrations of a cylindrical shell in a linear viscoelastic medium", IOP Conference Series: Materials Science and Engineering, 883:1 (2020), 12-19.*

37. M. K. Teshaev, I. I. Safarov, M. Mirsaidov, "Oscillations of multilayer viscoelastic composite toroidal pipes", Journal of the Serbian Society for Computational Mechanics, 13:2 (2019), 104-115.

38. M. K. Teshaev, "Realization of servo-constraints by electromechanical servosystems",Russian Mathematics., 13:2 (2019), 104-115.

39. V. G. Romanov, A. L. Karchevsky, "Determination of permittivity and conductivity of medium in a vicinity of a well having complex profile", Eurasian Journal of Mathematicaland Computer Applications, 6:4 (2018), 62-72.

40. A. L. Karchevsky, "A numerical solution to a system of elasticity equations for layered anisotropic media", Russian Geology and Geophysics, 46:3 (2005), 339-351.

41. A. L. Karchevsky, "The direct dynamical problem of seismics for horizontally stratified media", Sib.Electron. Mat. Izv., 2:2 (2005), 23-61.

42. A. L. Karchevsky, "Numerical solution to the one-dimensional inverse problem for an elastic system", Dokl. Earth Sci., 3:8 (2000), 23-61.

43. A. L. Karchevsky, "Simultaneous reconstruction of permittivity and conductivity", Inverse and III-Posed Probl., 17:4 (2009), 387-404.

44. E. Kurpinar, A. L. Karchevsky, "Numerical solution of the inverse problem for the elasticity system for horizontally stratified media", Inverse and III-Posed Probl., 20:3 (2004), 953-976.

45. A. L. Karchevsky, "Numerical reconstruction of medium parameters of member of thin anisotropic layers", Inverse and III-Posed Probl., 12:5 (2004), 519-534.

46. U. D. Durdiev, "Numerical method for determining the dependence of the dielectric permittivity on the frequency in the equation of electrodynamics with memory", Sib. Electron. Mat. Izv., 17 (2020), 179-189.

47. Z. R. Bozorov, "Numerical determining a memory function of a horizontally-stratified elastic medium with aftereffect", Eurasian journal of mathematical and computer applications, 8:2 (2020), 4-16.

48. D. K. Durdiev, X. X. Turdiev, "Obratnaya zadacha dlya giperbolicheskoy sistemi per-vogo poryadka s pamyatyu", Differentsialnie uravneniya, 56:12 (2020), 1666-1675.

49. D. K. Durdiev, Kh. Kh. Turdiev, "The problem of finding the kernels in the system of integro-differential Maxwell's equations", Sib. Zh. Ind. Math., 24:2 (2021), 38-61.

50. A. N. Kolmogorov, S. V. Fomin, Elementi teorii funktsiy i funktsionalnogo ana- liza, Nauka, Gl. red. fiz.-mat. lit., M, 1989