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MUNDARIJA * СОДЕРЖАНИЕ *** CONTENTS**

ANIQ VA TABIIY FANLAR *** EXACT AND NATURAL SCIENCES *** ТОЧНЫЕ И ЕСТЕСТВЕННЫЕ НАУКИ

Rahmonova Ch.F., Niyozova Sh.A., Zokirov B.U., Ashirov O.N., Hasanov Sh.Sh., Abduraxmanov J.M.	Mahalliy sharoitlarda an'anaviy usullarda tayyorlangan sut-qatiq mahsulotlarini mikrobiologik tadqiq qilish	4
Imomova Sh.M., Rahmonqulova Z.B.	Funksiyalarni Mathcad muhitida sonli integrallash	9
Jumaev J.J., Atoev D.D.	Investigation of the integro-differential equation of parabolic type with nonlocal condition	15
Muxtarov Y., Xudoyberdiyev S.S.	Chiziqli bir jinsli tenglamalarni yechishda darajali qatorlarni qo'llash	20
Nuriddinov J.Z., Xasanova M.A., Qarshiboyeva Sh.Q.	O'zgaruvchan koeffitsiyentli parabolik tipdagi integro-differensial tenglama yadrosining yagonaligi to'g'risida	24
Muxtarov Y., O'roqov N.O'.,	Chiziqli differensial tenglamalarni yechishda operator usulini qo'llash	33
Primov T.I.	Iqtisodiy jarayonlarning eng sodda matematik modellarini axborot texnologiyalarini qo'llash orqali tuzish	37
Quldoshova M.J., Rahmonov E.S.	Issiqlik almashish jarayonini boshqarish masalasi	40
Rahmatov S.E., Quronboyev D.D.	The cross section of recoil emission process of Ag(110) crystal	44
Rahmonov E.S., Jamolov Sh.J.	Birinchi skalyar ko'paytmaga ko'ra ortogonal aks ettirishlar $O(1,Y)$ gruppasi	50
Tolipov N.I.	Doira tashqarisida to'rtınchi tartibli ellıptik tipdagi differensial tenglama uchun qo'yilgan ichki - chegaraviy masala	54
Мухаммадиева Д.К., Муродиллоева З.Ҳ.	Применение свёрточных нейронных сетей для диагностирования рака лёгких на ранних стадиях	61
Tursunov A.R., Axmedov A.A., Toshboboyev Sh.M.	Yupqa plyonkalar strukturasini tahlil qilish	67
Turapova A.U.	Ikki o'lchamli panjarada lokal bo'lmagan Shredinger operatorining xos qiymati uchun asimptotik yoyilmalar	72
Дурдиев Д.Қ., Турдиев Х.Х., Тошев Д.А.	Решение задачи типа Коши-Гурса для уравнения гиперболического типа	89
Raximova S.M., Suvonova S.A.	Maydon tranzistoriga optik signallar ta'sirini o'rganish	96

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INVESTIGATION OF THE INTEGRO-DIFFERENTIAL EQUATION OF PARABOLIC TYPE WITH NONLOCAL CONDITION

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Abstract: *This paper studied the existence and uniqueness of a solution of the second order integro-differential parabolic equation with nonlocal initial-boundary conditions. Firstly, the problem is replaced with the equivalent integral equation with respect to unknown function u(x,t) using the Fourier method. Then, using Schauder principle and Gronwall inequalities, the local existence and uniqueness are proven for equivalent integral equation.*

Keywords: *integro-differential equation, nonlocal initial-boundary problem, inverse problem, integral equation, Schauder principle.*

ИССЛЕДОВАНИЕ ИНТЕГРОДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ПАРАБОЛИЧЕСКОГО ТИПА С НЕЛОКАЛЬНЫМ УСЛОВИЕМ

Аннотация: *В работе исследовано существование и единственность решения интегро-дифференциального параболического уравнения второго порядка с нелокальными начально-краевыми условиями. Во-первых, задача заменяется эквивалентным интегральным уравнением относительно неизвестной функции* $u(x,t)$ *методом Фурье. Затем с помощью принципа Шаудера и неравенств Гронуолла доказывается локальное существование и единственность решения эквивалентного интегрального уравнения.*

Ключевые слова: *интегро-дифференциальное уравнение, нелокальная начально-краевая задача, обратная задача, интегральное уравнение, принцип Шаудера.*

NOLOKAL SHARTLI PARABOLIK TURLI INTEGRO-DIFFERENSIAL TENGLAMANI TEKSHIRISH

Annotatsiya: *Ushbu maqolada nolokal boshlang'ich-chegaraviy shartli ikkinchi tartibli integro-differensial parabolik tenglama yechimining mavjudligi va yagonaligi o'rganilgan. Dastlab, masala Furye usuli yordamida noma'lum* $u(x,t)$ *funksiyaga nisbatan ekvivalent integral tenglama bilan almashtiriladi. Keyin Shauder prinsipi va Gronwall tengsizliklari yordamida ekvivalent integral tenglama uchun mavjudlik va yagonalik isbotlanadi.*

Kalit so'zlar: *integro-differensial tenglama, nolokal boshlang'ich-chegaraviy masala, teskari masala, integral tenglama, Shaulder prinsipi.*

1. Introduction and setting up the problem

Today in theory of mathematical physics equations, investigations devoted to the direct and inverse problems took an important place. This problems arise in situations, when the structure of the mathematical model of the studying process is known and it is necessary to set the problems of determining the parameters of the mathematical model itself. Such problems include the problems of determining the various kernel, leading and lower coefficients of the equations, nonlocal initial and boundary conditions, and so on (see [1]).

Problems with nonlocal conditions for partial differential equations have been studied by many authors. In the articles [2]-[5] were considered with the study of the unique solvability of a nonlocal inverse boundary value problems for second-order hyperbolic equation with overdetermination conditions. These problems the existence and uniqueness theorem for the classical solution of the inverse coefficient problem is proved.

The inverse problem of determining the time-dependent thermal diffusivity and the temperature distribution in a parabolic equation in the case of nonlocal initial-boundary conditions containing a real parameter and integral overdetermi-nation conditions are investigated in the works [6]-[12].

The problem of determining the kernel $k(t)$ of the integral term in an integro-differential heat equation were studied in many publications [13]–[17] , in which both one- and multidimensional inverse problems with classical initial, initial-boundary conditions were investigated. There is proven existence and uniqueness of inverse problem solutions.

In this article we study an inverse problem in integro-differential equation for second-order parabolic equation with nonlocal initial-boundary condition. The inverse problem of determination of the kernel $k(t)$ function in the one - dimensional integro – differential parabolic equation existence and uniqueness of this problem solution is studied.

Let $T > 0$ is fixed number and $D_{Tl} = \{(x,t): 0 < x < l, 0 < t \leq T\}$. Consider the inverse problem of determining of functions $u(x,t), k(t)$ such that it satisfies the equation

$$u_t - u_{xx} = \int_0^t k(t-\tau)u(x,\tau)d\tau, \quad (x,t) \in D_{Tl}, \tag{1.1}$$

with the nonlocal initial condition

$$u(x,0) + \lambda u(x,T) + \int_0^T p(\tau)u(x,\tau)d\tau = \varphi(x), \quad x \in [0,l], \tag{1.2}$$

the boundary conditions

$$u|_{x=0} = u|_{x=l} = 0, \quad t \in [0,T], \tag{1.3}$$

here $\lambda \geq 0$ is a given number greater than zero, $\varphi(x), p(t)$ are given functions of $x \in [0,l]$ and $t \in [0,T]$.

In the direct problem, for given numbers l, T, λ and sufficiently smooth functions $k(t), \varphi(x)$, it required to find a function $u(x,t) \in C^{2,1}(D_T)$ satisfying nonlocal initial-boundary problem (1.1)-(1.3) for $(x,t) \in D_T$.

Let $C^m(0;l)$ be the class of m times continuously differentiable with all derivatives up to the m −th order (inclusive) in $(0;l)$ functions. In the case $m = 0$ this space coincides with the class of continuous functions. $C^{m,k}(D_T)$ is the class of m times continuously differentiable with respect to x and k times continuously differentiable with respect to t all derivatives in the domain D_T functions.

2. Solvability of the problem

The solution of equation (1.1) with the nonlocal initial condition (1.2) and the boundary conditions (1.3) satisfies the relation

$$u(x,t) = \Phi(x,t) + \int_0^t \int_0^l G(x,\xi,t-\beta) \int_0^\beta k(\beta-\tau)u(\xi,\tau)d\tau d\xi d\beta -$$

$$-\lambda \int_0^T \int_0^l G_0(x,\xi,t+T-\beta) \int_0^\beta k(\beta-\tau)u(\xi,\tau)d\tau d\xi d\beta -$$

$$-\int_0^T \int_0^\mu \int_0^l p(\mu)G_0(x,\xi,t+\mu-\beta) \int_0^\beta k(\beta-\tau)u(\xi,\tau)d\tau d\xi d\beta d\mu, \tag{2.1}$$

where

$$\Phi(x,t) = \int_0^l \varphi(\xi)G_0(x,\xi,t)d\xi,$$

$$G(x,\xi,t) = \frac{2}{l}\sum_{n=1}^{\infty} e^{-(\frac{\pi n}{l})^2 t} sin\frac{\pi n}{l}\xi sin\frac{\pi n}{l}x,$$

$$G_0(x,\xi,t) = \frac{2}{l(1+\lambda e^{-(\frac{\pi n}{l})^2 T} + \int_0^T p(\tau)e^{-(\frac{\pi n}{l})^2 \tau}d\tau)}\sum_{n=1}^{\infty} e^{-(\frac{\pi n}{l})^2 t} sin\frac{\pi n}{l}\xi sin\frac{\pi n}{l}x.$$

Now we write property of Green function which will be needed in the future.

Remark. *The integral of the Green function does not exceed 1:*

$$\int_0^l G_0(x,\xi,t)d\xi \le \int_0^l G(x,\xi,t)d\xi \le 1, x \in (0,l), t \in (0,T].$$

Denote the operator taking the function $u(x,t)$ to the right-hand side of (2.1) by A. Then (2.1) is written as the operator equation

$$u = Au, \tag{2.2}$$

Let

$\Phi_0 = \max_{(x,t)\in \overline{D}_T} |\Phi(x,t)|,\quad k_0 = \max_{t\in[0,T]} |k(t)|,\quad p_0 = \max_{t\in[0,T]} |p(t)|,$

$S_d(0) = \{u: \| u \| \le d\}$, d be some positive number.

We use the Schauder principle to the existence of solution of the operator equation (2.2).

Theorem (Schauder principle)[see 10, p. 411]: *Let the operator A map a closed bounded convex set $S_d(0)$ of a Banach space X into itself. Then if A is completely continuous on $S_d(0)$, then it has a fixed point on $S_d(0)$.*

Lemma 2.1. *Suppose that the following conditions are satisfied: $\varphi(x) \in C[0,l]$, $k(t) \in C^1[0,T]$, $\varphi(0) = \varphi(l) = 0$. Then for all T satisfying the estimate*

$0 < T \le T_1$, (2.3)

where T_1 is a positive root of the equation

$$2p_0T^3 + 3k_0d(1+\lambda)T^2 + 6(\Phi_0 - d) = 0,$$

the operator A is uniformly bounded and equicontinuous.

Then there exists a classical solution of problem (1.1)-(1.3) in the space $C^{2,1}(D_T)$.

Proof. First, we establish the uniform boundedness of the operator A. To this end, we show that there exists a $\rho \in (0,d]$ such that $\| Au \| \le \rho$, where $\| Au \| = \max_{(x,t)\in \overline{D}_T} |Au|$. For $u \in S_d(0)$ and $(x,t) \in \overline{D}_T$, we find estimate

$$\| Au \| \le \max_{(x,t)\in \overline{D}_T} |\Phi(x,t)| + \max_{(x,t)\in \overline{D}_T} \left| \int_0^t \int_0^l G(x,\xi,t-\beta) \int_0^\beta k(\beta-\tau)\big(u_1(\xi,\tau) - u_2(\xi,\tau)\big) d\tau d\xi d\beta \right|$$

$$+ \max_{(x,t)\in \overline{D}_T} |\lambda \int_0^T \int_0^l G_0(x,\xi,t+T-\beta) \int_0^\beta k(\beta-\tau)(u_1(\xi,\tau) - u_2(\xi,\tau)) d\tau d\xi d\beta| +$$

$$+ \max_{(x,t)\in \overline{D}_T} | \int_0^T \int_0^\mu \int_0^l p(\mu) G_0(x,\xi,t+\mu-\beta) \int_0^\beta k(\beta-\tau)(u_1(\xi,\tau) - u_2(\xi,\tau)) d\tau d\xi d\beta d\mu| \le$$

$$\le \Phi_0 + k_0 d\frac{T^2}{2} + \lambda k_0 d\frac{T^2}{2} + p_0 k_0 d\frac{T^3}{3} = \Phi_0 + k_0 d(1+\lambda)\frac{T^2}{2} + p_0 k_0 d\frac{T^3}{3} \equiv d.$$

For T that satisfy the estimate (2.3), the operator A is uniformly bounded.

Definition. *An operator A is said to be equicontinuous if for each $\varepsilon > 0$ there exists a $\delta_0 = \delta_0(\varepsilon) > 0$ such that the inequality*

$$\| Au_1 - Au_2 \| \le \varepsilon \tag{2.3}$$

holds for all $u_1, u_2 \in S_d(0)$ with $\| u_1 - u_2 \| \le \delta_0$.

We consider the estimates

$\| Au_1 - Au_2 \| \le$

$$\le \max_{(x,t)\in \overline{D}_T} | \int_0^t \int_0^l G(x,\xi,t-\beta) \int_0^\beta k(\beta-\tau)(u_1(\xi,\tau) - u_2(\xi,\tau)) d\tau d\xi d\beta| +$$

$$+ \max_{(x,t)\in \overline{D}_T} |\lambda \int_0^T \int_0^l G_0(x,\xi,t+T-\beta) \int_0^\beta k(\beta-\tau)(u_1(\xi,\tau) - u_2(\xi,\tau)) d\tau d\xi d\beta| +$$

$$+ \max_{(x,t)\in \overline{D}_T} | \int_0^T \int_0^\mu \int_0^l p(\mu) G_0(x,\xi,t+\mu-\beta) \int_0^\beta k(\beta-\tau)(u_1(\xi,\tau) - u_2(\xi,\tau)) d\tau d\xi d\beta d\mu| \le$$

$$\leq (1+\lambda+p_0\frac{2T}{3})k_0\frac{T^2}{2} \parallel u_1-u_2 \parallel \leq (1+\lambda+p_0\frac{2T}{3})k_0\frac{T^2}{2}\delta.$$

Consequently, if we take $\delta_0=\frac{2\varepsilon}{(1+\lambda+p_0\frac{2T}{3})k_0T^2}$, then inequality (2.3) will hold for $\delta\in(0,\delta_0]$, the operator A is equicontinuous. Then the operator A is completely continuous on S_d, and it has at least one fixed point on S_d by the Schauder principle. The proof of the lemma is complete.

Thus, Lemma 2.1 imply the following assertion on the existence of a solution of the operator equation (2.2).

Now show that this solution is the only one. Suppose that there are two solutions $U^1(x,t)$ and $U^2(x,t)$. Then their difference $Z(x,t)=U^2(x,t)-U^1(x,t)$ is a solution to the equation

$$Z(x,t)=\int_0^t\int_0^l G(x,\xi,t-\beta)\int_0^\beta k(\beta-\tau)Z(\xi,\tau)d\tau d\xi d\beta -$$

$$-\lambda\int_0^T\int_0^l G_0(x,\xi,t+T-\beta)\int_0^\beta k(\beta-\tau)Z(\xi,\tau)d\tau d\xi d\beta -$$

$$-\int_0^T\int_0^\mu\int_0^l p(\mu)G_0(x,\xi,t+\mu-\beta)\int_0^\beta k(\beta-\tau)Z(\xi,\tau)d\tau d\xi d\beta d\mu,$$

Let $\tilde{Z}(t)$ denote the supremum of the module of the function $Z(x,t)$ for $x\in(0,l)$ at each fixed $t\in(0,T)$. Then we have the inequality

$$\tilde{Z}(t)\leq 2k_0(1+\lambda+p_0T)T\int_0^T\tilde{Z}(\tau)d\tau, \qquad t\in[0,T].$$

***Lemma (**Gronwall**) [see 18].** Let $u(x)$ and $v(x)$ nonnegative piecewise continuous functions on $[x_0,x]$ for which the inequality*

$$u(x)=D+\left|\int_{x_0}^x u(t)v(t)dt\right|,$$

Holds, where D and x_0 are nonnegative constants. Then,

$$u(x)\leq De^{\left|\int_{x_0}^x v(t)dt\right|}.$$

Applying the above lemma 2.2 here, we obtain that $\tilde{Z}(t)=0$ for $t\in(0,T)$, which means that $Z(x,t)=0$ in D_T, i.e. $U^2(x,t)=U^1(x,t)$ in D_T. Therefore, equation (2.1) has a unique solution in D_T. The Lemma 2.1 is proved.

Conclusion

In this work, it is studied the existence and uniqueness of a solution of the second order integro-differential parabolic equation with nonlocal initial-boundary conditions. Firstly, the problem is replaced with the equivalent integral equation with respect to unknown function u(x,t) using the Fourier method. Then, using Schauder principle and Gronwall inequalities, the local existence and uniqueness are proved for equivalent integral equation.

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