Министерство науки и высшего образования Республики Казахстан
Комитет науки
Институт математики и математического моделирования
Механико-математический факультет
Казахского национального университета имени аль-Фараби

Традиционная международная апрельская математическая конференция
в честь Дня науки Республики Казахстан

ТЕЗИСЫ ДОКЛАДОВ

Алматы 2024

Funding: This research is funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (grant No. AP19675193).

Keywords: differential-algebraic equation, boundary value problem, Weierstrass canonical form, method of parameterization.

2010 Mathematics Subject Classification: 34A09, 34B05

References

[1] Kunkel P., Mehrmann V. *Differential-Algebraic Equations: Analysis and Numerical Solution*, European Mathematical Society, 2006.

[2] Dzhumabaev D. S. Criteria for the unique solvability of a linear boundary value problem for an ordinary differential equation, *USSR Comp. Math. Math. Phys.*, **29**:1 (1989), 34–46.

Determination of coefficients of fractional differential equations with the Generalized Riemann-Liouville Time Derivative

D.K. DURDIEV[1,2,a], H.H. TURDIEV[1,2,b]

[1]*Bukhara branch of the institute of Mathematics named after V.I. Romanovskiy at the Academy of sciences of the Republic of Uzbekistan, Bukhara, Uzbekistan*

[2]*Bukhara State University, Bukhara, Uzbekistan*

E-mail: [a]*d.durdiev@mathinst.uz*, [b]*h.h.turdiev@buxdu.uz*

Let $T > 0$, $l > 0$ be fixed numbers and $\Omega_{lT} := \{(x,t) : 0 < x < l, 0 < t \leq T\}$. Consider the time-fractional diffusion equation

$$D^{\alpha,\beta}_{0+,t}u(x,t) - u_{xx} + q(t)u(x,t) = p(t)f(x,t), \quad x \in (0,l),\ t \in (0,T], \tag{1}$$

the initial conditions of Cauchy type

$$I^{(2-\alpha)(1-\beta)}_{0+,t}u(x,t)\big|_{t=0} = \varphi_1(x),$$

$$\frac{\partial}{\partial t}\left(I^{(2-\alpha)(1-\beta)}_{0+,t}u\right)(x,t)\big|_{t=0} = \varphi_2(x),\ x \in [0,l], \tag{2}$$

the boundary conditions

$$u(0,t) = u(l,t) = 0, \quad 0 \leq t \leq T, \tag{3}$$

and the nonlocal additional condition

$$\int_0^l w_i(x)u(x,t)dx = h_i(t),\ i = 1,2,\ t \in [0,T]. \tag{4}$$

Here the generalized Riemann-Liouville (Hilfer) fractional differential operator $D^{\alpha,\beta}_{0+,t}$ of the order $1 < \alpha < 2$ and type $0 \leq \beta \leq 1$ (see, [1], pp. 112-118, [2], pp. 62-65).

Assume that throughout this article, given functions φ_1, φ_2, f, w and h satisfy the following assumptions:

A1) $\varphi_i \in C^3[0,l]$, $\varphi_i^{(4)} \in L_2[0,l]$, $\varphi_i(0) = \varphi_i(l) = 0$, $\varphi''{}_i(0) = \varphi''{}_i(l) = 0$, $i = 1,2$;

A2) $f(x,\cdot) \in C[0,T]$ *and for* $t \in [0,T]$, $f(\cdot,t) \in C^3[0,l]$, $f(\cdot,t)^{(4)} \in L_2[0,l]$, $f(0,t) = f(l,t) = 0$, $f_{xx}(0,t) = f_{xx}(l,t) = 0$;

A3) $w(x) \in C^2[0,l]$ *and* $w(0) = w(l) = 0$ *and* $w''(0) = w''(l) = 0$;

A4) $\left(D^{\alpha,\beta}_{0+,t}h\right)(t) \in C[0,T]$, $|h(t)| \geq h_0 > 0$, h_0 *is a given number,*

$$\int_0^l w_i(x)\varphi_1(x)dx = I^{(2-\alpha)(1-\beta)}_{0+,t}h_i(t)\big|_{t=0+},$$

$$\int_0^l w_i(x)\varphi_2(x)dx = \frac{\partial}{\partial t}\left(I_{0+,t}^{(2-\alpha)(1-\beta)}h_i(t)\right)(t)\Big|_{t=0+},\ i=1,2.$$

We consider the weighted spaces of continuous functions [[3], pp. 4-5, 162-163].

$$C_\gamma^{2,\alpha,\beta}(\Omega) = \Big\{u(x,t):\ u(\cdot,t)\in C^2(0,1);\ t\in[0,T] \text{ and}$$

$$D_{0+,t}^{\alpha,\beta}u(x,\cdot)\in C_\gamma(0,T];\ x\in[0,1],\ 1<\alpha\le 2,\ 0\le\beta\le 1\Big\},$$

where $\overline{\Omega}_{lT} := \{(x,t): 0\le x\le l,\ 0\le t\le T\}$.

The papers [3] and [4] study inverse problems of finding time-dependent source terms, respectively, in time-fractional diffusion equation by using eigenfunction expansion of the non-self adjoint spectral problem along the generalized Fourier method. The main results of these studies comprise the existence and uniqueness theorems, as well as a stability estimate for the solution of the problem of determining the coefficient in a time-fractional diffusion and wave equation.

Using the above results, we obtain the following assertion.

Lemma. Let $p(t)$, $q(t)\in C[0,T]$, A1)-A2) are satisfied, then there exists a unique solution of the direct problem (1)-(3) $u(x,t)\in C_\gamma^{2,\alpha,\beta}(\overline{\Omega}_{lT})$.

Theorem Let A1)-A4) are satisfied. Then there exists a number $T^*\in(0,T)$, such that there exists a unique solution $p(t),\ q(t)\in C[0,T^*]$ of the inverse problem (1)-(4).

Keywords: Hilfer fractional differential equation, Riemann-Liouville fractional derivative, inverse problem, initial conditions, boundary conditions.

2010 Mathematics Subject Classification: 34A08, 34K10, 34K29, 34K37, 34M50, 35R11.

References

[1] Hilfer R. *Applications of Fractional Calculus in Physics*, World Scientific, Singapore (2000).

[2] Podlubny I. *Fractional Differential Equations, of Mathematics in Science and Engineering, vol. 198*, Academic Press, New York, NY, USA (1999).

[3] Kilbas A.A., Srivastava H.M., Trujillo J.J. *Theory and application of fractional differetial equations Mathematical Studies*, Elsevier , Amsterdam (2006).

[4] Turdiev H.H. Inverse coefficient problems for a time-fractional wave equation with the generalized Riemann-Liouville time derivative, *Russian Mathematics (Izvestiya VUZ. Matematika)*, **10** (2023), 46–59.

[5] Durdiev D.K., Turdiev H.H. Inverse coefficient problem for fractional wave equation with the generalized Riemann–Liouville time derivative, *Indian J. Pure Appl. Math.*, https://doi.org/10.1007/s13226-023-00517-9 (2023)

Self-similar solutions for the membrane transverse vibration equation

A. HASANOV[a], S.G. RASHIDOV[b]

V.I.Romanovskiy Institute of Mathematics, Tashkent, Uzbekistan;

E-mail: [a]anvarhasanov@yahoo.com, [b]sardorrashidov1995@mail.ru

In our modern life, many problems of modern mathematics and theoretical physics lead to the investigation of hypergeometric functions of one and several variables, (for example) partial differential equations are obtainable with the help of such hypergeometric functions [1].In particular, the energy absorbed by some nonferromagnetic conductor sphere included in an internal magnetic field can be calculated with the help of such functions [2]. Hypergeometric functions of several variables are used in physical and quantum chemical applications as well [3]. Especially, many problems in gas dynamics lead to solutions of degenerate second-order partial differential equations which are then solvable in terms of multiple hypergeometric functions [5-6].

We consider and establish the solutions of the degenerating model equation in terms of the hypergeometric function ${}_2F_1\left(a,b;c;z\right)$.

When solving vibration problems, the model is obtained by calculating the transverse displacement $u\left(r,t\right)$ of a symmetrically deformed membrane. To process inhomogeneous waves $u\left(r,t\right)$ representing the frequency, the following equation is modeled and considered by J.Kastillo, C.Jiménez and R.Meléndez [4].

$$a^{-2}u_{tt}\left(r,t\right)=r^{2-2c}u_{rr}\left(r,t\right)+\left(1-2l\right)r^{1-2c}u_{r}\left(r,t\right)+\left(l^{2}-c^{2}\nu^{2}\right)r^{-2c}u\left(r,t\right) \tag{8}$$
$$\left(\nu,l,c=const>0\right),$$

where $a^2=\frac{T}{D}$, T membrana tension and D mass per unit area of the membrane.

We obtain the following special solutions of equation (8):

$$u_1\left(r,t\right)=r^{l-\nu c}t^{\nu}{}_2F_1\left(-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},1-\nu,\frac{r^{2c}}{a^2c^2t^2}\right), \tag{9}$$

$$u_2\left(r,t\right)=r^{l+c\nu}\left(\frac{1}{a^2c^2t}\right)^{\nu}\delta{}_2F_1\left(-\frac{\nu}{2}-\frac{1}{2},-\frac{\nu}{2},1+\nu,\frac{r^{2c}}{a^2c^2t^2}\right), \tag{10}$$

where ${}_2F_1\left(a,b;c;z\right)$ is Gaussian hypergeometric function with two numerator parameters and one denominator parameter.

Keywords: Parabolic PDE of degenerate type; Self-made solution; Linearly independent solution, Generalized hypergeometric function, Integral representation.

2010 Mathematics Subject Classification: 35L80, 33C05, 35C06.

References

[1] H. M. Srivastava., P. W. Karlsson *Multiple Gaussian Hypergeometric Series* Halsted, New York, Chichester, Brisbane, Toronto, 1985

[2] A. Erdelyi., W. Magnus., F. Oberhettinger., F. G. Tricomi *Higher Transcendental Functions*, McGrawHill, New York, Toronto, London, Vol.1. (1953).

[3] A. W. Niukkanen Generalized hypergeometric series $N\ F\left(x_1,...,x_N\right)$ arising in physical and quantum chemical applications, *J. Phys. A: Math. Gen*, **16**:2 (1983), 1813–1825.

[4] J. Kastillo., C.Jiménez., R.Meléndez Una transformada finita de Hankel generalizada *Matematicas: Ensenanza Universitaria Escuela Regional de Matematicas Universidad del Valle – Colombia* **17**:1 (2009), 13-21.

[5] A. Hasanov., E. T. Karimov Fundamental solutions for a class of three-dimensional elliptic equations with singular coefficients *Appl. Math. Lett.* **22**,(2009) 1828–1832.

[6] M. S. Salakhitdinov.,A. Hasanov The fundamental solution for one class of degenerate elliptic equations, *More Progresses in Analysis, Proceedings of the 5th International ISAAC Congress* (World Scientific, Singapore, 2009), 521–531.

Solvability of an inverse coefficient problem for a time-fractional diffusion equation with periodic boundary and integral overdetermination conditions

Jonibek JUMAEV

Institute of Mathematics named after V.I. Romanovskiy, Bukhara, Uzbekistan
Bukhara state university, Bukhara, Uzbekistan
E-mail: jonibekjj@mail.ru

We consider the initial-periodic boundary problem for the fractional diffusion equation

$$\partial_t^{\alpha}u-u_{xx}+a(t)u=f(x,t)g(t),\quad (x,t)\in D_T, \tag{1}$$

$$u(x,0)=\varphi(x),x\in[0,1], \tag{2}$$

$$u(0,t) = u(1,t), \quad u_x(0,t) = u_x(1,t), \varphi(0) = \varphi(1), \quad \varphi'(0) = \varphi'(1), t \in [0,T], \tag{3}$$

where ∂_t^α is the Caputo fractional derivative of order $0 < \alpha \leq 1$ in the time variable (see [1, pp. 90-94]), $a(t), g(t), t > 0$ are the source control terms, $f(x,t)$ is known source term, $\varphi(x)$ is the initial temperature, T is arbitrary positive number and $D_T := \{(x,t) : 0 < x < 1, 0 < t \leq T\})$.
.

The problem of determining a function $u(x,t), (x,t) \in D_T$, that satisfies (1)-(3) with known functions $a(t), g(t), f(x,t)$ and $\varphi(x)$ will be called the direct problem.

In the inverse problem, it is required to determine the coefficients $a(t), g(t), t > 0$, in (1) using over-determination conditions about the solution of the direct problem (1)-(3):

$$\int_0^1 \omega_i(x) u(x,t) dx = h_i(t), \; i = 1,2, \quad x \in [0,1], \tag{4}$$

where $\omega_i(x), h_i(t), i = 1,2$ are given functions.

By $C^{2,\alpha}(D_T)$ we denote the class of 2 times continuously differentiable with respect to x and α times continuously differentiable with respect to t in the domain D_T functions.

Definition 1. *The triple of functions $\{u(x,t), a(t), g(t)\}$ from the class $C^{2,\alpha}(D_T) \cap C^{1,0}(\overline{D}_T) \times C[0,T] \times C[0,T]$ is said to be a classical solution of problem (1)-(4), if the functions $u(x,t), a(t)$ and $g(t)$ satisfy the following conditions:*

(1) The function $u(x,t)$ and its derivatives $\partial_t^\alpha u(x,t), u_{xx}(x,t)$ are continuous in the domain D_T ;

(2) the function $a(t), g(t)$ is continuous on the interval $[0,T]$;

(3) equation (3) and conditions (2)-(4) are satisfied in the classical sense.

Throughout this article the functions φ, f, ω_i and h_i $(i := 1,2)$ are assumed to satisfy the following conditions:

$(A1)$ $\varphi(x) \in C^2(0,1)$; $\varphi^{(3)}(x) \in L_2(0,1)$; $\varphi(0) = \varphi(1)$; $\varphi'(0) = \varphi'(1)$; $\varphi''(0) = \varphi''(1)$; $\varphi^{(3)}(0) = \varphi^{(3)}(1)$;

$(A2)$ $f(x,t) \in C(\overline{D}_T) \cap C^{2,1}(D_T)$; $f^{(3)}(x,t) \in L_2(D_T)$; $f(0,t) = f(1,t)$; $f'(0,t) = f'(1,t)$; $f''(0,t) = f''(1,t)$;

$(A3)$ $h_i(t) \in AC[0,T]$; $\omega_i(x) \in C^2[0,1]$; $\omega_i^{(3)}(x,t) \in L_2[0,l]$; $\int_0^1 \omega_i(x)\varphi(x)dx = h_i(0)$; $\omega_i(0) = \omega_i(1)$; $\omega_i'(0) = \omega_i'(1)$; $\omega_i''(0) = \omega_i''(1), i = 1,2$.

Lemma 1. *Let $\{g(t), a(t)\} \in C[0,T], (A1), (A2)$ are satisfied, then there exists a unique solution of the direct problem (1)-(3) $u(x,t) \in C^{2,\alpha}(D_T) \cap C^{1,0}(\overline{D}_T)$.*

The main result of this work is presented as follows:

Theorem 1. *Let (A1)-(A4) are satisfied. Then there exists a number $T^* \in (0,T)$, such that there exists a unique solution $a(t)$, $g(t) \in C[0,T^*]$ of the inverse problem (1)-(4).*

For proving this theorem, inverse problem (1)-(4) reduces to the equivalent integral equations with respect unknown functions $u(x,t), a(t), g(t)$. For solving this equation the contracted mapping principle is applied. The local existence and uniqueness results are proven.

Funding: No funds, grants, or other support was received.

Keywords: time-fractional diffusion equation, periodic boundary conditions, inverse problem, integral equation.

2010 Mathematics Subject Classification: 35A01; 35A02; 35L02; 35L03; 35R03.

References

[1] Kilbas A.A., Srivastava H.M., Trujillo J.J., *Theory and Applications of Fractional Differential Equations.* Elsevier, Amsterdam, (2006).

[2] Kolmogorov A., Fomin S., *Elements of function theory and functional analysis,* Moscow: Nauka, (1972).(In Russian)

[3] Durdiev D.K., Jumaev J.J., Inverse Coefficient Problem for a Time-Fractional Diffusion Equation in the Bounded Domain, *Lobachevskii Journal of Mathematics*, **2**:44 (2023), 548-557.