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ABSTRACTS

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Section 6: Mathematical analysis and its applications

Let $E \subset \partial D$ be some subset of boundary of the domain $D \subset \mathbb{C}^n$ and ψ is a bounded and non-positive function in E. $\mathcal{U}(E, D, \psi)$ the class of all functions $u(z) \in sh_m(D)$, such that $u^*|_E \leq \psi|_E$, $u|_D < 0$ and let

$$\omega(z, E, D, \psi) = \sup\{u(z) : u(z) \in \mathcal{U}(E, D, \psi)\}.$$

Definition 1. The function

$$\omega^*(z, E, D, \psi) = \overline{\lim_{w \to z}} \omega(w, E, D, \psi)$$

is called a $(m, \psi)-$subharmonic measure of boundary set E with respect to D.

As can be seen from the definition, $\omega^*(z, E, D, \psi)$ is $m-$subharmonic and the inequality $\omega^*(z, E, D, \psi) \leq 0$ holds for all $z \in D$. Also, from the definition of the $(m, \psi)-$subharmonic of the boundary set, the following equivalence relation is valid.

$$-\inf_{\xi \in E} \psi(\xi) \cdot \omega^*(z, E, D) \leq \omega^*(z, E, D, \psi) \leq -\sup_{\xi \in E} \psi(\xi) \cdot \omega^*(z, E, D), \forall z \in D \quad (1)$$

holds for any set $E \subset \partial D$.

Let $K \subset \partial D$ be a compact.

Definition 2. A point $\xi \in K$ is said to be globally $(m, \psi)-$regular if $(\omega^*(\xi, K, D, \psi))^* = \psi(\xi)$. It is said to be locally $(m, \psi)-$regular if for any neighborhood B, $\xi \in B \subset \mathbb{C}^n$, the intersection $K \bigcap \bar{B}$ is globally $(m, \psi)-$ regular at the point ξ, i.e. $\left(\omega^*\left(\xi, K \bigcap \overline{B}, D, \psi\right)\right)^* = \psi(\xi)$. If all points of a compact K are $(m, \psi)-$regular, then the compact K is called a $(m, \psi)-$regular compact.

Theorem 1. Let $\psi \in C(K)$. A fixed point $\xi \in K \subset \partial D$ is locally $(m, \psi)-$regular if and only if it is locally $m-$regular, $\left(\omega^*\left(\xi, K \bigcap \overline{B}, D\right)\right)^* = -1$.

The proof of this theorem uses the inequality (1).

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About dynamic systems of a QnSO

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Consider the following QnSO on S^2:

$$W_0 : \begin{cases} x' = ax^2 + 2bxy + cy^2 \\ y' = (1-a)x^2 + 2(1-b)xy + (1-c)y^2 \\ z' = z(2-z). \end{cases} \quad (1)$$

where

$$a, c \in [0, 1], \quad b \in [-\sqrt{ac}, \quad 1 + \sqrt{(1-a)(1-c)}]. \quad (2)$$

Denote

$$\mathcal{Z} = \{(x, y, z) \in S^2 : z = 0\}.$$

It is easy to see that $\mathcal{Z}$ is an edge of the simplex and it is invariant: $z = 0 \quad \Rightarrow z' = 0$.

The fixed points are solutions to the system (1)

$$\begin{cases} x = ax^2 + 2bxy + cy^2 \\ y = (1-a)x^2 + 2(1-b)xy + (1-c)y^2 \\ z = z(2-z) \end{cases} \tag{3}$$

From the third equation of the system we find $z = 0$ and $z = 1$.

1) Case: If $z = 1$, then according to the definition of a simplex, i.e. $x + y + z = 1$, it follows that $x = y = 0$. Thus, the point $e_1 = (0, 0, 1)$ will be a fixed point of the operator W_0. That is, there is only one fixed point outside $\mathcal{Z}$.

2) Case: If $z = 0$, then the restriction of operator (1) to $\mathcal{Z}$ is an arbitrary one-dimensional QnSO in S^1:

$$\begin{cases} x' = ax^2 + 2bxy + cy^2 \\ y' = (1-a)x^2 + 2(1-b)xy + (1-c)y^2, \end{cases} \tag{4}$$

Since $x + y = 1$ the fixed point equation is reduced to one-dimensional form:

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Carleman's formula for a second kind matrix polydisk

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Consider the second kind classic domain [1,2]

$$D_2 = \left\{ Z_2 \in \mathbb{C}[m \times m] : Z_2 \overline{Z_2} < I \right\},$$

where $Z_2 - m \times m$ square matrix, $\overline{Z_2}-$conjugate matrix of the matrix Z_2, $I - m \times m$ identity matrix. Also take the skeleton [1,2] of the domain D_2

$$S_2 = \left\{ \xi_2 \in \mathbb{C}[m \times m] : \xi_2 \overline{\xi_2} < I \right\}.$$

Cartesian product of n copies of the aboveD_2 domain $\underbrace{D_2 \times D_2 \times \ldots D_2}_{n\,ta}$, is called a second kind matrix polydisk and denoted as T_2^n , i.e.

$$T_2^n = \left\{ Z = \left(Z_2^{(1)}, Z_2^{(2)}, \ldots, Z_2^{(n)} \right) \in \mathbb{C}^n[m \times m] : \; Z_2^{(j)} \overline{Z_2^{(j)}} < I, \, Z_2^{(j)} \in D_2 \right\}.$$

Similarly, we write the skeleton of this second kind matrix polydisk as the Cartesian product of n copies of S_2 and denote as $S\left(T_2^n\right)$, i.e.

$$S\left(T_2^n\right) = \left\{ \xi = \left(\xi_2^{(1)}, \xi_2^{(2)}, \ldots, \xi_2^{(n)} \right) \in \mathbb{C}^n[m \times m] : \; \xi_2^{(j)} \overline{\xi_2^{(j)}} = I, \, \xi_2^{(j)} \in S_2 \right\}.$$

Let $f\left(Z\right)$ be a function on T_2^n.