ABSTRACTS

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MATHEMATICAL ANALYSIS AND ITS APPLICATIONS IN MODERN MATHEMATICAL PHYSICS

PART I

Samarkand
September 23-24, 2022

MINISTRY OF HIGHER AND SECONDARY SPECIAL
EDUCATION OF THE REPUBLIC OF UZBEKISTAN

SAMARKAND STATE UNIVERSITY NAMED AFTER SH.RASHIDOV
MATHEMATICS INSTITUTE OF THE ACADEMY OF SCIENCE OF
UZBEKISTAN

INTERNATIONAL CONFERENCE

MATHEMATICAL ANALYSIS AND ITS APPLICATIONS IN MODERN MATHEMATICAL PHYSICS

September 23-24, 2022; Samarkand, Uzbekistan

PART I

SAMARKAND – 2022

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Mathematical analysis and its applications in modern mathematical physics: international scientific conference (September 23-24, 2022 y. Samarkand).

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The collection of abstracts of the reports of the international scientific conference "Mathematical analysis and its applications in modern mathematical physics" contains scientific reports on the following areas: Mathematical analysis, differential equations, mathematical physics, algebra, geometry, numerical mathematics, mathematical modeling, probability theory, mathematical statistics, mathematical engineering and information technologies .

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holds for $|p - p'| \to 0$ *and* $z \to -0$.

We remark that Theorems 1, 2 and 3 are play key role in the spectral analysis of the family of 3×3 operator matrices, associated with the lattice systems describing two identical bosons and one particle, another nature in interactions, without conservation of the number of particles.

Reference:

1. *S.N.Lakaev.* Some spectral properties of the generalized Friedrichs model, (Russian), Trudy Sem. Petrovsk. 11 (1986), pp. 210–238, Translation in J. Soviet Math. 45:6 (1989), pp. 1540–1565.

On invariant sets of a quadratic non-stochastic operator

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Non-linear dynamical systems arise in many problems of biology, physics and other sciences. In particular, quadratic dynamical systems describe the behavior of populations of different species with population models [1, 2, 3]. Let $E = \{1, 2, \dots, m\}$. A distribution on the set E is a probability measure $x = (x_1, \dots, x_m)$, i.e., an element of the simplex:

$$S^{m-1} = \{x \in R : x_i \geq 0, \sum_{i=1}^{m} x_i = 1\}.$$

In general, a quadratic operator V, $V : x \in R^m \to x' = V(x) \in R^m$ is defined by:

$$V : x_k' = \sum_{i,j=1}^{m} P_{ij,k} x_i x_j, \quad k = 1, \dots, m \tag{1}$$

In this talk we are interested to a non-stochastic quadratic mapping of simplex to itself, i.e. $V : S^{m-1} \to S^{m-1}$.

Definition. [3] A quadratic operator (1), preserving a simplex, is called non-stochastic (QnSO) if at least one of its coefficients $P_{ij,k}$, $i \neq j$ is negative.

Consider the following example of QnSO on the two-dimensional simplex S^2.

$$\begin{cases} x' = \frac{1}{2}(z-y)^2 + \frac{3}{2}x(y+z) \\ y' = \frac{1}{2}(x-z)^2 + \frac{3}{2}y(x+z) \\ z' = \frac{1}{2}(y-x)^2 + \frac{3}{2}z(x+y). \end{cases} \tag{2}$$

Fixed points. The fixed points are solutions to the system (2)

$$\begin{cases} x = \frac{1}{2}(z-y)^2 + \frac{3}{2}x(y+z) \\ y = \frac{1}{2}(x-z)^2 + \frac{3}{2}y(x+z) \\ z = \frac{1}{2}(y-x)^2 + \frac{3}{2}z(x+y). \end{cases}$$

By full analysis this system one obtains the following family of fixed points:

$$a_1 = (0, \frac{1}{2}, \frac{1}{2}), \quad a_2 = (\frac{1}{2}, 0, \frac{1}{2}), \quad a_3 = (\frac{1}{2}, \frac{1}{2}, 0), \quad a_4 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}).$$

Thus a_1, a_2 and a_3 are saddle, but a_4 is an attracting fixed point.

Invariant sets. Recall that a set M is called invariant with respect to an operator V if $V(M) \subseteq M$.

Introduce the following sets:

$$\begin{aligned}
M_1 &= \{(x,y,z) \in S^2 : x > y > z > 1/6\},\\
M_2 &= \{(x,y,z) \in S^2 : x > z > y > 1/6\},\\
M_3 &= \{(x,y,z) \in S^2 : y > x > z > 1/6\},\\
M_4 &= \{(x,y,z) \in S^2 : y > z > x > 1/6\},\\
M_5 &= \{(x,y,z) \in S^2 : z > x > y > 1/6\},\\
M_6 &= \{(x,y,z) \in S^2 : z > y > x > 1/6\}.
\end{aligned}$$

$$l_1 := \begin{cases} x = y \\ x+y+z = 1, \end{cases} \quad l_2 := \begin{cases} x = z \\ x+y+z = 1, \end{cases} \quad l_3 := \begin{cases} y = z \\ x+y+z = 1. \end{cases}$$

Theorem. The sets $M_i, i = 1,2,3,4,5,6$ are invariant with respect to the operator (2). Moreover, each median of the simplex S^2 is an invariant.

References

1. *Lyubich Yu.I.* Mathematical structures in population genetics, Springer-Verlag, 1992.
2. *Rozikov U.A.* Population dynamics: algebraic and probabilistic approach, *World Sci. Publ.* Singapore. 2020, 460 pp.
3. *Rozikov U.A., Xudayarov S.S.* Quadratic non-stochastic operators: examples of splitted chaos, Ann. Funct. Anal., **13**(1) (2022), Paper No. 17, 17 pp..

Comments on Chernoff and Trotter-Kato product formulæ

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The Chernoff $\sqrt{n}$-Lemma:

Lemma 1. *Let bounded operator C on a Banach space $\mathfrak{X}$ be a contraction, i.e., $\|C\| \le 1$. Then one has the estimate*

$$\|(C^n - e^{n(C-I)}x\| \le \sqrt{n}\,\|(C-I)x\|\,,\ x \in \mathfrak{X},\ n \in \mathbb{N}.$$

The following propositions revise the Chernoff $\sqrt{n}$-Lemma:

Proposition 1. Let C be contraction on a Banach space $\mathfrak{X}$. Then $\{e^{t(C-I)}\}_{t\ge 0}$ is a norm-continuous contraction semigroup on $\mathfrak{X}$ and one has the estimate

$$\|(C^n - e^{n(C-I)})\,x\| \le \frac{n}{\epsilon_n^2}\,2\,\|x\| + \epsilon_n\,\|(I-C)\,x\|\,, \quad n \in \mathbb{N}\setminus\{0\}\,,$$

for all $x \in \mathfrak{X}$ and $\epsilon_n > 0$. For optimal value of the *splitting* parameter ϵ_n one gets:

$$\|(C^n - e^{n(C-I)})\,\| \le \frac{3}{2}\,\sqrt[3]{n}\,\|2\,(I-C)\,\|^{2/3}\,,$$

which is called the $\sqrt[3]{n}$ -Lemma.

Proposition 2. *Let $C \in \mathcal{L}(\mathfrak{X})$ be contruction on a Banach space $\mathfrak{X}$. Then following estimate*

$$\|(C^n - e^{n(C-1)})\,x\| \le \frac{n}{2}\,(\|(C-1)^2\,x\| + \frac{e^2}{3}\,\|(C-1)^3\,x\|)\,,$$