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МАТЕМАТИКА, ФИЗИКА ВА АХБОРОТ ТЕХНОЛОГИЯЛАРИНИНГ ДОЛЗАРБ МУАММОЛАРИ

МАВЗУСИДАГИ РЕСПУБЛИКА МИҚЁСИДАГИ ОНЛАЙН ИЛМИЙ-АМАЛИЙ АНЖУМАНИ

ТЕЗИСЛАР ТЎПЛАМИ

ЎЗБЕКИСТОН РЕСПУБЛИКАСИ ОЛИЙ ВА
ЎРТА МАХСУС ТАЪЛИМ ВАЗИРЛИГИ

БУХОРО ДАВЛАТ УНИВЕРСИТЕТИ

ФИЗИКА-МАТЕМАТИКА ФАКУЛЬТЕТИ

“МАТЕМАТИКА, ФИЗИКА ВА АХБОРОТ ТЕХНОЛОГИЯЛАРИНИНГ ДОЛЗАРБ МУАММОЛАРИ”

мавзусидаги
Республика миқёсидаги онлайн илмий-амалий анжумани

ТЕЗИСЛАР ТЎПЛАМИ

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Анжуман Ўзбекистон Республикаси Президенти Шавкат Мирзиёевнинг 2020 йил 24 январдаги Олий Мажлисга йўллаган Мурожаатномаси ҳамда Ўзбекистон Республикаси Вазирлар Маҳкамасининг 2020 йил 7 февралдаги 56-Ф-сон фармойишига мувофиқ ташкил этилмоқда.

1. Дастурий қўмита

1. О.Х.Хамидов – университет ректори, раис.
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5. Т.Ҳ.Расулов – Математика кафедраси мудири, аъзо.
6. Ш.Ш.Файзиев – Физика кафедраси мудири, аъзо.
7. О.И.Жалолов – Ахборот технологиялари кафедраси мудири, аъзо.

2. Ташкилий қўмита

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10. Н.Ҳ.Маматова – Математика кафедраси доценти, аъзо.

Тезисларда келтирилган фактлар, иқтибосларнинг ҳаққонийлиги ва савияси ҳамда имло хатолари учун муаллифларнинг ўзлари масъулдирлар.

$$V_{a,b}(z):\begin{cases}V_a(z), \text{ if } x\le 1/2,\\ V_b(z), \text{ if } x>1/2.\end{cases}$$

We consider the dynamical system generated by the evolution operator $V_{a,b}$. Using the equality $x+y=1$, this operator can be reduced to the function $f_{a,b}:[0,1]\to[0,1]$ defined by

$$f_{a,b}(x)=\begin{cases}(a-1)x^2+1, \text{ if } x\le 1/2,\\ x(1+b-bx), \text{ if } x>1/2,\end{cases} \quad (5)$$

where $a\in[0,1], b\in[-1,1]$.

The following three cases are possible

(i) $0\le a\le 1$, $0<b\le 1$;

(ii) $0\le a\le 1, b=0$;

(iii) $0\le a\le 1$, $-1\le b<0$.

In this work we discuss the case (iii).

The case $0\le a\le 1$, $-1\le b<0$. In this case, we have the following

Theorem. *For the dynamical system generated by function (5) the following hold:*

1) $f_{a,b}$ *has unique fixed point* $x=1$;

2) if $0\le a<1$, *then* $f_{a,b}$ *has not a periodic point of period two;*

3) if $a=1$, *then for any initial point* $x^{(0)}$ *the trajectory* $x^{(n)}$ *has the following limit* $\lim\limits_{n\to\infty}x^{(n)}=1$.

4) if $0\le a\le \dfrac{-b^2+2\sqrt{b^4+2\sqrt{b^2+1}b^2-b^2}+2b}{b^2}$, *then* $f_{a,b}$ *has a periodic point of period three.*

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A CLASS OF QUADRATIC STOCHASTIC PROCESSES

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Denote by S the set of all possible kinds of stochasticity ([1],[2]) and denote by $\mathbf{M}$ the set of all possible multiplication rules of cubic matrices.

Let $M^{[s,t]}=\left(a_{ijk}^{[s,t]}\right)_{ijk=1}^m$ be a cubic matrix with two parameters $s\ge 0, t\ge 0$

Definition ([1]). A family $\{M^{[s,t]}:s,t\in R_+\}$ is called a quadratic stochastic process (QSP) of type $(\sigma\mid\mu)$ if for each time s and t the cubic matrix $M^{[s,t]}$ is stochastic in sense $\sigma\in S$ and satisfies the Kolmogorov-Chapman equation

$$M^{[s,t]}=M^{[s,\tau]} *_\mu M^{[\tau,t]}, \text{ for all } 0\le s<\tau<t \quad (1)$$

with respect to the multiplication $\mu\in\mathbf{M}$.

On the set $I=\{1,2,...,m\}$ consider binary operation $a(i,j)=\max\{i,j\}$. Let σ is a fixed stochasticity of cubic matrices then the QSP corresponding to Maksimov's multiplication (see section 2.2 of [2]) of $\max$ operation is denoted as type $(\sigma\,|\max)$. Here we give some examples of such QSP.

For simplicity we take $m=2$ and solve the equation (1) for matrix $M^{[s,t]}=\left(a_{ijk}^{[s,t]}\right)_{ijk=1}^{2}$. In this case (1) is in the following form

$$\begin{cases} a_{111}^{[s,t]}=a_{111}^{[s,\tau]}a_{111}^{[\tau,t]}+a_{112}^{[s,\tau]}a_{211}^{[\tau,t]} \\ a_{112}^{[s,t]}=a_{111}^{[s,\tau]}a_{112}^{[\tau,t]}+a_{112}^{[s,\tau]}a_{212}^{[\tau,t]} \\ a_{211}^{[s,t]}=a_{211}^{[s,\tau]}a_{111}^{[\tau,t]}+a_{212}^{[s,\tau]}a_{211}^{[\tau,t]} \\ a_{212}^{[s,t]}=a_{211}^{[s,\tau]}a_{112}^{[\tau,t]}+a_{212}^{[s,\tau]}a_{212}^{[\tau,t]} \\ a_{111}^{[s,t]}=a_{111}^{[s,\tau]}a_{121}^{[\tau,t]}+a_{112}^{[s,\tau]}a_{221}^{[\tau,t]}+a_{122}^{[s,\tau]}a_{221}^{[\tau,t]}+a_{121}^{[s,\tau]}a_{111}^{[\tau,t]}+a_{121}^{[s,\tau]}a_{121}^{[\tau,t]}+a_{122}^{[s,\tau]}a_{211}^{[\tau,t]} \\ a_{122}^{[s,t]}=a_{111}^{[s,\tau]}a_{122}^{[\tau,t]}+a_{121}^{[s,\tau]}a_{112}^{[\tau,t]}+a_{121}^{[s,\tau]}a_{122}^{[\tau,t]}+a_{112}^{[s,\tau]}a_{222}^{[\tau,t]}+a_{122}^{[s,\tau]}a_{212}^{[\tau,t]}+a_{122}^{[s,\tau]}a_{222}^{[\tau,t]} \\ a_{221}^{[s,t]}=a_{211}^{[s,\tau]}a_{121}^{[\tau,t]}+a_{212}^{[s,\tau]}a_{221}^{[\tau,t]}+a_{221}^{[s,\tau]}a_{111}^{[\tau,t]}+a_{221}^{[s,\tau]}a_{121}^{[\tau,t]}+a_{222}^{[s,\tau]}a_{211}^{[\tau,t]}+a_{222}^{[s,\tau]}a_{221}^{[\tau,t]} \\ a_{222}^{[s,t]}=a_{212}^{[s,\tau]}a_{222}^{[\tau,t]}+a_{221}^{[s,\tau]}a_{112}^{[\tau,t]}+a_{221}^{[s,\tau]}a_{122}^{[\tau,t]}+a_{222}^{[s,\tau]}a_{212}^{[\tau,t]}+a_{222}^{[s,\tau]}a_{222}^{[\tau,t]}+a_{211}^{[s,\tau]}a_{122}^{[\tau,t]} \end{cases} \tag{2}$$

Denoting

$$b_{ij}^{[s,t]}=a_{i1j}^{[s,t]}+a_{i2j}^{[s,t]},\quad B^{[s,t]}=(b_{ij}^{[s,t]}) \tag{3}$$

one can reduce the system (2) to the following one

$$\begin{cases} b_{11}^{[s,t]}=b_{11}^{[s,\tau]}b_{11}^{[\tau,t]}+b_{12}^{[s,\tau]}b_{21}^{[\tau,t]} \\ b_{12}^{[s,t]}=b_{11}^{[s,\tau]}b_{12}^{[\tau,t]}+b_{12}^{[s,\tau]}b_{22}^{[\tau,t]} \\ b_{21}^{[s,t]}=b_{21}^{[s,\tau]}b_{11}^{[\tau,t]}+b_{22}^{[s,\tau]}b_{21}^{[\tau,t]} \\ b_{22}^{[s,t]}=b_{21}^{[s,\tau]}b_{12}^{[\tau,t]}+b_{22}^{[s,\tau]}b_{22}^{[\tau,t]} \end{cases} \tag{4}$$

Note that 1-4 equations of the system (2) can be solved independently from 5-8 equations. Therefore if we solve system of 1-4 equations of (2) and solve system (4) then by (3) we can find all unknown functions of (2).

Denote $c_{ij}^{[s,t]}=a_{i1j}^{[s,t]},\ C^{[s,t]}=\left(c_{ij}^{[s,t]}\right)$ then 1-4 equations of the system (2) is

$$\begin{cases} c_{11}^{[s,t]}=c_{11}^{[s,\tau]}c_{11}^{[\tau,t]}+c_{12}^{[s,\tau]}c_{21}^{[\tau,t]} \\ c_{12}^{[s,t]}=c_{11}^{[s,\tau]}c_{12}^{[\tau,t]}+c_{12}^{[s,\tau]}c_{22}^{[\tau,t]} \\ c_{21}^{[s,t]}=c_{21}^{[s,\tau]}c_{11}^{[\tau,t]}+c_{22}^{[s,\tau]}c_{21}^{[\tau,t]} \\ c_{22}^{[s,t]}=c_{21}^{[s,\tau]}c_{12}^{[\tau,t]}+c_{22}^{[s,\tau]}c_{22}^{[\tau,t]} \end{cases} \tag{5}$$

Both system of equations (4) and (5) are Kolmogorov-Chapman equations for square matrices. Using known solutions for these equations, one can give concrete solutions of the system (2).

Namely, if $B^{[s,t]}=(b_{ij}^{[s,t]})$ is a solution to (4) and $C^{[s,t]}=(c_{ij}^{[s,t]})$ is a solution to (5) then corresponding solution to the system (2) is

$$M^{[s,t]}=\left(\begin{array}{cc|cc} c_{11}^{[s,t]} & c_{12}^{[s,t]} & c_{21}^{[s,t]} & c_{22}^{[s,t]} \\ b_{11}^{[s,t]}-c_{11}^{[s,t]} & b_{12}^{[s,t]}-c_{12}^{[s,t]} & b_{21}^{[s,t]}-c_{21}^{[s,t]} & b_{22}^{[s,t]}-c_{22}^{[s,t]} \end{array}\right) \tag{6}$$

Theorem. Let $B^{[s,t]}=(b_{ij}^{[s,t]})$ be a solution to (4) and $C^{[s,t]}=(c_{ij}^{[s,t]})$ be a solution to (5) with $c_{ij}^{[s,t]}\in[0,1]$ and for any $i,j=1,2,\ 0\le s<t$ then the family of matrices $M^{[s,t]}$ given in (6) is a QSP of type

- $(12\,|\max)$ iff $B^{[s,t]}$ is left stochastic for any $0\le s<t$.

- $(13|\max)$ iff $B^{[s,t]}$ (resp. $C^{[s,t]}$) with non negative elements with sum of all elements equals to 2 (resp. 1).

- $(23|\max)$ iff $B^{[s,t]}$ is right stochastic for any $0 \le s < t$.

- $(1|\max)$ iff $B^{[s,t]}$ (resp. $C^{[s,t]}$) with non negative elements with sum of all elements of each column equals to 2 (resp. left stochastic).

- $(2|\max)$ never.

- $(3|\max)$ iff $B^{[s,t]}$ (resp. $C^{[s,t]}$) with non negative elements with sum of all elements of each row equals to 2 (resp. right stochastic).

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ON THE MOORE-PENROSE AND CORE GENERALIZED INVERSES OF MATRICES

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The Moore-Penrose inverse celebrates its 100th birthday in 2020, as the notion standing behind the term was first defined by Moore in 1920 [1]. Its rediscovery by Penrose in 1955 [2] can be considered as a caesura after which the inverse attracted the attention it deserves and has henceforth been exploited in various research areas of applied origin. As specified by Penrose [2], the Moore-Penrose inverse of a matrix $K \in \mathbb{C}_{m,n}$ is the matrix $K^\dagger \in \mathbb{C}_{n,m}$ satisfying the equations

$$KK^\dagger K = K, \qquad K^\dagger K K^\dagger = K^\dagger, \qquad KK^\dagger = (KK^\dagger)^*, \qquad K^\dagger K = (K^\dagger K)^*.$$

An important fact is that the Moore-Penrose inverse of any matrix always exists and is unique. Another relevant feature is that the Moore-Penrose inverse can be used to represent orthogonal projectors, as the matrices $KK^\dagger$, $K^\dagger K$, $I_m - KK^\dagger$ and $I_n - K^\dagger K$ represent the orthogonal projectors, which project onto $\mathcal{R}(K)$, $\mathcal{R}(K^*)$, $\mathcal{N}(K^*)$, and $\mathcal{N}(K)$, respectively. In these expressions, I_p stands for the identity matrix of order p, whereas the symbols K^*, $\mathcal{R}(K)$, and $\mathcal{N}(K)$ stand for the conjugate transpose, range (column space), and null space of $K \in \mathbb{C}_{m,n}$. The Moore-Penrose inverse has proved to have many applications in various research areas of diverse origin, e.g., physics, statistics, robotics, neural networks, or digital image restoration.

Another generalized inverse of interest is the core inverse, which was first defined in [3] and since then attracted considerable attention. The core inverse exists only for square matrices of index one, and the core inverse of an index one matrix $K \in \mathbb{C}_{n,n}$ is the matrix $K^{\oplus} \in \mathbb{C}_{n,n}$ satisfying the conditions

$$KK^{\oplus} = KK^\dagger \text{ and } KK^\dagger K^{\oplus} = K^{\oplus}.$$

It is seen that the definition of the core inverse establishes its link with the Moore-Penrose inverse. The core inverse is also related with other important generalized inverses of matrices, such as the group inverse (a particular Drazin inverse) and the Bott-Duffin inverse, each of which has many relevant applications. Similarly as in the case of the Moore-Penrose inverse, also the core inverse is unique.

The talk will begin with a brief introduction to the theory of generalized inverses and afterwards will focus on selected, emerging problems. Considerable attention will be paid to various representations of the inverses. Possible areas of further research will be indicated as well.

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МУНДАРИЖА