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Section 1: Mathematical analysis and its applications

Foydalanilgan adabiyotlar

1. M. Muminov, H. Neidhardt, and T. Rasulov, “On the spectrum of the lattice spin-boson Hamiltonian for any coupling:1D case,” J.Math.Phys. 56, 53507 (2015).
2. T. Kh. Rasulov, “Branches of the essential spectrum of the lattice spin-boson model with at most two photons”, Theor. Math. Phys. 186, 251-267 (2016).

LOGISTIK MAP OF A NON-STOCHASTIC QUADRATIC OPERATOR

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Abstract. *Many practical works are effectively used to determine the dynamics of evolutionary operators, most often defined by stochastic and non-stochastic cubic matrices. The time-dependent family of cubic matrices is used to express mathematical models of problems in biology and physics.*

Scientific research is being conducted in the world to determine the dynamical systems formed by stochastic and non-stochastic quadratic operators and quadratic non-stochastic operators.

In this article, we show that a quadratic non-stochastic operator in a two-dimensional (2D) simplex forms a chaotic dynamical system.

Key words: *simplex; quadratic stochastic operator, quadratic non-stochastic operator, fixed point, repelling, attracting, saddle.*

STOXASTIK BO'LMAGAN KVADRATIK OPERATORNING LOGISTIK XARITASI

Annotatsiya. *Ko‘plab amaliy ishlar aksariyat hollarda stoxastik va nostoxastik kubik matritsalar orqali aniqlangan evalyutsion operatorlar dinamikasini aniqlashda samarali qo‘llanilmoqda. Kubik matritsalarning vaqtga bog‘liq oilasi biologiya va fizika masalalarini matematik modelini ifodalashda foydalaniladi. [1-3]*

Dunyoda xaotik dinamik sistema hosil qiluvchi stoxastik va nostoxastik kvadratik operatorlar hamda kvadratik nostoxastik operatorlar hosil qilgan dinamik sistemalarni aniqlashga doir ilmiy izlanishlar olib borilmoqda.

Ushbu maqolada biz ikki o‘lchovli (2D) simpleksda kvadratik stoxastik bo‘lmagan operator xaotik dinamik sistema tashkil etishi ko’rsatilgan.

Kalit so‘zlar: *simpleks; kvadratik stoxastik operator, kvadratik nostoxastik operator, qo‘zg‘almas nuqta, tortuvchi, itaruvchi, egar, logistic map.*

ЛОГИСТИЧЕСКОЕ ОТОБРАЖЕНИЕ НЕСТОХАСТИЧЕСКОГО КВАДРАТИЧНОГО ОПЕРАТОРА

Абстрактный. *Многие практические работы эффективно используются для определения динамики эволюционных операторов, чаще всего определяемых стохастическими и нестохастическими кубическими матрицами. Семейство зависящих от времени кубических матриц используется для выражения математических моделей задач в биологии и физике.*

В мире ведутся научные исследования по определению динамических систем, образованных стохастическими и нестохастическими квадратичными операторами и квадратичными нестохастическими операторами.

В этой статье мы показываем, что квадратичный нестохастический оператор в двумерном (2D) симплексе образует хаотическую динамическую систему.

Ключевые слова: *симплекс; квадратичный стохастический оператор, квадратичный нестохастический оператор, неподвижная точка, атталкивающей, притагивающей, седловой,*

Let $S^2 = \{(x,y,z) \in R^3 : x \geq 0, y \geq 0, z \geq 0, x+y+z = 1\}$ be two-dimensional simplex. Consider the following quadratic non-stochastic operator

$$K : (x,y,z) \rightarrow (x', y', z')$$

$$\begin{cases} x' = ax^2 + 2bxy + cy^2 \\ y' = (1-a)x^2 + 2(1-b)xy + (1-c)y^2 \\ z' = z(2-z). \end{cases} \qquad (1)$$

where

$$a, c \in [0,1], \quad d \in [-\sqrt{ac}, 1 + \sqrt{(1-a)(1-c)}\ ] \qquad (2)$$

For arbitrary given vector $x^0 \in S^2$ the discrete-time dynamical system of x^0 under operator W is the sequence of vectors of S^2

$$x_0, \ x_1 = K(x^0), \ x^2 = K^2(x^0), \ x^3 = K^3(x^0), \qquad (3)$$

K here $K^n(x)$ is the n-fold composition of K with itself.

The main problem: is to know what ultimately happens with the sequence (3). Does the $\lim_{n\to\infty} x^n$ exist? This is very difficult problem, in general (see [1],[2]).

Remark. *We do not know any quadratic* stochastic *operator with chaotic behavior of trajectories. In the (above considered) case:* $a = c = 1$, $b \in [-1, 0)$, *the operator* (1) *has the form*

$$\begin{aligned} x' &= x^2 + 2bxy + y^2 \\ y' &= 2(1-b)xy. \end{aligned} \qquad (4)$$

$$f(x) = (a - 2b + c)x^2 + 2(b-c)x + c. \qquad (5)$$

Since $P_{12,1} = b < 0$, *this operator is non-stochastic. For arbitrary initial point* $(1-y_0, y_0) \in S^1$*, its trajectory has the form* $(1-y_n, y_n)$, *where* y_n *is defined by* (4). *Therefore, from above-mentioned properties of the logistic map, it follows that when* $-1 \leq b < -0.784975$ *the operator* (4) *generates a chaotic dynamical system on the one-dimensional simplex.*

References

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We consider a family of the generalized Friedrichs model $H_{\mu_1\mu_2}(p), \mu_1, \mu_2 \in \mathbb{R}, \mu_1^2 + \mu_2^2 \neq 0, p \in \mathbb{T}^2$ with perturbation of rank 2, which is a generalization of the family of two-particle Schrödinger operators $H_{\mu_1\mu_2}(p), p \in \mathbb{T}^2 = (-\pi, \pi]^2$ associated with a system of two arbitrary (identical) quantum mechanical particles moving on the two dimensional lattice $\mathbb{Z}^2$ and interacting via zero range attractive or repulsive potential as follows:

$$H_{\mu_1\mu_2}(p) = H_0(p) + \mu_1 V_1 + \mu_2 V_2, \qquad \mu_1, \mu_2 \in \mathbb{R}, \mu_1^2 + \mu_2^2 > 0,$$

where $H_0(p), p \in \mathbb{T}^2$ is a multiplication operator by a real-analytic function $w_p(\cdot) := w(p, \cdot)$ defined on $(\mathbb{T}^2)^2$, i.e.,

$$(H_0(p)f)(q) = w_p(q) f(q), \quad f \in L^2(\mathbb{T}^2)$$

and $V_i : L^2(\mathbb{T}^2) \rightarrow L^2(\mathbb{T}^2)$ is the perturbation operator of the form