V.I.ROMANOVSKIY INSTITUTE
OF MATHEMATICS

UNIVERSITY OF EXACT
AND SOCIAL SCIENCES

ABSTRACTS
OF THE INTERNATIONAL CONFERENCE
GIBBS MEASURES AND THE THEORY OF DYNAMICAL SYSTEMS

MAY 20 – 21, 2024

TASHKENT, UZBEKISTAN

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May 20–21, 2024, Tashkent, Uzbekistan

TASHKENT – 2024

Gibbs measures and the theory of dynamical systems: Abstract of the international conference, May 20–21, 2024, Tashkent, Uzbekistan.

MAIN SECTIONS OF THE CONFERENCE:

- Algebra and analysis,
- Continuous time dynamical systems (Differential equations),
- Discrete-time dynamical systems,
- Stochastic processes.

The aim of the scientific conference is to discuss the current state and prospects for the development of the main directions of Gibbs measures and the theory of dynamical systems, as well as their applications in mathematical physics, mathematical biology and the theory of stochastic processes.

The conference languages: Uzbek, English and Russian.

This conference was organized on the basis of resolution №76 of the Ministry of Higher Education, Science and Innovation of the Republic of Uzbekistan dated March 20, 2024.

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Contents

ABOUT DYNAMIC SYSTEMS OF A QnSO

Khudayarov S.S.[1], **Dilmurodov E.B.**[2]
Bukhara State University, Bukhara, Uzbekistan;
s.s.xudayarov@buxdu.uz
Bukhara Branch Institute of Mathematics named after V.I. Romanovsky;
e.b.dilmurodov@buxdu.uz

Let $I = \{1, 2, \dots, m\}$. A distribution of the set I is a probability measure $x = (x_1, \dots, x_m)$, i.e. an element of the simplex:

$$S^{m-1} = \left\{ x \in \mathbb{R}^m : x_i \geq 0,\ \sum_{i=1}^{m} x_i = 1 \right\}. \tag{1}$$

The quadratic stochastic operator (QSO) is a mapping of the simplex S^{m-1} into itself, of the form

$$V : x'_k = \sum_{i,j=1}^{m} P_{ij,k} x_i x_j, \quad k = 1, \dots, m. \tag{2}$$

where $P_{ij,k}$ are coefficients of heredity and

$$P_{ij,k} \geq 0, \ \ P_{ij,k} = P_{ji,k}, \ \ \sum_{k=1}^{m} P_{ij,k} = 1, \ \ i, j, k = 1, \dots, m. \tag{3}$$

Thus, each quadratic stochastic operator V can be uniquely defined by a cubic matrix $(P_{ij,k})_{i,j,k=1}^{m}$ with conditions (3).

For a given $x^{(0)} \in S^{m-1}$ the trajectory (orbit) $\{x^{(n)}\}$ of $x^{(0)}$ under the action of QSO (2) is defined by

$$x^{(n+1)} = V(x^{(n)}), \ \ n = 0, 1, 2...$$

In general, a quadratic operator V, $V : x \in \mathbb{R}^m \to x' = V(x) \in \mathbb{R}^m$ corresponding to a cubic matrix $(P_{ij,k})_{i,j,k=1}^{m}$ is defined by (2):

Definition.[1] A quadratic operator (2) preserving a simplex, is called non-stochastic (QnSO) if at least one of its coefficients $P_{ij,k}$, $i \neq j$ is negative.

Theorem 1. [1] For a quadratic operator V (given by (2)), to preserve a simplex S^{m-1} it is **sufficient** that

i) $\sum\limits_{k=1}^{m} P_{ij,k} = 1, \ \ i, j = 1, \dots, m;$

ii) $0 \leq P_{ii,k} \leq 1, \ \ i, k = 1, \dots, m;$

iii) $-\frac{1}{m-1}\sqrt{P_{ii,k}P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1 - P_{ii,k})(1 - P_{jj,k})}$

and **necessary** that the conditions (i), (ii) and

iii′) $-\sqrt{P_{ii,k}P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1 - P_{ii,k})(1 - P_{jj,k})}$

are satisfied.

Consider the following QnSO on S^2:

$$W_0 : \begin{cases} x' = ax^2 + 2bxy + cy^2 \\ y' = (1-a)x^2 + 2(1-b)xy + (1-c)y^2 \\ z' = z(2 - z). \end{cases} \tag{4}$$

where

$$a, c \in [0,1], \quad b \in [-\sqrt{ac}, \quad 1+\sqrt{(1-a)(1-c)}]. \tag{5}$$

Theorem 2. For the initial point $(x_0, y_0, z_0) \in S^2$, $z_0 \neq 0$ the trajectory (x_n, y_n, z_n) approaches the point $e_1 = (0,0,1)$.

Denote

$$\mathcal{Z} = \{(x,y,z) \in S^2 : z = 0\}.$$

It is easy to see that $\mathcal{Z}$ is an edge of the simplex and it is invariant: $z = 0 \quad \Rightarrow z' = 0$.

Remark. If $z = 0$, then the restriction of operator (4) to $\mathcal{Z}$ is an arbitrary one-dimensional QnSO in S^1:

$$\begin{cases} x' = ax^2 + 2bxy + cy^2 \\ y' = (1-a)x^2 + 2(1-b)xy + (1-c)y^2. \end{cases} \tag{6}$$

Some special cases a operator the form (6) were studied in [1].

References

1. Rozikov U.A., Xudayarov S.S. Quadratic non-stochastic operators: examples of splitted chaos // Ann. Funct. Anal., 2022. Vol. 13, №.17, pp. 1–17.