



MINISTRY OF HIGHER EDUCATION,
SCIENCE AND INNOVATIONS OF
THE REPUBLIC OF UZBEKISTAN



V.I.ROMANOVSKIY
INSTITUTE
OF MATHEMATICS



UNIVERSITY OF EXACT
AND SOCIAL SCIENCES



OXUS
UNIVERSITY

INTERNATIONAL SCIENTIFIC CONFERENCE

**MATHEMATICAL ANALYSIS AND
DYNAMICAL SYSTEMS**

CONFERENCE PROGRAMME

MAY 20 - 21, 2025

TASHKENT, UZBEKISTAN

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On the fixed point and invariant set QnSO

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Let $I = \{1, 2, \dots, m\}$. A distribution of the set I is a probability measure $x = (x_1, \dots, x_m)$, i.e. an element of the simplex:

$$S^{m-1} = \left\{ x \in \mathbb{R}^m : x_i \geq 0, \sum_{i=1}^m x_i = 1 \right\}. \quad (1)$$

The quadratic stochastic operator (QSO) is a mapping of the simplex S^{m-1} into itself, of the form

$$V : x'_k = \sum_{i,j=1}^m P_{ij,k} x_i x_j, \quad k = 1, \dots, m. \quad (2)$$

where $P_{ij,k}$ are coefficients of heredity and

$$P_{ij,k} \geq 0, \quad P_{ij,k} = P_{ji,k}, \quad \sum_{k=1}^m P_{ij,k} = 1, \quad i, j, k = 1, \dots, m. \quad (3)$$

Thus, each quadratic stochastic operator V can be uniquely defined by a cubic matrix $(P_{ij,k})_{i,j,k=1}^m$ with conditions (3).

For a given $x^{(0)} \in S^{m-1}$ the trajectory (orbit) $\{x^{(n)}\}$ of $x^{(0)}$ under the action of QSO (2) is defined by

$$x^{(n+1)} = V(x^{(n)}), \quad n = 0, 1, 2, \dots \quad (4)$$

The main problem: is to know what ultimately happens with the sequence (4). Does the limit $\lim_{n \rightarrow \infty} x_n = ?$ This is very difficult problem, in general (see [1],[2]).

Definition 1. [1] A quadratic operator (2) preserving a simplex, is called non-stochastic (QnSO) if at least one of its coefficients $P_{ij,k}$, $i \neq j$ is negative.

Theorem 1. [3] For a quadratic operator V (given by (2)), to preserve a simplex S^{m-1} it is **sufficient** that

- i) $\sum_{k=1}^m P_{ij,k} = 1, \quad i, j = 1, \dots, m;$
 - ii) $0 \leq P_{ii,k} \leq 1, \quad i, k = 1, \dots, m;$
 - iii) $-\frac{1}{m-1} \sqrt{P_{ii,k} P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1 - P_{ii,k})(1 - P_{jj,k})}$
- and **necessary** that the conditions (i), (ii) and
- iii') $-\sqrt{P_{ii,k} P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1 - P_{ii,k})(1 - P_{jj,k})}$
- are satisfied.

Let's consider the following quadratic stochastic operator S^2 :

$$W : \begin{cases} x' = x^2 + y^2 + z^2 - axy - axz - ayz \\ y' = (2 + a)(x + z)y \\ z' = (2 + a)zx. \end{cases} \quad (5)$$

where

$$a \in [-2, 2] \quad (6)$$

If this operator W takes a value in the interval $a \in [0, 2]$ for the parameter a , then W is a quadratic non-stochastic operator (QnSO) according to Theorem 3.

$$W_0 : \begin{cases} x' = x^2 + y^2 + z^2 - axy - axz - ayz \\ y' = (2 + a)(x + z)y \\ z' = (2 + a)zx. \end{cases} \quad (7)$$

where

$$a \in [0, 2] \quad (8)$$

Fixed points. The fixed points of this operator are solutions to the system $W_0(x, y, z) = (x, y, z)$. The following lemma gives all fixed points of operator W_0 .

Lemma. The set of all fixed points of operator (7) is

$$\text{Fix}(W) = \left\{ A_1 = (1, 0, 0), A_2 = \left(\frac{1}{2+a}, \frac{1+a}{2+a}, 0 \right), A_3 = \left(\frac{1}{2+a}, 0, \frac{1+a}{2+a} \right) \right\}$$

Definition 2. A fixed point x^* of a mapping W is called hyperbolic point if its Jacobian J_W at x^* has no eigenvalues on the unit circle; **attracting** point if all the eigenvalues of the Jacobi matrix $J_W(x^*)$ are less than 1 in absolute value; **repelling** point if all the eigenvalues of the Jacobi matrix $J_W(x^*)$ are greater than 1 in absolute value; a **saddle** point otherwise. The following theorem gives type (see [1] for definitions) of each fixed point of the operator W_2 :

Theorem 2. For the operator W_2 , the points e_1, e_2, e_3 are fixed points. Moreover,

- 1) the point e_2 is non-hyperbolic;
- 2) if $a \in [0, 1)$, then the point e_1 is a repeller, point e_3 is a saddle;
- 3) if $a = 1$, then e_1 and e_3 are non-hyperbolic;
- 4) if $a \in (1, 2]$, then e_1 is a saddle and point e_3 is a repeller.

On invariant sets. Recall that a set W is called invariant with respect to an operator V if $V(W) \subset W$.

Denote

$$\mathcal{I}_1 = \{(x, y, z) \in S^2 : y = 0\}, \quad \mathcal{I}_2 = \{(x, y, z) \in S^2 : z = 0\},$$

It is easy to see that \mathcal{I}_1 and \mathcal{I}_2 is an edge of the simplex and it is invariant: $y = 0 \Rightarrow y' = 0, z = 0 \Rightarrow z' = 0$.

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