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$$\times e^{i\lambda F(x_1^c,x_2^c,s_1^c,s_2^c,s_3.y_1,y_2,y_3)}\left(A(x_1^c,x_2^c,s_1^c,s_2^c,s_3)+O(\lambda^{-1})\right)ds_3 \quad \lambda\to+\infty \qquad (10)$$

Yuqoridagi tenglik bilan aniqlangan integralni baholaymiz.

$$\left|\frac{\lambda}{2\pi}\int_{\mathbb{R}}\chi_1(s_3)\,e^{i\lambda F(x_1^c,x_2^c,s_1^c,s_2^c,s_3.y_1,y_2,y_3)}\left(A(x_1^c,x_2^c,s_1^c,s_2^c,s_3)+O(\lambda^{-1})\right)ds_3\right|\le$$

$$\le\left|\frac{\lambda}{2\pi}\right|\int_{\mathbb{R}}|\chi_1(s_3)|\left|e^{i\lambda F(x_1^c,x_2^c,s_1^c,s_2^c,s_3.y_1,y_2,y_3)}\right|\left|\left|A(x_1^c,x_2^c,s_1^c,s_2^c,s_3)+O(\lambda^{-1})\right|\right|ds_3\right|\le$$

$$\le C|\lambda|,\ \ \lambda\to+\infty \qquad (11)$$

Shunday qilib, (10) tenglikka va (11) tengsizlikka ko‘ra quyidagi bahoga ega bo‘ldik.

$$\left|\mu^{\lambda}(y)\right|\le C|\lambda|,\ \ \lambda\to+\infty$$

Adabiyotlar ro‘yxati.

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Quadratic stochastic processes of permutation matrix

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Denote $I=\{1,2,\dots,m\}$. Let $\mathcal{C}$ be the set of all m^3-dimensional cubic matrices over the field of real numbers. Denote by E_{ijk}, $i,j,k\in I$ the basis cubic matrices in $\mathcal{C}$.(see [1])

Following [2] define the following multiplications for basis matrices E_{ijk} :

$$E_{ijk} *_0 E_{lnr} = \delta_{kl}\delta_{jn}E_{ijr}, \tag{1}$$

where δ_{kl} is the Kronecker symbol, i.e.

$$\delta_{kl} = \begin{cases} 0, & k \neq l \\ 1, & k = l \end{cases}$$

Then for any two cubic matrices $A = (a_{ijk}), B = (b_{ijk}) \in \mathcal{C}$ the matrix $A *_0 B = (c_{ijk})$ is defined by

$$c_{ijr} = \sum_{k=1}^{m} a_{ijk} b_{kjr}. \tag{2}$$

The following results of this section are proven in [2] (see also [1] for detailed proofs)

Proposition. *The algebra of cubic matrices* $(\mathcal{C}, *_0)$ *is a direct sum of algebras of square matrices.*

Define multiplication:

$$E_{ijk} *_a E_{lnr} = \delta_{kl} E_{ia(j,n)r}, \tag{3}$$

where $a : I \times I \to I$, $(j,n) \mapsto a(j,n) \in I$, is an arbitrary associative binary operation.

Stochasticity. Define several kinds of cubic stochastic matrices (see [2,4]): a cubic matrix $P = (p_{ijk})_{i,j,k=1}^{m}$ is called

- $(1,2)$-*stochastic* if

$$p_{ijk} \geq 0, \qquad \sum_{i,j=1}^{m} p_{ijk} = 1, \textit{ for all } k.$$

- $(1,3)$-*stochastic* if

$$p_{ijk} \geq 0, \qquad \sum_{i,k=1}^{m} p_{ijk} = 1, \textit{ for all } j.$$

- $(2,3)$-*stochastic* if

$$p_{ijk} \geq 0, \qquad \sum_{j,k=1}^{m} p_{ijk} = 1, \textit{ for all } i.$$

- 3-*stochastic* if

$$p_{ijk} \geq 0, \qquad \sum_{k=1}^{m} p_{ijk} = 1, \textit{ for all } i, j .$$

The last one can be also given with respect to first and second index.

Denote by S the set of all possible kinds of stochasticity and denote by M the set of all possible multiplication rules of cubic matrices.

Let parameters $s \geq 0$, $t \geq 0$, are considered as time.

Denote by $M^{[s,t]} = \left(P_{ijk}^{[s,t]}\right)_{i,j,k=1}^{m}$ a cubic matrix with two parameters.

Definition.[3]. *A family* $\{M^{[s,t]} : s,t \in R_+\}$ *is called a Markov process of cubic matrices (or a quadratic stochastic process (QSP)) of type* $(\sigma \mid \mu)$ *if for each time* s *and* t *the cubic matrix* $M^{[s,t]}$ *is stochastic in sense* $\sigma \in \mathrm{S}$ *and satisfies the Kolmogorov-Chapman equation (for cubic matrices):*

$$M^{[s,t]} = M^{[s,\tau]} *_{\mu} M^{[\tau,t]}, \textit{for all } 0 \leq s < \tau < t \qquad (4)$$

with respect to the multiplication $\mu \in \mathrm{M}$.

In the work M. Ladra and U.A Rozikov proved the following theorem.

Theorem 1. [3]. *Let* $\{A^{[t]} = (a_{ij}^{[t]}, t \geq 0)\}$ *be a family of invertible (for all t),* $m \times m$ *square matrices and let* $\left(A^{[t]}\right)^{-1} = (b_{ij}^{[t]})$ *denote the inverse of* $A^{[t]}$. *Let* $\mathrm{B}^{(s)} = \left(\beta_{ijk}^{(s)}\right)$, *be a cubic matrix, where* $\beta_{ijk}^{(s)}, i, j, k = 1, \ldots, m,$ *are arbitrary functions such that*

$$\sum_{j=1}^{m} \beta_{ijk}^{(s)} = a_{ik}^{[s]}, \textit{for any } i, k \textit{ and } s.$$

Then cubic matrix

$$M^{[s,t]} = \left(\sum_{k=1}^{m} \beta_{ijk}^{(s)} b_{kr}^{[t]} \right)_{i,j,r=1}^{m} \qquad (5)$$

generates an flow of algebras (i.e. satisfies equation (4) *of type* (D)).

In general, the matrix (1) does not generate a QSP. Here our aim is to find conditions on matrices $A^{[t]}$ and $B^{(s)}$ (mentioned in Theorem 1) ensuring that the matrix (5) generates a QSP.

QSP of type $(\sigma \mid D)$.

Lemma 1. *Let* $A^{[t]} = \left(a_{ij}^{[t]}\right)_{i,j=1}^{m}$ *be a right stochastic and invertible matrix and let* $(A^{[t]})^{-1} = \left(b_{ij}^{[t]}\right)_{i,j=1}^{m}$ *be its inverse matrix. If* $(A^{[t]})^{-1}$ *is not a stochastic matrix,then it has a least one negative element.*

Proposition 1. *If* $A^{[t]}$ *and* $\mathrm{B}^{(s)} = (\beta_{ijk}^{(s)})_{i,j,k=1}^{m}$ *in Theorem 1 satisfies the following conditions:*

– $(A^{[t]})^{-1}$ *is a left stochastic*;

– $\mathrm{B}^{(s)}$ is a $(1,2)$-*stochastic*.

Then the matrix $M^{[s,t]}$ (*given by* (5)) $(12\,|\,D)$.

Example 1. *Let*

$$A^{[t]} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$\mathrm{B}^{(s)} = \left(\begin{array}{cc|cc} f(s) & 0 & 0 & g(s) \\ 1-f(s) & 0 & 0 & 1-g(s) \end{array}\right)$$

Here $0 \le f(s), g(s) \le 1$.

Then $A^{[t]}$ and $(A^{[t]})^{-1}$ satisfy all conditions of Theorem 1.

In this case the QSP is defined by the matrix

$$M^{[s]} = \left(\begin{array}{cc|cc} f(s) & 0 & 0 & g(s) \\ 1-f(s) & 0 & 0 & 1-g(s) \end{array}\right)$$

Theorem 2. *If in Theorem 1 both* $A^{[t]}$ *and* $(A^{[t]})^{-1}$ *are stochastic for some* $t > 0$ *and* $(\beta_{ijk}^{(s)})_{i,j,k=1}^{m}$ *is 3-stochastic for some* $s < t$, *then* $M^{[s,t]}$ *is not 3-stochastic.*

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On the Discrete Spectrum of the One-dimensional Discrete Schrödinger Operator Depending on Three Parametres

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Abstract: We consider a family of Schrödinger operators $H_{\lambda\mu k} = -\Delta - \lambda\delta_{kx} - \mu\delta_{0x}$ the one-dimensional lattice Z^1, where Δ is a standard discrete Laplacian, $\delta_{\cdot,\cdot}$ is a Kronecker delta function, and $\lambda, \mu \in \mathrm{R}^1$ and $k \in \mathrm{Z}^1$ are parameters. Behaviors of eigenvalues, resonances and their dependence on the parameters of the operator are explicitly derived.
Keywords: Discrete Laplacian, Discrete Shcrödinger operator, eigenvalues, Fredholm determinant, resonances

1 Introduction

Behavior of eigenvalues below the essential spectrum of standard Schrödinger operators of the form $-\Delta + \varepsilon V$ defined on $L^2(\mathrm{R}^n)$ has been considerably studied so far. Here V is a negative potential and $\varepsilon \geq 0$ is a parameter which is varied. When ε approaches to some critical point $\varepsilon_c \geq 0$, the negative eigenvalues approach to the left edge of the essential spectrum, and consequently they are absorbed into it. A crucial mathematical problem is to specify whether the edge of the essential spectrum is an eigenvalue or a threshold resonance at the critical point ε_c. Their behaviors depend on the spatial dimension n.

The discrete Schrödinger operators have attracted considerable attention for both combinatorial Laplacians and quantum graphs; for some recent summaries refer to [1, 2, 3, 5, 6, 7, 8] and the references therein. Particularly, eigenvalue behavior of discrete Schrödinger operators are discussed in e.g. [9, 10, 11, 12] and are briefly discussed in [12,13,14] when potentials are delta functions with a single point mass. In [9] an explicit example of a $-\Delta - V$ on the three-dimensional lattice Z^3, which possesses both a *lower* threshold resonance and a *lower* threshold eigenvalue, is constructed, where $-\Delta$ stands for the standard discrete Laplacian in $\ell^2(\mathrm{Z}^n)$ and V is a multiplication operator by the function