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$$\times e^{i\lambda F(x_1^c, x_2^c, s_1^c, s_2^c, s_3, y_1, y_2, y_3)} (A(x_1^c, x_2^c, s_1^c, s_2^c, s_3) + O(\lambda^{-1})) ds_3 \quad \lambda \rightarrow +\infty \quad (10)$$

Yuqoridagi tenglik bilan aniqlangan integralni baholaymiz.

$$\begin{aligned} & \left| \frac{\lambda}{2\pi} \int_{\mathbb{R}} \chi_1(s_3) e^{i\lambda F(x_1^c, x_2^c, s_1^c, s_2^c, s_3, y_1, y_2, y_3)} (A(x_1^c, x_2^c, s_1^c, s_2^c, s_3) + O(\lambda^{-1})) ds_3 \right| \leq \\ & \leq \left| \frac{\lambda}{2\pi} \right| \int_{\mathbb{R}} |\chi_1(s_3)| |e^{i\lambda F(x_1^c, x_2^c, s_1^c, s_2^c, s_3, y_1, y_2, y_3)}| |A(x_1^c, x_2^c, s_1^c, s_2^c, s_3) + O(\lambda^{-1})| ds_3 \leq \\ & \leq C|\lambda|, \quad \lambda \rightarrow +\infty \end{aligned} \quad (11)$$

Shunday qilib, (10) tenglikka va (11) tengsizlikka ko'ra quyidagi bahoga ega bo'ldik.

$$|\mu^\lambda(y)| \leq C|\lambda|, \quad \lambda \rightarrow +\infty$$

Adabiyotlar ro'yxati.

1. Федорюк М. В. - Метод перевала. - 1977. - 368с
2. I. M. Vinogradov, The Method of Trigonometric Sums in the Theory of Numbers (Nauka, Moscow, 1971) [in Russian].
3. Hua Loo-keng, "On the number of solutions of Tarry's problem," Acta Sci. Sinica 1 (1), 1–76 (1952).
4. I. A. Ikromov and D. Muller, "Fourier Restriction for Hypersurfaces in Three Dimensions and Newton Polyhedra," in Ann. of Math. Stud. (Princeton Univ. Press, Princeton, NJ, 2016), Vol. 194. 8.
5. I. A. Ikromov, "Summability of oscillatory integrals over parameters and the boundedness problem for Fourier transforms on curves," Math. Notes 87 (5), 700–719 (2010).

Quadratic stochastic processes of permutation matrix

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Denote $I = \{1, 2, \dots, m\}$. Let \mathcal{C} be the set of all m^3 -dimensional cubic matrices over the field of real numbers. Denote by E_{ijk} , $i, j, k \in I$ the basis cubic matrices in \mathcal{C} . (see [1])

Following [2] define the following multiplications for basis matrices E_{ijk} :

$$E_{ijk} *_0 E_{lmr} = \delta_{kl} \delta_{jn} E_{ijr}, \quad (1)$$

where δ_{kl} is the Kronecker symbol, i.e.

$$\delta_{kl} = \begin{cases} 0, & k \neq l \\ 1, & k = l \end{cases}$$

Then for any two cubic matrices $A = (a_{ijk}), B = (b_{ijk}) \in \mathcal{C}$ the matrix $A *_0 B = (c_{ijk})$ is defined by

$$c_{ijr} = \sum_{k=1}^m a_{ijk} b_{kjr}. \quad (2)$$

The following results of this section are proven in [2] (see also [1] for detailed proofs)

Proposition. *The algebra of cubic matrices $(\mathcal{C}, *_0)$ is a direct sum of algebras of square matrices.*

Define multiplication:

$$E_{ijk} *_a E_{lmr} = \delta_{kl} E_{ia(j,n)r}, \quad (3)$$

where $a: I \times I \rightarrow I$, $(j,n) \mapsto a(j,n) \in I$, is an arbitrary associative binary operation.

Stochasticity. Define several kinds of cubic stochastic matrices (see [2,4]): a cubic matrix $P = (p_{ijk})_{i,j,k=1}^m$ is called

- (1,2)-stochastic if

$$p_{ijk} \geq 0, \quad \sum_{i,j=1}^m p_{ijk} = 1, \text{ for all } k.$$

- (1,3)-stochastic if

$$p_{ijk} \geq 0, \quad \sum_{i,k=1}^m p_{ijk} = 1, \text{ for all } j.$$

- (2,3)-stochastic if

$$p_{ijk} \geq 0, \quad \sum_{j,k=1}^m p_{ijk} = 1, \text{ for all } i.$$

- 3-stochastic if

$$p_{ijk} \geq 0, \quad \sum_{k=1}^m p_{ijk} = 1, \text{ for all } i, j.$$

The last one can be also given with respect to first and second index.

Denote by S the set of all possible kinds of stochasticity and denote by M the set of all possible multiplication rules of cubic matrices.

Let parameters $s \geq 0, t \geq 0$, are considered as time.

Denote by $M^{[s,t]} = (P_{ijk}^{[s,t]})_{i,j,k=1}^m$ a cubic matrix with two parameters.

Definition.[3]. A family $\{M^{[s,t]} : s, t \in R_+\}$ is called a Markov process of cubic matrices (or a quadratic stochastic process (QSP)) of type $(\sigma | \mu)$ if for each time s and t the cubic matrix $M^{[s,t]}$ is stochastic in sense $\sigma \in S$ and satisfies the Kolmogorov-Chapman equation (for cubic matrices):

$$M^{[s,t]} = M^{[s,\tau]} *_{\mu} M^{[\tau,t]}, \text{ for all } 0 \leq s < \tau < t \quad (4)$$

with respect to the multiplication $\mu \in M$.

In the work M. Ladra and U.A Rozikov proved the following theorem.

Theorem 1. [3]. Let $\{A^{[t]} = (a_{ij}^{[t]}, t \geq 0)\}$ be a family of invertible (for all t), $m \times m$ square matrices and let $(A^{[t]})^{-1} = (b_{ij}^{[t]})$ denote the inverse of $A^{[t]}$. Let $B^{(s)} = (\beta_{ijk}^{(s)})$, be a cubic matrix, where $\beta_{ijk}^{(s)}, i, j, k = 1, \dots, m$, are arbitrary functions such that

$$\sum_{j=1}^m \beta_{ijk}^{(s)} = a_{ik}^{[s]}, \text{ for any } i, k \text{ and } s.$$

Then cubic matrix

$$M^{[s,t]} = \left(\sum_{k=1}^m \beta_{ijk}^{(s)} b_{kr}^{[t]} \right)_{i,j,r=1}^m \quad (5)$$

generates an flow of algebras (i.e. satisfies equation (4) of type (D)).

In general, the matrix (1) does not generate a QSP. Here our aim is to find conditions on matrices $A^{[t]}$ and $B^{(s)}$ (mentioned in Theorem 1) ensuring that the matrix (5) generates a QSP.

QSP of type $(\sigma | D)$.

Lemma 1. Let $A^{[t]} = (a_{ij}^{[t]})_{i,j=1}^m$ be a right stochastic and invertible matrix and let $(A^{[t]})^{-1} = (b_{ij}^{[t]})_{i,j=1}^m$ be its inverse matrix. If $(A^{[t]})^{-1}$ is not a stochastic matrix, then it has at least one negative element.

Proposition 1. If $A^{[t]}$ and $B^{(s)} = (\beta_{ijk}^{(s)})_{i,j,k=1}^m$ in Theorem 1 satisfies the following conditions:

- $(A^{[t]})^{-1}$ is a left stochastic;
- $B^{(s)}$ is a (1,2)-stochastic.

Then the matrix $M^{[s,t]}$ (given by (5)) (12 | D).

Example 1. Let

$$A^{[t]} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$B^{(s)} = \left(\begin{array}{cc|cc} f(s) & 0 & 0 & g(s) \\ 1-f(s) & 0 & 0 & 1-g(s) \end{array} \right)$$

Here $0 \leq f(s), g(s) \leq 1$.

Then $A^{[t]}$ and $(A^{[t]})^{-1}$ satisfy all conditions of Theorem 1.

In this case the QSP is defined by the matrix

$$M^{[s,t]} = \left(\begin{array}{cc|cc} f(s) & 0 & 0 & g(s) \\ 1-f(s) & 0 & 0 & 1-g(s) \end{array} \right)$$

Theorem 2. If in Theorem 1 both $A^{[t]}$ and $(A^{[t]})^{-1}$ are stochastic for some $t > 0$ and $(\beta_{ijk}^{(s)})_{i,j,k=1}^m$ is 3-stochastic for some $s < t$, then $M^{[s,t]}$ is not 3-stochastic.

References

1. Rozikov U.A. // Population dynamics: algebraic and probabilistic approach. World Sci. Publ. Singapore. P. 1-460. 2020.
2. Maksimov V.M. // Cubic stochastic matrices and their probability interpretations Theory Probab. Appl, 41:1 (1996), P. 55-69.
3. Ladra M., Rozikov U.A. // Flow of finite-dimensional algebras. Jour. Algebra, 470 (2017), P.263-288.
4. Mukhamedov F.M., Ganikhodjaev N.N. // Quantum quadratic operators and

processes. Lecture Notes in Mathematics book 2133, Springer, Cham, (2015).

On the Discrete Spectrum of the One-dimensional Discrete Schrödinger Operator Depending on Three Parametres

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Abstract: We consider a family of Schrödinger operators $H_{\lambda,\mu,k} = -\Delta - \lambda\delta_{kx} - \mu\delta_{0x}$ the one-dimensional lattice Z^1 , where Δ is a standard discrete Laplacian, δ_{\cdot} is a Kronecker delta function, and $\lambda, \mu \in \mathbb{R}^1$ and $k \in Z^1$ are parameters. Behaviors of eigenvalues, resonances and their dependence on the parameters of the operator are explicitly derived.

Keywords: Discrete Laplacian, Discrete Schrödinger operator, eigenvalues, Fredholm determinant, resonances

1 Introduction

Behavior of eigenvalues below the essential spectrum of standard Schrödinger operators of the form $-\Delta + \varepsilon V$ defined on $L^2(\mathbb{R}^n)$ has been considerably studied so far. Here V is a negative potential and $\varepsilon \geq 0$ is a parameter which is varied. When ε approaches to some critical point $\varepsilon_c \geq 0$, the negative eigenvalues approach to the left edge of the essential spectrum, and consequently they are absorbed into it. A crucial mathematical problem is to specify whether the edge of the essential spectrum is an eigenvalue or a threshold resonance at the critical point ε_c . Their behaviors depend on the spatial dimension n .

The discrete Schrödinger operators have attracted considerable attention for both combinatorial Laplacians and quantum graphs; for some recent summaries refer to [1, 2, 3, 5, 6, 7, 8] and the references therein. Particularly, eigenvalue behavior of discrete Schrödinger operators are discussed in e.g. [9, 10, 11, 12] and are briefly discussed in [12,13,14] when potentials are delta functions with a single point mass. In [9] an explicit example of a $-\Delta - V$ on the three-dimensional lattice Z^3 , which possesses both a *lower* threshold resonance and a *lower* threshold eigenvalue, is constructed, where $-\Delta$ stands for the standard discrete Laplacian in $\ell^2(Z^n)$ and V is a multiplication operator by the function

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