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ABOUT FIXED POINT OF A QnSO

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Let $I = \{1, 2, \dots, m\}$. A distribution of the set I is a probability measure $x = (x_1, \dots, x_m)$, i.e. an element of the simplex:

$$S^{m-1} = \left\{ x \in \mathbb{R}^m : x_i \geq 0, \sum_{i=1}^m x_i = 1 \right\}. \quad (1)$$

The quadratic stochastic operator (QSO) is a mapping of the simplex S^{m-1} into itself, of the form

$$V : x'_k = \sum_{i,j=1}^m P_{ij,k} x_i x_j, \quad k = 1, \dots, m. \quad (2)$$

where $P_{ij,k}$ are coefficients of heredity and

$$P_{ij,k} \geq 0, \quad P_{ij,k} = P_{ji,k}, \quad \sum_{k=1}^m P_{ij,k} = 1, \quad i, j, k = 1, \dots, m. \quad (3)$$

Thus, each quadratic stochastic operator V can be uniquely defined by a cubic matrix $(P_{ij,k})_{i,j,k=1}^m$ with conditions (3).

For a given $x^{(0)} \in S^{m-1}$ the trajectory (orbit) $\{x^{(n)}\}$ of $x^{(0)}$ under the action of QSO (2) is defined by

$$x^{(n+1)} = V(x^{(n)}), \quad n = 0, 1, 2, \dots$$

In general, a quadratic operator V , $V : x \in \mathbb{R}^m \rightarrow x' = V(x) \in \mathbb{R}^m$ corresponding to a cubic matrix $(P_{ij,k})_{i,j,k=1}^m$ is defined by (2):[1],[2]

Definition A quadratic operator (2) preserving a simplex, is called non-stochastic (QnSO) if at least one of its coefficients $P_{ij,k}$, $i \neq j$ is negative.

Quadratic dynamical systems describe the behavior of populations of different species with population models [3],[4],[5].

Theorem[6]. For a quadratic operator V (given by (2)), to preserve a simplex S^{m-1} it is **sufficient** that

- i) $\sum_{k=1}^m P_{ij,k} = 1, \quad i, j = 1, \dots, m;$
 - ii) $0 \leq P_{ii,k} \leq 1, \quad i, k = 1, \dots, m;$
 - iii) $-\frac{1}{m-1} \sqrt{P_{ii,k} P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1 - P_{ii,k})(1 - P_{jj,k})}$
- and **necessary** that the conditions (i), (ii) and
- iii') $-\sqrt{P_{ii,k} P_{jj,k}} \leq P_{ij,k} \leq 1 + \sqrt{(1 - P_{ii,k})(1 - P_{jj,k})}$
- are satisfied.

Consider the following QnSO on S^2 :

$$W_0 : \begin{cases} x' = ax^2 + 2bxy + cy^2 \\ y' = (1-a)x^2 + 2(1-b)xy + (1-c)y^2 \\ z' = z(2-z). \end{cases} \quad (4)$$

where

$$a, c \in [0, 1], \quad b \in [-\sqrt{ac}, 1 + \sqrt{(1-a)(1-c)}]. \quad (5)$$

Denote

$$\mathcal{Z} = \{(x, y, z) \in S^2 : z = 0\}.$$

It is easy to see that $\mathcal{Z}$ is an edge of the simplex and it is invariant: $z = 0 \Rightarrow z' = 0$.

The fixed points are solutions to the system (4)

$$\begin{cases} x = ax^2 + 2bxy + cy^2 \\ y = (1-a)x^2 + 2(1-b)xy + (1-c)y^2 \\ z = z(2-z) \end{cases} \quad (6)$$

From the third equation of the system we find $z = 0$ and $z = 1$.

1) Case: If $z = 1$, then according to the definition of a simplex, i.e. $x + y + z = 1$, it follows that $x = y = 0$. Thus, the point $e_1 = (0, 0, 1)$ will be a fixed point of the operator W_0 . That is, there is only one fixed point outside $\mathcal{Z}$.

2) Case: If $z = 0$, then the restriction of operator (4) to $\mathcal{Z}$ is an arbitrary one-dimensional QnSO in S^1 :

$$\begin{cases} x' = ax^2 + 2bxy + cy^2 \\ y' = (1-a)x^2 + 2(1-b)xy + (1-c)y^2, \end{cases} \quad (7)$$

Since $x + y = 1$ the fixed point equation is reduced to one-dimensional form:

$$x = f(x) := ax^2 + 2bx(1-x) + c(1-x)^2. \quad (8)$$

From this we get

$$(a - 2b + c)x^2 + (2b - 2c - 1)x + c = 0,$$

$$D = (2b - 2c - 1)^2 - 4c(a - 2b + c) = (2b - 1)^2 + 4c(1 - a),$$

$$x_{1,2} = \frac{1 + 2c - 2b \pm \sqrt{(2b - 1)^2 + 4c(1 - a)}}{2 \cdot (a - 2b + c)}.$$

These fixed points are possible if the parameters a, b, c satisfying (5) are in the following sets:

$$A_1 = \{(a, b, c) : 0 \leq x_1 = \frac{1 + 2c - 2b + \sqrt{(2b - 1)^2 + 4c(1 - a)}}{2 \cdot (a - 2b + c)} \leq 1\}$$

or

$$A_2 = \{(a, b, c) : 0 \leq x_2 = \frac{1 + 2c - 2b - \sqrt{(2b - 1)^2 + 4c(1 - a)}}{2 \cdot (a - 2b + c)} \leq 1\}$$

Since $(2b - 1)^2 + 4c(1 - a) > 0$ and $f : [0, 1] \rightarrow [0, 1]$, we note that f has always at least one fixed point.

Adabiyotlar

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SHTOLTS-CHEZARO TEOREMASI VA UNING TADBIQLARI

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Annotatsiya. Ushbu tezida Shtolts teoremasidan foydalanib ketma-ketliklarning yaqinlashuvchiligini isbotlash va limitini topish usullari keltirilgan.

Kalit soʻzlar: ketma-ketlik va uning limiti, Shtolts teoremasi, Teylor formulasi, Sterling formulasi.

Sonli ketma-ketlikning limitini topishning turli usullari mavjud boʻlib[1]-[3], shular orasida Shtolts-Chezarov teoremasi[4] alohida oʻrin tutadi. Quyidagi dastlab Shtolts-Chezarov teoremasini keltiramiz, soʻngra ushbu teorema yordamida baʼzi ketma-ketliklarning limitini hisoblaymiz.

Teorema(Shtolts-Chezarov teoremasi). Aytaylik, $\{a_n\}_{n \geq 1}$ va $\{b_n\}_{n \geq 1}$ haqiqiy ketma-ketliklar berilgan boʻlsin. Agar $\{b_n\}$ ketma-ketlik qatʼiy monoton va $\lim_{n \rightarrow \infty} b_n = \infty$ shartlar oʻrinli boʻlib, ushbu

$$\lim_{n \rightarrow \infty} \frac{a_{n+1} - a_n}{b_{n+1} - b_n} = l < \infty$$

limit mavjud boʻlsa, u holda $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = l$ tenglik oʻrinli boʻladi.

1-misol. Quyidagi limitni hisoblang:

$$\lim_{n \rightarrow \infty} \frac{1}{\left(\sqrt[n]{(2n-1)!}\right)^2} \sum_{k=1}^n \left[\left(\sqrt[2k]{k!} + \sqrt[2(k+1)]{(k+1)!} \right)^2 \right].$$

Bu yerda $[x] - x$ sonning butun qismini bildiradi.

Yechimi. Shtolts-Chezarov teoremasining barcha shartlari qanoatlantirilganini eʼtiborga olgan holda, limitni hisoblaymiz:

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{1}{\left(\sqrt[n]{(2n-1)!}\right)^2} \sum_k \left[\left(\sqrt[2k]{k!} + \sqrt[2(k+1)]{(k+1)!} \right)^2 \right] = \\ & \lim_{n \rightarrow \infty} \frac{\sum_{k=1}^{n+1} \left[\left(\sqrt[2k]{k!} + \sqrt[2(k+1)]{(k+1)!} \right)^2 \right] - \sum_{k=1}^n \left[\left(\sqrt[2k]{k!} + \sqrt[2(k+1)]{(k+1)!} \right)^2 \right]}{\left(\sqrt[n+1]{(2n+1)!}\right)^2 - \left(\sqrt[n]{(2n-1)!}\right)^2} = \\ & = \lim_{n \rightarrow \infty} \frac{\left[\left(\sqrt[2(n+1)]{(n+1)!} + \sqrt[2(n+2)]{(n+2)!} \right)^2 \right]}{\left(\sqrt[n+1]{(2n+1)!}\right)^2 - \left(\sqrt[n]{(2n-1)!}\right)^2}. \end{aligned}$$

Soʻnggi limitni hisoblash uchun quyidagi Sterling formulasidan foydalanamiz:

$$n! \sim \left(\frac{n}{e}\right)^n.$$

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