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We recall that each solution of (3) corresponds to a Gibbs measure. Using Lemma 1 we obtain the following theorem.

**Theorem.** For the Ising model in a second-order Cayley tree the following assertions hold:

1) If  $\alpha \in (0; 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}; \infty)$ , then there are two non-translation-invariant Gibbs measure.

2) If  $\alpha \in (0; \sqrt{2} + 1 - \sqrt{2 + 2\sqrt{2}})$ , then there is non-translation-invariant Gibbs measure.

3) If  $\alpha \in (3 - 2\sqrt{2}; 1) \cup (1; 3 + 2\sqrt{2})$ , then there is no Gibbs measure.

**Remark.**

If  $\alpha = 3 - 2\sqrt{2}$ ,  $\alpha = 3 + 2\sqrt{2}$  and  $\alpha = 1$  the measure is translation-invariant. This has been studied in previous works (see [2]).

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## FORMULA FOR THE NUMBER OF EIGENVALUES OF A SOLUBLE MODEL OF COUPLED MOLECULAR AND NUCLEAR HAMILTONIANS

**Rasulov T.H.<sup>1</sup>, Dilmurodov E.B.<sup>2</sup>**

<sup>1</sup>t.h.rasulov@buxdu.uz, <sup>2</sup>e.b.dilmurodov@buxdu.uz

<sup>1,2</sup>Bukhara state university

**Abstract.** *The special type of a soluble model of coupled molecular and nuclear Hamiltonians  $H$  so-called generalized Friedrichs model is considered in this note. We determine the well known Faddeev operator  $T(z)$  corresponding to  $H$  and we obtain the formula for the multiplicity of discrete eigenvalues of  $H$ .*

We consider the complex Hilbert space  $\mathcal{H} := \mathcal{H}_0 \oplus \mathcal{H}_1$  (two-channel), which is the direct sum of  $\mathcal{H}_0 := \mathbb{C}$  (channel 1 – a one-dimensional "molecular" space) and  $\mathcal{H}_1$  (channel 2 – a "nuclear" space). The elements of  $\mathcal{H}$  are vectors with two coordinates:  $f = (f_0, f_1)$  with  $f_0 \in \mathcal{H}_0$  and  $f_1 \in \mathcal{H}_1$ . For convinience we remind that the scalar product of  $f = (f_0, f_1) \in \mathcal{H}$  and  $g = (g_0, g_1) \in \mathcal{H}$  is defined

$$\langle f, g \rangle_{\mathcal{H}} := f_0 \bar{g}_0 + \langle f_1, g_1 \rangle$$

using the scalar products in the spaces  $\mathcal{H}_0$  and  $\mathcal{H}_1$ .

In the Hilbert space  $\mathcal{H}$  we consider the operator matrix

$$H := \begin{pmatrix} \omega & \langle \cdot, v \rangle \\ v & h \end{pmatrix}, \quad (1)$$

where  $\omega \in \mathbb{R}$  is a trial molecular energy, a vector  $v \in \mathcal{H}_1$  provides the coupling between the channels and  $h$  is the (self-adjoint) nuclear Hamiltonian in  $\mathcal{H}_1$ . The Hamiltonian (1) can be considered as a soluble model of coupled molecular Hamiltonian and nuclear Hamiltonian. It is also well known as the generalized Friedrichs model [1, 2].

We determine the special type of nuclear Hamiltonian  $h$  so-called Friedrichs model. Let  $D_i, i = 1, \dots, n$  ( $n \in \mathbb{N}$ ) be the bounded domains with Euclidian measure in the  $d$ -dimensional space  $\mathbb{R}^d$ , such that  $D_i \cap D_j = \emptyset, i \neq j$  and  $D := \bigcup_{i=1}^n \overline{D}_i$ . In the nuclear space  $\mathcal{H}_1 := L_2(D)$  we introduce Hamiltonian  $h$  as

$$(hf_1)(p) = u(p)f_1(p) - \mu \int_D K(p, t)f_1(t)dt, \quad (2)$$

in the latter formula  $\mu > 0$  is a coupling constant,  $K(\cdot, \cdot)$  is a real-valued bounded symmetric function on  $D^2 := D \times D$  and  $u(\cdot)$  is a real-valued piecewise continuous bounded function on  $D$ . Let  $v(\cdot)$  be also a real-valued bounded function on  $D$ . Under these conditions one can easily show that the model Hamiltonian  $H$  is bounded and self-adjoint.

For the further convinience we rewrite the Hamiltonian (1) with (2) in the following form

$$H := \begin{pmatrix} H_{00} & H_{01} \\ H_{01}^* & H_{11} \end{pmatrix}, \quad (3)$$

with the entries  $H_{ij}: \mathcal{H}_j \rightarrow \mathcal{H}_i, i \leq j, i, j = 0, 1$  defined by

$$\begin{aligned} H_{00}f_0 &= \omega f_0, & H_{01}f_1 &= \int_D v(t)f_1(t)dt, \\ H_{11} &:= H_{11}^0 - K, & (H_{11}^0 f_1)(p) &= u(p)f_1(p), \\ (Kf_1)(p) &= \mu \int_D K(p, t)f_1(t)dt. \end{aligned}$$

Here  $f_i \in \mathcal{H}_i, i = 0, 1$ .

**Lemma 1.** *The relation*

$$(K^{1/2}f)(p) = \sqrt{\mu} \int_D \tilde{K}(p, t)f(t)dt$$

is valid. Here the kernel of  $K^{1/2}$  is formally denoted by  $\tilde{K}(\cdot, \cdot)$  and it is a square-integrable on  $D^2$ .

Let  $I_i$  be the identity operator on  $\mathcal{H}_i, i = 0, 1$  and  $R_{11}^0(z) := (H_{11}^0 - zI_1)^{-1}$ . We consider

$$T(z) := \begin{pmatrix} T_{00}(z) & T_{01}(z) \\ T_{10}(z) & T_{11}(z) \end{pmatrix}, \quad z \in \mathbb{C} \setminus \sigma(H_{11}^0)$$

in the Hilbert space  $\mathcal{H}$ , where  $T_{ij}(z): \mathcal{H}_j \rightarrow \mathcal{H}_i, i, j = 0, 1$  has form

$$\begin{aligned} T_{00}(z) &:= (1+z)I_0 - H_{00} + H_{01}R_{11}^0(z)H_{01}^*, \\ T_{01}(z) &:= -H_{01}R_{11}^0(z)K^{1/2}, \\ T_{10}(z) &:= -K^{1/2}R_{11}^0(z)H_{01}^*, & T_{11}(z) &:= K^{1/2}R_{11}^0(z)K^{1/2}. \end{aligned}$$

In the complex Hilbert space  $\mathcal{H}$  we consider the bounded self-adjoint operator  $B$ . For the real number  $\lambda$  we set  $\mathcal{H}_B(\lambda) \subset \mathcal{H}$  a subspace such that  $\langle Bf, f \rangle_{\mathcal{H}} > \lambda \|f\|^2$  for any  $f \in \mathcal{H}_B(\lambda)$  and we determine [3] the quantity  $n(\lambda, B)$  by

$$n(\lambda, B) := \sup_{\mathcal{H}_B(\lambda)} \dim \mathcal{H}_B(\lambda).$$

The quantity  $n(\lambda, B)$  is equal to infinity if  $\lambda$  smaller than  $\max \sigma_{\text{ess}}(B)$ ; if  $n(\lambda, B)$  is finite quantity, then it is equal to the number of the eigenvalues of  $B$  bigger than  $\lambda$ .

The number of discrete eigenvalues of  $H$  belonging to the interval  $(a, b) \subset \mathbb{R} \setminus \sigma(H_{11}^0)$  is denote by  $N_{(a,b)}(H)$ .

Now we formulate the main theorem of the present note.

**Theorem 1.** *The quantity  $z_0 \in \mathbb{R} \setminus \sigma(H_{11}^0)$  is a discrete eigenvalue of  $H$  iff  $n(1, T(\cdot))$  is discontinuous at  $z = z_0$ . Moreover, the multiplicity  $k$  of the discrete eigenvalue  $z_0$  satisfies the identity*

$$k = \lim_{\xi \rightarrow +0} [n(1, T(z_0 + \xi)) - n(1, T(z_0))] + \lim_{\xi \rightarrow +0} [n(1, T(z_0 - \xi)) - n(1, T(z_0))].$$

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## ON THE FIXED POINTS OF A QUADRATIC OPERATOR DEFINED BY A NON-STOCHASTIC CUBIC MATRIX

**Rozikov U.A.<sup>1</sup>, Jumayev J.N.<sup>2</sup>**

<sup>1</sup>rozikovu@yandex.ru, <sup>2</sup>jahongirjumayev@mail.ru

<sup>1</sup>V.I. Romanovskiy Institute of Mathematics

<sup>2</sup>Karshi State University

**Annotatsiya** *We consider a quadratic operator defined by a non-stochastic cubic matrix which maps a two-dimensional simplex to itself. Let us give main definitions.*

A quadratic operator  $V : x \in \mathbb{R}^m \rightarrow x' = V(x) \in \mathbb{R}^m$  is defined by:

$$V : x'_k = \sum_{i,j=1}^m P_{ij,k} x_i x_j, \quad k = 1, \dots, m. \quad (1)$$

Denote  $E = \{1, 2, \dots, m\}$ . A distribution on the set  $E$  is a probability measure  $x = (x_1, x_2, \dots, x_m)$ , i.e., an element of the simplex:

$$S^{m-1} = \left\{ x \in \mathbb{R}^m : x_i \geq 0, \sum_{i=1}^m x_i = 1 \right\}.$$