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We recall that each solution of (3) corresponds to a Gibbs measure. Using Lemma 1 we obtain the following theorem.

Theorem. For the Ising model in a second-order Cayley tree the following assertions hold:

1) If $\alpha \in (0;3-2\sqrt{2}) \cup (3+2\sqrt{2};\infty)$, then there are two non-translation-invariant Gibbs measure.

2) If $\alpha \in (0;\sqrt{2}+1-\sqrt{2+2\sqrt{2}})$, then there is non-translation-invariant Gibbs measure.

3) If $\alpha \in (3-2\sqrt{2};1) \cup (1;3+2\sqrt{2})$, then there is no Gibbs measure.

Remark.

If $\alpha = 3-2\sqrt{2}, \alpha = 3+2\sqrt{2}$ and $\alpha = 1$ the measure is translation-invariant. This has been studied in previous works (see [2]).

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FORMULA FOR THE NUMBER OF EIGENVALUES OF A SOLUBLE MODEL OF COUPLED MOLECULAR AND NUCLEAR HAMILTONIANS

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Abstract. *The special type of a soluble model of coupled molecular and nuclear Hamiltonians H so-called generalized Friedrichs model is considered in this note. We determine the well known Faddeev operator $T(z)$ corresponding to H and we obtain the formula for the multiplicity of discrete eigenvalues of H.*

We consider the complex Hilbert space $\mathcal{H} := \mathcal{H}_0 \oplus \mathcal{H}_1$ (two-channel), which is the direct sum of $\mathcal{H}_0 := \mathbb{C}$ (channel 1 – a one-dimensional "molecular" space) and $\mathcal{H}_1$ (channel 2 – a "nuclear" space). The elements of $\mathcal{H}$ are vectors with two coordinates: $f = (f_0, f_1)$ with $f_0 \in \mathcal{H}_0$ and $f_1 \in \mathcal{H}_1$. For convinience we remind that the scalar product of $f = (f_0, f_1) \in \mathcal{H}$ and $g = (g_0, g_1) \in \mathcal{H}$ is defined

$$\langle f, g \rangle_{\mathcal{H}} := f_0 \overline{g}_0 + \langle f_1, g_1 \rangle$$

using the scalar products in the spaces $\mathcal{H}_0$ and $\mathcal{H}_1$.

In the Hilbert space $\mathcal{H}$ we consider the operator matrix

$$H := \begin{pmatrix} \omega & \langle \cdot, v \rangle \\ v & h \end{pmatrix}, \tag{1}$$

where $\omega \in \mathbb{R}$ is a trial molecular energy, a vector $v \in \mathcal{H}_1$ provides the coupling between the channels and h is the (self-adjoint) nuclear Hamiltonian in $\mathcal{H}_1$. The Hamiltonian (1) can be considered as a soluble model of coupled molecular Hamiltonian and nuclear Hamiltonian. It is also well known as the generalized Friedrichs model [1, 2].

We determine the special type of nuclear Hamiltonian h so-called Friedrichs model. Let D_i, $i = 1, \ldots, n$ $(n \in N)$ be the bounded domains with Eucledian measure in the d-dimensional space $\mathbb{R}^{\mathrm{d}}$, such that $D_i \cap D_j = \emptyset, i \neq j$ and $D := \bigcup_{i=1}^{n} \overline{D}_i$. In the nuclear space $\mathcal{H}_1 := L_2(D)$ we introduce Hamiltonian h as

$$(hf_1)(p) = u(p)f_1(p) - \mu \int_D K(p,t)f_1(t)dt, \qquad (2)$$

in the latter formula $\mu > 0$ is a coupling constant, $K(\cdot,\cdot)$ is a real-valued bounded symmetric function on $D^2 := D \times D$ and $u(\cdot)$ is a real-valued piecewise continuous bounded function on D. Let $v(\cdot)$ be also a real-valued bounded function on D. Under these conditions one can easily show that the model Hamiltonian H is bounded and self-adjoint.

For the further convinience we rewrite the Hamiltonian (1) with (2) in the following form

$$H := \begin{pmatrix} H_{00} & H_{01} \\ H_{01}^* & H_{11} \end{pmatrix}, \qquad (3)$$

with the entries $H_{ij} : \mathcal{H}_j \to \mathcal{H}_i$, $i \leq j$, $i, j = 0{,}1$ defined by

$$H_{00}f_0 = \omega f_0, \quad H_{01}f_1 = \int_D v(t)f_1(t)dt,$$
$$H_{11} := H_{11}^0 - K, \quad (H_{11}^0 f_1)(p) = u(p)f_1(p),$$
$$(Kf_1)(p) = \mu \int_D K(p,t)f_1(t)dt.$$

Here $f_i \in \mathcal{H}_i$, $i = 0{,}1$.

Lemma 1. *The relation*

$$(K^{1/2}f)(p) = \sqrt{\mu} \int_D \widetilde{K}(p,t)f(t)dt$$

is valid. Here the kernel of $K^{1/2}$ is formally denoted by $\widetilde{K}(\cdot,\cdot)$ and it is a square-integrable on D^2.

Let I_i be the identity operator on $\mathcal{H}_i$, $i = 0{,}1$ and $R_{11}^0(z) := (H_{11}^0 - zI_1)^{-1}$. We consider

$$T(z) := \begin{pmatrix} T_{00}(z) & T_{01}(z) \\ T_{10}(z) & T_{11}(z) \end{pmatrix}, \quad z \in \mathbb{C} \backslash \sigma(H_{11}^0)$$

in the Hilbert space $\mathcal{H}$, where $T_{ij}(z) : \mathcal{H}_j \to \mathcal{H}_i$, $i, j = 0{,}1$ has form

$$T_{00}(z) := (1+z)I_0 - H_{00} + H_{01}R_{11}^0(z)H_{01}^*,$$
$$T_{01}(z) := -H_{01}R_{11}^0(z)K^{1/2},$$
$$T_{10}(z) := -K^{1/2}R_{11}^0(z)H_{01}^*, \quad T_{11}(z) := K^{1/2}R_{11}^0(z)K^{1/2}.$$

In the complex Hilbert space $\mathcal{H}$ we consider the bounded self-adjoint operator B. For the real number λ we set $\mathcal{H}_B(\lambda) \subset \mathcal{H}$ a subspace such that $\langle Bf, f \rangle_{\mathcal{H}} > \lambda \| f \|^2$ for any $f \in \mathcal{H}_B(\lambda)$ and we determine [3] the quantity $n(\lambda, B)$ by

$$n(\lambda, B) := \sup_{\mathcal{H}_B(\lambda)} \dim \mathcal{H}_B(\lambda).$$

The quantity $n(\lambda, B)$ is equal to infinity if λ smaller than $\max\sigma_{\text{ess}}(B)$; if $n(\lambda, B)$ is finite quantity, then it is equal to the number of the eigenvalues of B bigger than λ.

The number of discrete eigenvalues of H belonging to the interval $(a, b) \subset \mathbb{R}\setminus\sigma(H_{11}^0)$ is denote by $N_{(a,b)}(H)$.

Now we formulate the main theorem of the present note.

Theorem 1. *The quantity $z_0 \in \mathbb{R}\setminus\sigma(H_{11}^0)$ is a discrete eigenvalue of H iff $n(1, T(\cdot))$ is discontinuous at $z = z_0$. Moreover, the multiplicity k of the discrete eigenvalue z_0 satisfies the identity*

$$k = \lim_{\xi\to+0}[n(1, T(z_0+\xi)) - n(1, T(z_0))] + \lim_{\xi\to+0}[n(1, T(z_0-\xi)) - n(1, T(z_0))].$$

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ON THE FIXED POINTS OF A QUADRATIC OPERATOR DEFINED BY A NON-STOCHASTIC CUBIC MATRIX

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Annotatsiya *We consider a quadratic operator defined by a non-stochastic cubic matrix which maps a two-dimensional simplex to itself. Let us give main definitions.*

A quadratic operator $V : x \in \mathbb{R}^m \to x' = V(x) \in \mathbb{R}^m$ is defined by:

$$V : x'_k = \sum_{i,j=1}^{m} P_{ij,k} x_i x_j, \quad k = 1, \dots, m. \tag{1}$$

Denote $E = \{1, 2, \dots, m\}$. A distribution on the set E is a probability measure $x = (x_1, x_2, \dots, x_m)$, i.e., an element of the simplex:

$$S^{m-1} = \left\{ x \in \mathbb{R}^m : x_i \ge 0, \sum_{i=1}^{m} x_i = 1 \right\}.$$