O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

1930 BUXORO DAVLAT UNIVERSITETI

FIZIKA, MATEMATIKA VA MEXANIKANING DOLZARB MUAMMOLARI XALQARO ILMIY-AMALIY ANJUMANI

MATERIALLARI

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Buxoro, O‘zbekiston, 24-25-may, 2023-yil

МИНИСТЕРСТВО ВЫСШЕГО ОБРАЗОВАНИЯ, НАУКИ И ИННОВАЦИЙ РЕСПУБЛИКИ УЗБЕКИСТАН
БУХАРСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

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ABSTRACTS

(Part I)

of the international scientific and practical conference

ACTUAL PROBLEMS OF PHYSICS, MATHEMATICS AND MECHANICS

Bukhara, Uzbekistan, May 24-25, 2023

Remark. *There is a difference between Theorem 1 and Theorem 1 of [2], i.e., for any $\sigma_b \in C_5$, there exists a configuration φ (generally not periodic) on the Cayley tree such that $\varphi_{b'} \in C_5$ for any $b' \in M$ and $\varphi_b = \sigma_b$.*

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SOME PROPERTIES OF THE FIRST SCHUR COMPLEMENT CORRESPONDING TO A LATTICE SPIN-BOSON MODEL WITH AT MOST TWO PHOTONS

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The spin-boson model is a well-known quantum-mechanical model which describes the interaction between a two-level atom and a photon field. We refer to [1] and [2] for excellent reviews respectively from physical and mathematical perspectives.

Let us introduce a lattice spin-boson model with at most two photons. Let T^{d} be the d–dimensional torus, $L_2((T^{\mathrm{d}})$ be the Hilbert space of square integrable (complex) functions defined on T^{d}, $\mathbb{C}^2$ be the state of the two-level atom and $\mathcal{F}_{\mathrm{b}}(L_2(T^{\mathrm{d}}))$ be the symmetric Fock space for bosons, that is,

$$\mathcal{F}_{\mathrm{b}}(L_2(T^{\mathrm{d}})) := \mathbb{C} \oplus L_2(T^{\mathrm{d}}) \oplus L_2^{sym}((T^{\mathrm{d}})^2) \oplus \ldots.$$

Here $L_2^{sym}((T^{\mathrm{d}})^n)$ is the Hilbert space of symmetric functions of $n \geq 2$ variables. For $m = 1,2$ we denote $\mathcal{L}_m := \mathbb{C}^2 \otimes \mathcal{F}_{\mathrm{b}}^{(m)}(L_2(T^{\mathrm{d}}))$, where

$$\mathcal{F}_{\mathrm{b}}^{(1)}(L_2(T^{\mathrm{d}})) := \mathbb{C} \oplus L_2(T^{\mathrm{d}}), \quad \mathcal{F}_{\mathrm{b}}^{(2)}(L_2(T^{\mathrm{d}})) := \mathbb{C} \oplus L_2(T^{\mathrm{d}}) \oplus L_2^{sym}((T^{\mathrm{d}})^2).$$

We write elements F of the space L_2 in the form $F = \{f_0^{(\mathrm{s})}, f_1^{(\mathrm{s})}(k_1), f_2^{(\mathrm{s})}(k_1,k_2); \mathrm{s} = \pm\}$. Then the norm in $\mathcal{L}_2$ is given by

$$\| F \|^2 := \sum_{\mathrm{s}=\pm} \left(|f_0^{(\mathrm{s})}|^2 + \int_{T^{\mathrm{d}}} |f_1^{(\mathrm{s})}(k_1)|^2 dk_1 + \frac{1}{2} \int_{(T^{\mathrm{d}})^2} |f_2^{(\mathrm{s})}(k_1,k_2)|^2 dk_1 dk_2 \right). \quad (1)$$

We recall that the lattice spin-boson model with at most two photons A_2 is acting in $\mathcal{L}_2$ as the 3×3 tridiagonal block operator matrix

$$A_2 := \begin{pmatrix} A_{00} & A_{01} & 0 \\ A_{01}^* & A_{11} & A_{12} \\ 0 & A_{12}^* & A_{22} \end{pmatrix},$$

where matrix elements A_{ij} are defined by

$$A_{00} f_0^{(\mathrm{s})} = \mathrm{s}\varepsilon f_0^{(\mathrm{s})}, \quad A_{01} f_1^{(\mathrm{s})} = \alpha \int_{T^{\mathrm{d}}} v(t) f_1^{(-\mathrm{s})}(t) dt,$$

$$(A_{11} f_1^{(\mathrm{s})})(k_1) = (\mathrm{s}\varepsilon + w(k_1)) f_1^{(\mathrm{s})}(k_1), \quad (A_{12} f_2^{(\mathrm{s})})(k_1) = \alpha \int_{T^{\mathrm{d}}} v(t) f_2^{(-\mathrm{s})}(k_1,t) dt,$$

$$(A_{22} f_2^{(\mathrm{s})})(k_1,k_2) = (\mathrm{s}\varepsilon + w(k_1) + w(k_2)) f_2^{(\mathrm{s})}(k_1,k_2),$$

$$f = \{f_0^{(\mathrm{s})}, f_1^{(\mathrm{s})}, f_2^{(\mathrm{s})}; \mathrm{s} = \pm\} \in \mathcal{L}_2.$$

Here A_{ij}^* denotes the adjoint operator to A_{ij} for $i < j$ with $i,j = 0,1,2$; $w(k)$ is the dispersion of the free field, $\alpha v(k)$ is the coupling between the atoms and the field modes, $\alpha > 0$ is a real number, so-called the coupling constant, real number. We assume that $v(\cdot)$ and $w(\cdot)$ are the real-valued continuous functions on T^{d}. Under these assumptions the lattice spin-boson model with at most two photons A_2 is bounded and self-adjoint in the complex Hilbert space $\mathcal{L}_2$.

To study the spectral properties of A_2 we introduce the following two bounded self-adjoint operators $A_2^{(\mathrm{s})}$, $\mathrm{s}=\pm$, which acts in $\mathcal{F}_{\mathrm{b}}^{(2)}(L_2(T^{\mathrm{d}}))$ as

$$A_2^{(\mathrm{s})} := \begin{pmatrix} \hat{A}_{00}^{(\mathrm{s})} & \hat{A}_{01} & 0 \\ \hat{A}_{01}^* & \hat{A}_{11}^{(\mathrm{s})} & \hat{A}_{12} \\ 0 & \hat{A}_{12}^* & \hat{A}_{22}^{(\mathrm{s})} \end{pmatrix}$$

with the entries

$$\hat{A}_{00}^{(\mathrm{s})} f_0 = \mathrm{s}\varepsilon f_0, \quad \hat{A}_{01} f_1 = \alpha \int_{T^{\mathrm{d}}} v(t) f_1(t) dt,$$

$$(\hat{A}_{11}^{(\mathrm{s})} f_1)(k_1) = (-\mathrm{s}\varepsilon + w(k_1)) f_1(k_1), \quad (\hat{A}_{12} f_2)(k_1) = \alpha \int_{T^{\mathrm{d}}} v(t) f_2(k_1, t) dt,$$

$$(\hat{A}_{22}^{(\mathrm{s})} f_2)(k_1, k_2) = (\mathrm{s}\varepsilon + w(k_1) + w(k_2)) f_2(k_1, k_2), \quad (f_0, f_1, f_2) \in \mathcal{F}_{\mathrm{b}}^{(2)}(L_2(T^{\mathrm{d}})).$$

It is easy to check that

$$(\hat{A}_{01}^* f_0)(k_1) = \alpha v(k_1) f_0;$$

$$(\hat{A}_{12}^* f_1)(k_1, k_2) = \alpha (v(k_1) f_1(k_2) + v(k_2) f_1(k_1)), \quad (f_0, f_1) \in \mathcal{F}_{\mathrm{b}}^{(1)}(L_2(T^{\mathrm{d}})).$$

In order to describe the essential spectrum of A_2 we define an analytic function $\Delta^{(\mathrm{s})}(\cdot)$ in $\mathbb{C} \backslash [\mathrm{s}\varepsilon + m; \mathrm{s}\varepsilon + M]$ by

$$\Delta^{(\mathrm{s})}(\lambda) := -\mathrm{s}\varepsilon - \lambda - \alpha^2 \int_{T^{\mathrm{d}}} \frac{v^2(t) dt}{\mathrm{s}\varepsilon + w(t) - \lambda},$$

where the numbers m and M are defined by

$$m := \min_{p \in T^{\mathrm{d}}} w(p), \quad M := \max_{p \in T^{\mathrm{d}}} w(p).$$

Let $\sigma^{(\mathrm{s})}$ be the set of all complex numbers $\lambda \in \mathbb{C}$ such that the equality $\Delta^{(\mathrm{s})}(\lambda - w(k_1)) = 0$ holds for some $k_1 \in T^{\mathrm{d}}$. Then for the essential spectrum of $A_2^{(\mathrm{s})}$ we have

$$\sigma_{ess}(A_2^{(\mathrm{s})}) = \sigma^{(\mathrm{s})} \cup [\mathrm{s}\varepsilon + 2m; \mathrm{s}\varepsilon + 2M].$$

Next, we represent the space $\mathcal{F}_{\mathrm{b}}^{(2)}(L_2(T^{\mathrm{d}}))$ as a direct sum of two Hilbert spaces $\mathcal{F}_{\mathrm{b}}^{(1)}(L_2(T^{\mathrm{d}}))$ and $L_2^{sym}((T^{\mathrm{d}})^2)$, that is, $\mathcal{F}_{\mathrm{b}}^{(2)}(L_2(T^{\mathrm{d}})) = \mathcal{F}_{\mathrm{b}}^{(1)}(L_2(T^{\mathrm{d}})) \oplus L_2^{sym}((T^{\mathrm{d}})^2)$. Then the first Schur complement of the operator $A_2^{(\mathrm{s})}$ with respect to this decomposition is defined as

$$S_1^{(\mathrm{s})}(\lambda): \mathcal{F}_{\mathrm{b}}^{(1)}(L_2(T^{\mathrm{d}})) \to \mathcal{F}_{\mathrm{b}}^{(1)}(L_2(T^{\mathrm{d}})), \quad \lambda \in \rho(\hat{A}_{22}^{(\mathrm{s})});$$

$$S_1^{(\mathrm{s})}(\lambda) := \begin{pmatrix} \hat{A}_{00}^{(\mathrm{s})} & \hat{A}_{01} \\ \hat{A}_{01}^{*} & \hat{A}_{11}^{(\mathrm{s})} \end{pmatrix} - \lambda - \begin{pmatrix} 0 \\ \hat{A}_{12} \end{pmatrix} (\hat{A}_{22}^{(\mathrm{s})} - \lambda)^{-1} (0 \quad \hat{A}_{12}^{*}).$$

Define

$$S_{00}^{(\mathrm{s})}(\lambda) := \hat{A}_{00}^{(\mathrm{s})} - \lambda, \quad S_{01}^{(\mathrm{s})}(\lambda) := \hat{A}_{01};$$

$$S_{10}^{(\mathrm{s})}(\lambda) := \hat{A}_{01}^{*}, \quad S_{11}^{(\mathrm{s})}(\lambda) := \hat{A}_{11}^{(\mathrm{s})} - \lambda - \hat{A}_{12}(\hat{A}_{22}^{(\mathrm{s})} - \lambda)^{-1}\hat{A}_{12}^{*}.$$

Then the operator $S_1^{(\mathrm{s})}(\lambda)$ has form

$$S_1^{(\mathrm{s})}(\lambda) = \begin{pmatrix} S_{00}^{(\mathrm{s})}(\lambda) & S_{01}^{(\mathrm{s})}(\lambda) \\ S_{10}^{(\mathrm{s})}(\lambda) & S_{11}^{(\mathrm{s})}(\lambda) \end{pmatrix}.$$

We study some important properties of the first Schur complement

$$S_1(\lambda) := \mathrm{diag}\{S_1^{(+)}(\lambda), S_1^{(-)}(\lambda)\}$$

for the lattice spin-boson model with at most two photons A_2.

Proposition 1. *The number $\lambda \in \mathbb{C}\backslash\sigma_{ess}(A_2)$ is an eigenvalue of the operator A_2 if and only if the operator $S_1(\lambda)$ has an eigenvalue equal to zero. Moreover, the eigenvalues λ and 0 have the same multiplicities.*

Proposition 2. $\lambda \in \sigma_{ess}(A_2)\backslash\sigma(A_{22})$ *if and only if* $0 \in \sigma_{ess}(S_1(\lambda))$.

From Propositions 1 and 2 we obtain the following two corollaries.

Corollary 1. *Let $\lambda \in \mathbb{C}\backslash\sigma_{ess}(A_2)$. Then $\lambda \in \rho(A_2) \Leftrightarrow 0 \in \rho(S_1(\lambda))$.*

Corollary 2. *Let $\lambda_0 \in \mathbb{R}\backslash\sigma_{ess}(A_2)$. If $(\lambda_0; \lambda_0 + \gamma) \in \rho(A_2)$ (resp. $(\lambda_0 - \gamma; \lambda_0) \in \rho(A_2)$) for some $\gamma > 0$, then there exists a number $\delta = \delta(\gamma) > 0$ such that $(0; \delta) \in \rho(S_1(\lambda_0))$ (resp. $(-\delta; 0) \in \rho(S_1(\lambda_0))$).*

We note that a lattice spin-boson model A_m with $m = 1,2$ is considered in [3, 4]. In particular, in [3] the location of the essential spectrum of A_2 is described; for any coupling constant the finiteness of the number of eigenvalues below the bottom of the essential spectrum of A_2 is established (with sketch of proof). The paper [4] is devoted to the study of the geometrical structure of the branches of the essential spectrum of A_2.

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THE TWO ORTHOGONAL PROJECTORS IN SEPARABLE GILBERT SPACE

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The theory of unitary representations of groups dates back to the 19-th century and is associated with the names of G. Frobenius, I. Schur, W. Burnside, F.E. Molina

MUNDARIJA
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