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FERMIONLI FOK FAZODAGI UCHINCHI TARTIBLI OPERATORLI MATRITSAGA MOS KANAL OPERATOR VA UNING SPEKTRI

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Annotatsiya. *Mazkur maqolada fermionli Fok fazoning nol zarrachali, bir zarrachali va ikki zarrachali qism fazolarining to'g'ri yig'indisida ta'sir qiluvchi $\mathcal{A}_{\mu,\lambda}$, $\mu,\lambda > 0$ uchinchi tartibli operatorli matritsa qaraladi. Bu operatorli matritsa panjaradagi soni saqlanmaydigan va uchtadan oshmaydigan zarrachalar sistemasiga mos keladi hamda chiziqli, chegaralangan va o'z-o'ziga qo'shma operator bo'ladi. $\mathcal{A}_{\mu,\lambda}$ operatorli matritsaga mos kanal operator quriladi. Uning spektrini aniqlash masalasi to'g'ri integralga yoyish orqali umumlashgan Fridrixs modelining spektrini o'rganish masalasiga keltiriladi. Kanal operator sof muhim spektrga ega ekanligi ko'rsatiladi va uni aniqlovchi kesmalarning maksimal soni topiladi.*

Kalit so'zlar: *fermionli Fok fazo, antisimmetrik funksiya, operatorli matritsa, kanal operator, umumlashgan Fridrixs modeli, to'g'ri integralga yoyish, sof muhim spektr.*

Аннотация. *В настоящей статье рассматривается операторная матрица третьего порядка $\mathcal{A}_{\mu,\lambda}$, $\mu,\lambda > 0$, действующая в прямой сумме ноль частичного, одно частичного и двухчастичного подпространств фермионного пространства Фока. Эта операторная матрица соответствует системе частиц с несохраняющимся и не более трёх частиц на решётке и является линейным, ограниченным и самосопряжённым. Построен канальный оператор, соответствующий канальному оператору $\mathcal{A}_{\mu,\lambda}$. Задача определения его спектра с использованием разложения на прямой интеграл приводится к изучению спектра обобщённой модели Фридрихса. Показывается, что канальный оператор имеет чисто существенный спектр, и найдено максимальное число определяющих его отрезков.*

Ключевые слова: *фермионное пространство Фока, антисимметрические функции, операторная матрица, канальный оператор, обобщённая модель Фридрихса, разложение прямого интеграла, чисто существенный спектр.*

Abstract. *In this paper an operator matrix of order three $\mathcal{A}_{\mu,\lambda}$, $\mu,\lambda > 0$ acting in the zero-particle, one-particle and two-particle subspaces of fermionic Fock space is considered. This operator matrix is corresponding to a system of nonconserved and no more than three particles on a lattice and it is a linear, bounded and self-adjoint. The channel operator corresponding to the operator matrix $\mathcal{A}_{\mu,\lambda}$ is constructed. The problem to define its spectrum using direct integral decomposition is reduced to the problem of the studying the spectrum of the generalized Friedrich model. It is shown that the channel operator has a purely essential spectrum and the maximal number its segments is found.*

Key words: *fermionic Fock space, antisymmetric function, operator matrix, channel operator, generalized Friedrix model, direct integral decomposition, purely essential spectrum.*

Kirish. $\mathbb{R}^d$ haqiqiy Yevklid fazosidagi va $\mathbb{Z}^d$ butun sonli panjaradagi soni saqlanmaydigan chekli sondagi zarrachalar sistemasiga mos Hamiltonianlarning spektrini tadqiq qilish masalasi qattiq jismlar fizikasi [1], kvant maydon nazariyasi [2], statistik fizika [3] va zamonaviy matematik fizikaning yana ko'plab sohalarda ko'p o'rganiladigan masalalaridan biri hisoblanadi. $\mathbb{R}^d$ haqiqiy Yevklid fazosidagi soni saqlanmaydigan chekli sondagi zarrachalar sistemasiga mos Hamiltonianlar (spin-bozon modeli nomi bilan mashhur bo'lgan Hamiltonianlar) [4,5] maqolalarda tadqiq qilingan. $\mathbb{Z}^d$ butun sonli panjaradagi soni saqlanmaydigan chekli sondagi zarrachalar sistemasiga mos matritsaviy model operatorlarning muhim va diskret spektrlari esa [6-8] ishlarda batafsil o'rganilgan. [9,10] maqolalarda esa bunday turdagi model operatorlar uchun olingan ilmiy natijalar yordamida panjaradagi ko'pi bilan ikkita fotonga ega spin-bozon modelining muhim spektri, nuqtali spektri va diskret spektri haqidagi ma'lumotlar natija sifatida keltirib chiqarilgan.

Asosiy qism. Ushbu maqolada fermionli Fok fazosining nol zarrachali, bir zarrachali va ikki zarrachali qism fazolarining to'g'ri yig'indisida ta'sir qiluvchi hamda ikkita ta'sirlashish parametrlari

$\mu, \lambda > 0$ dan bog‘liq $\mathcal{A}_{\mu,\lambda}$ uchinchi tartibli operatorli matritsa chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator sifatida qaraladi. Ta’kidlash joizki, $\mathcal{A}_{\mu,\lambda}$ operatorli matritsa [11,12] maqolalarda o‘rganilgan operatorli matritsaning kompakt bo‘lmagan qo‘zg‘alishi hisoblanadi. $\mathcal{A}_{\mu,\lambda}$ operatorli matritsaga nisbatan sodda ko‘rinishga ega bo‘lgan kanal operator deb ataluvchi ikkinchi tartibli operatorli matritsa aniqlangan. Kanal operatorning spektrini topish masalasi ko‘pi bilan 3 o‘lchamli qo‘zg‘alishga ega umumlashgan Fridrixs modelining spektrini o‘rganish masalasiga keltirilgan. Umumlashgan Fridrixs modelining spektri yordamida kanal operatorning spektri topilgan.

2. Fermionli Fok fazodagi uchinchi tartibli operatorli matritsa. Faraz qilaylik, $\mathbb{T}^1 = (-\pi, \pi]$ bo‘lsin. $\mathbb{T}^1$ da qo‘shish va songa ko‘paytirish amallarini haqiqiy sonlarni 2π modul bo‘yicha qo‘shish va songa ko‘paytirish sifatida kiritamiz, masalan

$$\frac{\pi}{2} + \pi = \frac{3\pi}{2} = -\frac{\pi}{2} \,(\mathrm{mod}\, 2\pi);$$

$$6 \cdot \frac{\pi}{5} = 2\pi - \frac{4\pi}{5} = -\frac{4\pi}{5} \,(\mathrm{mod}\, 2\pi).$$

Ushbu to‘plamga bir o‘lchamli tor deyiladi. Istalgan d natural soni uchun $\mathbb{T}^d$ orqali d o‘lchamli torni, ya’ni $\mathbb{T}^d = \underbrace{\mathbb{T}^1 \times \mathbb{T}^1 \times \ldots \times \mathbb{T}^1}_{d\ marta}$ Dekart ko‘paytmani belgilaymiz.

$\mathbb{C}$ orqali bir o‘lchamli kompleks fazoni, $L_2(\mathbb{T}^d)$ orqali $\mathbb{T}^d$ da aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatlarni qabul qiluvchi) funksiyalarning Hilbert fazosini va $L_2^{as}\big((\mathbb{T}^d)^2\big)$ orqali $(\mathbb{T}^d)^2$ da aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatlarni qabul qiluvchi) antisimmetrik funksiyalarning Hilbert fazosini belgilaymiz.

Qulaylik uchun quyidagi fazolarni kiritamiz:

$$\mathcal{F}_a^{(1)}(L_2(\mathbb{T}^d)) := \mathbb{C} \oplus L_2(\mathbb{T}^d);$$

$$\mathcal{F}_a^{(2)}(L_2(\mathbb{T}^d)) := \mathbb{C} \oplus L_2(\mathbb{T}^d) \oplus L_2^{as}\big((\mathbb{T}^d)^2\big);$$

$$\mathcal{F}_a(L_2(\mathbb{T}^d)) := \mathbb{C} \oplus L_2(\mathbb{T}^d) \oplus L_2^{as}\big((\mathbb{T}^d)^2\big) \oplus \ldots .$$

1-ta’rif. *$\mathcal{F}_a(L_2(\mathbb{T}^d))$ fazoga $L_2(\mathbb{T}^d)$ fazo yordamida qurilgan fermionli Fok fazo, $\mathcal{F}_a^{(m)}(L_2(\mathbb{T}^d)), m = 1,2$ fazoga fermionli Fok fazoning qirqilgan $m+1$ zarrachali qism fazosi, $\mathcal{H}_n, n = 0,1,2$ fazoga fermionli Fok fazoning n zarrachali qism fazosi deyiladi.*

Foydalanishga qulaylik uchun

$$\mathcal{H}_0 := \mathbb{C},\ \mathcal{H}_1 := L_2(\mathbb{T}^d),\ \mathcal{H}_2 := L_2^{as}\big((\mathbb{T}^d)^2\big)$$

kabi belgilashlarni kiritamiz.

Eslatib o‘tish lozimki, $\mathcal{F}_a^{(2)}(L_2(\mathbb{T}^d))$ fazoning ixtiyoriy f elementi $f = (f_0, f_1, f_2)$ ko‘rinishdagi vektor-funksiya bo‘lib, bu fazodan olingan istalgan ikkita $f = (f_0, f_1, f_2)$ va $g = (g_0, g_1, g_2)$ elementlarning skalyar ko‘paytmasi

$$(f, g) = f_0 \cdot \overline{g_0} + \int_{\mathbb{T}^d} f_1(p) \cdot \overline{g_1(p)} dp + \int_{(\mathbb{T}^d)^2} f_2(p,q) \cdot \overline{g_2(p,q)} dp dq$$

kabi aniqlanadi.

$\mathcal{F}_a^{(2)}(L_2(\mathbb{T}^d))$ Hilbert fazosida

$$\mathcal{A}_{\mu,\lambda} := \begin{pmatrix} A_{00} & \mu A_{01} & 0 \\ \mu A_{01}^* & A_{11} & \mu A_{12} \\ 0 & \mu A_{12}^* & A_{22}^0 - \lambda V \end{pmatrix} \qquad (1)$$

kabi aniqlangan uchinchi tartibli operatorli matritsani qaraymiz. Bu yerda $A_{ij}: \mathcal{H}_j \to \mathcal{H}_i, i, j = 0,1,2$ matritsaviy elementlar quyidagi tengliklar yordamida aniqlanadi:

$$(A_{00} f_0) = \omega_0 f_0,\ \ (A_{01} f)_1 = \int_{\mathbb{T}^d} v(t) f_1(t) dt,$$

$$(A_{11} f_1)(p) = \omega_1(p) f_1(p),\ \ (A_{12} f_2)(p) = \int_{\mathbb{T}^d} v(t) f_2(p,t) dt,$$

$$(A_{22}^0 f_2)(p,q) = \omega_2(p,q) f_2(p,q), \quad V := V_1 + V_2\,,$$

$$(V_1 f_2)(p,q) = \varphi(q) \int_{\mathbb{T}^d} \varphi(t) f_2(p,t)dt, \quad (V_2 f_2)(p,q) = \varphi(p) \int_{\mathbb{T}^d} \varphi(t) f_2(t,q)dt\,.$$

Bunda μ, λ haqiqiy musbat sonlar, ω_0 tayinlangan haqiqiy son, $\omega_1(\cdot), v(\cdot)$ va $\varphi(\cdot)$ funksiyalar $\mathbb{T}^d$ da aniqlangan haqiqiy qiymatli uzluksiz funksiyalar, $\omega_2(\cdot,\cdot)$ esa $(\mathbb{T}^d)^2$ da aniqlangan haqiqiy qiymatli simmetrik uzluksiz funksiya.

Odatda zamonaviy matematik fizikada A_{01} va A_{12} operatorlarga yo'qotish operatorlari, ularga qo'shma bo'lgan A_{01}^* va A_{12}^* operatorlarga esa paydo qilish operatorlari deyiladi.

O'quvchiga qulaylik uchun A_{01}^* va A_{12}^* operatorlarni hisoblash bo'yicha ayrim mulohazalarni keltiramiz.

$$(A_{01} f_1, f_0) = \int_{\mathbb{T}^d} v(t) f_1(t)dt \cdot \overline{f_0} = \int_{\mathbb{T}^d} f_1(t)\overline{f_0 v(t)}dt = (f_1, A_{01}^* f_0)$$

$$(A_{12} f_2, f_1) = \int_{\mathbb{T}^d} \left(\int_{\mathbb{T}^d} v(s) f_2(t,s)ds \right) \cdot \overline{f_1(t)}dt = \int_{(\mathbb{T}^d)^2} v(s)\overline{f_1(t)} f_2(t,s)dsdt =$$

$$\int_{(T^d)^2} f_2(t,s) \cdot \overline{\frac{1}{2}\big(v(s) f_1(t) - v(t) f_1(s)\big)}dsdt = (f_2, A_{12}^* f_1).$$

Shunday qilib, quyidagi tengliklar o'rinli ekan:

$$(A_{01}^* f_0)(p) = v(p) f_0, f_0 \in \mathcal{H}_0;$$

$$(A_{12}^* f_1)(p,q) = \tfrac{1}{2}\big(v(p) f_1(q) - v(q) f_1(p)\big), f_1 \in \mathcal{H}_1.$$

1-lemma. $\mathcal{F}_a^{(2)}(L_2(\mathbb{T}^d))$ *Hilbert fazosida* (1)-*formula orqali ta'sir qiluvchi* $\mathcal{A}_{\mu,\lambda}$, $\mu, \lambda > 0$ *operatorli matritsa chiziqli, chegaralangan va o'z-o'ziga qo'shma operator bo'ladi.*

1-lemma Funksional analiz kursining tegishli ta'riflari va parametrlarga qo'yilgan shartlardan foydalanib oson isbotlanadi.

Ma'lumki, impuls ko'rinishdagi H uch zarrachali diskret Shryodinger operatori $L_2\big((\mathbb{T}^d)^3\big)$ Hilbert fazosida ta'sir qiladi. Sistemaning $K \in \mathbb{T}^3$ to'la kvaziimpulsini ajratishdan keyin H operator to'g'ri operatorli integralga yoyiladi (masalan [13, 14] maqolalarga qarang)

$$H = \int_{\mathbb{T}^d} \oplus H(K)dK$$

bu yerda chegaralangan o'z-o'ziga qo'shma $H(K), K \in \mathbb{T}^d$ operator $L_2(\Gamma_K)$ Hilbert fazosida ta'sir qiladi
($\Gamma_K \subset (\mathbb{T}^d)^2$ biror ko'pxillik).

Ta'kidlash joizki, $\mathcal{A}_{\mu,\lambda}$ operatorli matritsa $H(0)$ uch zarrachali diskret Shryodinger operator uchun o'rinli bo'lgan asosiy xossalarga ega, bu yerda ikki zarrachali diskret Shryodinger operator rolini umumlashgan Fridrixs modeli bajaradi [6-8]. Shu sababga ko'ra $\mathcal{F}_a^{(2)}(L_2(\mathbb{T}^d))$ Hilbert fazosi $\mathcal{F}_a(L_2(\mathbb{T}^d))$ fermionli Fok fazosining *uch zarrachali qirqilgan qism fazosi*, $\mathcal{A}_{\mu,\lambda}$ operatorli matritsaga esa panjaradagi soni saqlanmaydigan va uchtadan oshmaydigan zarrachalar sistemasiga mos operator deyiladi.

3. Kanal operator va uning spektri. Bu bo'limda to'g'ri operatorli integralga ([15], XIII.84-teorema) yoyish orqali kanal operatorning spektral xossalarini o'rganish masalasi umumlashgan Fridrixs modellari oilasining spektral xossalarini o'rganish masalasiga keltiriladi.

Quyidagicha belgilashlar kiritamiz:

$$\widehat{\mathcal{H}_1} := \mathcal{H}_1, \quad \widehat{\mathcal{H}}_2 := L_2((\mathbb{T}^d)^2), \quad \widehat{\mathcal{H}} := \widehat{\mathcal{H}_1} \oplus \widehat{\mathcal{H}_2}\,.$$

$\mathcal{A}_{\mu,\lambda}$ operatorli matritsa bilan bir qatorda kanal operator deb ataluvchi va $\widehat{\mathcal{H}}$ Hilbert fazosida

$$\mathcal{A}_{\mu,\lambda}^{\mathrm{ch}} := \begin{pmatrix} A_{11} & (\mu/\sqrt{2})A_{12} \\ (\mu/\sqrt{2})A_{12}^* & A_{22}^0 - \lambda V_1 \end{pmatrix}$$

kabi aniqlangan ikkinchi tartibli operatorli matritsani qaraymiz.

$\mathcal{A}_{\mu,\lambda}^{\mathrm{ch}}$ kanal operatorning $\widehat{\mathcal{H}}$ Hilbert fazodagi chiziqli, chegaralangan va o'z-o'ziga qo'shma ekanligini oson tekshirish mumkin.

$\mathcal{H}_0 \oplus \mathcal{H}_1$ fazoda ta'sir qiluvchi $\mathcal{A}_{\mu,\lambda}(k), k \in \mathbb{T}^d$ umumlashgan Fridrixs modelini

$$\mathcal{A}_{\mu,\lambda}(k) := \begin{pmatrix} A_{00}(k) & \mu A_{01} \\ \mu A_{01}^* & A_{11}^0(k) - \lambda v \end{pmatrix}$$

ko‘rinishda aniqlaymiz. Bu yerda

$$A_{00}(k)f_0=\omega_1(k)f_0,\qquad (A_{11}^0(k)f_1)(q)=\omega_2(k,q)f_1(k),\qquad (vf_1)(q)=\varphi(q)\int_{\mathbb{T}^d}\varphi(t)f_1(t)dt.$$

Ko‘rinib turibdiki, $\mathcal{A}_{\mu,\lambda}(k), k\in\mathbb{T}^d$ umumlashgan Fridrixs modeli $\mathcal{H}_0\oplus\mathcal{H}_1$ fazoda ikkinchi tartibli operatorli matritsa ko‘rinishidagi chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator bo‘ladi. Uning muhim spektrini aniqlash maqsadida $\mathcal{A}_{0,0}(k), k\in\mathbb{T}^d$ operatorli matritsani qaraymiz. Bu operatorli matritsa $\mathcal{H}_0\oplus\mathcal{H}_1$ fazoda $\mathcal{A}_{\mu,\lambda}(k)$ operatorli matritsada $\mu=\lambda=0$ deb olish hisobiga quyidagicha aniqlanadi:

$$\mathcal{A}_{0,0}(k)=\begin{pmatrix}A_{00}(k) & 0\\ 0 & A_{11}^{(0)}(k)\end{pmatrix},\qquad k\in\mathbb{T}^d.$$

$\mathcal{A}_{0,0}(k), k\in\mathbb{T}^d$ operatorli matritsaning qo‘zg‘alish operatori $\mathcal{A}_{\mu,\lambda}(k)-\mathcal{A}_{0,0}(k)$, $k\in\mathbb{T}^d$ ko‘pi bilan uch o‘lchamli chegaralangan va o‘z-o‘ziga qo‘shma operatorli matritsa bo‘ladi. Chekli o‘lchamli qo‘zg‘alishlashlarda muhim spektrning o‘zgarmasligi haqidagi mashhur Veyl teoremasiga ([15], XIII.14-teorema) ko‘ra, $\mathcal{A}_{\mu,\lambda}(k), k\in\mathbb{T}^d$ operatorli matritsaning muhim spektri $\mathcal{A}_{0,0}(k)$, $k\in\mathbb{T}^d$ operatorli matritsaning muhim spektri bilan ustma-ust tushishi kelib chiqadi. Ma’lumki,

$$\sigma_{\text{ess}}(\mathcal{A}_{0,0}(k)\,)\;=\;[m(k);\;M(k)],$$

Bu yerda $m(k)$ va $M(k)$ quyidagicha aniqlangan:

$$m(k)=\min\nolimits_{q\in\mathbb{T}^d} w_2(k,q)\,,\;M(k)=\max\nolimits_{q\in\mathbb{T}^d} w_2(k,q).$$

Oxirgi ikki tenglikdan $\mathcal{A}_{\mu,\lambda}(k), k\in\mathbb{T}^d$ operatorli matritsaning muhim spektri $\mu,\lambda>0$ ta’sirlashish parametrlaridan bog‘liq emasligi va

$$\sigma_{\text{ess}}(\mathcal{A}_{\mu,\lambda}(k))\;=\;[m(k);\;M(k)] \tag{2}$$

tenglik o‘rinli ekanligi kelib chiqadi.

1-izoh. Ta’kidlash joizki, $\mathcal{A}_{0,0}(k)$ operatorning muhim spektri ayrim $k\in\mathbb{T}^d$ uchun $\{m(k)\}$ yagona nuqtadan iborat to‘plam bo‘lishi mumkin va shu sababli har qanday $k\in\mathbb{T}^d$ uchun $\mathcal{A}_{0,0}(k)$ operatorning muhim spektrini uzluksiz spektr deb ayta olmaymiz. Misol uchun, agar $\omega_2(\cdot,\cdot)$ funksiya

$$\omega_2(k,q)\;=\varepsilon(k)+\varepsilon(k+q)+\varepsilon(q)$$

kabi ko‘rinishda bo‘lib, bu yerda $\varepsilon(\cdot)$ funksiya quyidagi

$$\varepsilon(q)=\sum_{i=1}^{d}(1-\cos q_i)\,,q=(q_1,\ldots,q_d)\in\mathbb{T}^d$$

tenglik bilan aniqlangan va

$$k=\underbrace{(\pi,\ldots,\pi)}_{d}\in\mathbb{T}^d$$

bo‘lsa, u holda $\sigma_{\text{ess}}(\mathcal{A}_{\mu,\lambda}(k))\;=\{4d\}$ tenglik o‘rinli bo‘ladi.

Har bir fiksirlangan $k\in\mathbb{T}^d$ uchun $\mathbb{C}\setminus\sigma_{\text{ess}}(\mathcal{A}_{\mu,\lambda}(k))$ sohada

$$\Delta(k,z)=\left(1-\lambda\int_{\mathbb{T}^d}\frac{\varphi^2(t)dt}{\omega_2(k,t)-z}\right)\left(\omega_1(k)-z-\frac{\mu^2}{2}\int_{\mathbb{T}^d}\frac{v^2(t)dt}{\omega_2(k,t)-z}\right)-\frac{\lambda\mu^2}{2\sqrt{2}}\left(\int_{\mathbb{T}^d}\frac{v(t)\varphi(t)dt}{\omega_2(k,t)-z}\right)^2.$$

Odatda $\Delta(k,\cdot), k\in\mathbb{T}^d$ funksiyaga $\mathcal{A}_{\mu,\lambda}(k)$ operatorli matritsaga mos Fredgolm determinanti deyiladi va bu funksiya qaralayotgan umumlashgan Fridrixs modelining xos qiymatlari soni hamda joylashuv o‘rnini aniqlashda muhim ahamiyatga ega. Endi $\mathcal{A}_{\mu,\lambda}(k), k\in\mathbb{T}^d$ operatorning xos qiymatlari va $\Delta(k,\cdot), k\in\mathbb{T}^d$ funksiyaning nollari o‘rtasidagi bog‘lanishni ifodalovchi natijani bayon qilamiz. Quyidagi lemma o‘rinli.

2-lemma. $z\in\mathbb{C}\setminus\sigma_{\text{ess}}(\mathcal{A}_{\mu,\lambda}(k))$ soni $\mathcal{A}_{\mu,\lambda}(k), k\in\mathbb{T}^d$ operatorli matritsaning xos qiymati bo‘lishi uchun $\Delta(k,z)=0$ bo‘lishi zarur va yetarlidir.

Isbot. Faraz qilaylik, $z\in\mathbb{C}\setminus\sigma_{\text{ess}}(\mathcal{A}_{\mu,\lambda}(k))$ soni $\mathcal{A}_{\mu,\lambda}(k), k\in\mathbb{T}^d$ operatorli matritsaning xos qiymati, $f=(f_0,f_1)\in\mathcal{H}_0\oplus\mathcal{H}_1$ esa shu xos qiymatga mos xos vektor-funksiya bo‘lsin. U holda f_0 va f_1 koordinatalar quyidagi tenglamalar sistemasini qanoatlantiradi:

$$\begin{cases}(\omega_1(k)-z)f_0+\dfrac{\mu}{\sqrt{2}}\displaystyle\int_{\mathbb{T}^d}v(t)\,f_1(t)dt=0;\\[2ex] \dfrac{\mu}{\sqrt{2}}v(q)f_0+(\omega_2(k,q)-z)f_1(q)-\lambda\varphi(q)\displaystyle\int_{\mathbb{T}^d}\varphi(t)f_1(t)dt=0.\end{cases} \tag{3}$$

(2) tenglikdan ixtiyoriy $z \in \mathbb{C} \setminus \sigma_{\text{ess}}(\mathcal{A}_{\mu,\lambda}(k))$ soni va $q \in \mathbb{T}^d$ uchun quyidagi munosabatga ega bo‘lamiz.

$$\omega_2(k,q) - z \neq 0, k \in \mathbb{T}^d .$$

(3) tenglamalar sistemasining ikkinchi tenglamasidan f_1 uchun quyidagi tenglik o‘rinli:

$$f_1(q) = \frac{\lambda\varphi(q)C_{f_1}}{\omega_2(k,q) - z} - \frac{\mu}{\sqrt{2}}\frac{v(q)f_0}{\omega_2(k,q) - z} \tag{4}$$

Bu yerda

$$C_{f_1} = \int_{\mathbb{T}^d} \varphi(t)f_1(t)dt. \tag{5}$$

(4) tenglik yordamida aniqlangan f_1 funksiyani (3) tenglamalar sistemasining birinchi tenglamasi va (5) tenglikka olib borib qo‘yib, (3) tenglamalar sistemasi nolmas yechimga ega bo‘lishi uchun

$$\left(\omega_1(k) - z - \frac{\mu^2}{2}\int_{\mathbb{T}^d} \frac{v^2(t)dt}{\omega_2(k,t) - z}\right) f_0 + \frac{\lambda\mu}{2}\int_{\mathbb{T}^d} \frac{v(t)\varphi(t)dt}{\omega_2(k,t) - z}\, C_{f_1} = 0;$$

$$\frac{\mu}{\sqrt{2}}\int_{\mathbb{T}^d} \frac{v(t)\varphi(t)dt}{\omega_2(k,t) - z} f_0 + \left(1 - \lambda\int_{\mathbb{T}^d} \frac{\varphi^2(t)dt}{\omega_2(k,t) - z}\right) C_{f_1} = 0$$

tenglamalar sistemasi nolmas yechimga ega bo‘lishi, ya’ni $\Delta(k,z) = 0$ bo‘lishi zarur va yetarli ekanligini hosil qilamiz. 2-lemma isbotlandi.

2-lemmadan quyidagi xulosa kelib chiqadi.

1-xulosa. $\mathcal{A}_{\mu,\lambda}(k), k \in \mathbb{T}^d$ operatorning diskret spektri $\sigma_{\text{disc}}(\mathcal{A}_{\mu,\lambda}(k))$ uchun quyidagi tenglik o‘rinlidir:

$$\sigma_{\text{disc}}(\mathcal{A}_{\mu,\lambda}(k)) = \{z \in \mathbb{C} \setminus \sigma_{\text{ess}}(\mathcal{A}_{\mu,\lambda}(k)) \colon \Delta(k,z) = 0\}, k \in \mathbb{T}^d$$

Quyidagi belgilashlarni kiritamiz:

$$\sigma_{\text{two}}(\mathcal{A}_{\mu,\lambda}^{\text{ch}}) = \bigcup_{k \in \mathbb{T}^d} \sigma_{\text{disc}}(\mathcal{A}_{\mu,\lambda}(k)), \qquad \sigma_{\text{three}}(\mathcal{A}_{\mu,\lambda}^{\text{ch}}) = [m; M].$$

Bu yerda

$$m := \min_{k,q \in \mathbb{T}^d} \omega_2(k,q), \qquad M = \max_{k,q \in \mathbb{T}^d} \omega_2(k,q).$$

1-teorema. $\mathcal{A}_{\mu,\lambda}^{\text{ch}}$ operatorning $\sigma(\mathcal{A}_{\mu,\lambda}^{\text{ch}})$ spektri quyidagi tenglikni qanoatlantiradi:

$$\sigma(\mathcal{A}_{\mu,\lambda}^{\text{ch}}) = \sigma_{\text{two}}(\mathcal{A}_{\mu,\lambda}^{\text{ch}}) \bigcup \sigma_{\text{three}}(\mathcal{A}_{\mu,\lambda}^{\text{ch}}).$$

Isbot. Ta’kidlash joizki, $\mathcal{A}_{\mu,\lambda}^{\text{ch}}$ kanal operator $\widehat{\mathcal{H}}$ Hilbert fazosidagi

$$U_\alpha \begin{pmatrix} g_1(q) \\ g_2(p,q) \end{pmatrix} = \begin{pmatrix} \alpha(p)g_1(q) \\ \alpha(p)g_2(p,q) \end{pmatrix}, \alpha(\cdot) \in C(\mathbb{T}^d), g_i \in \widehat{\mathcal{H}_\iota}, i = 1,2$$

ko‘paytirish operatori bilan o‘rin almashinish xossasiga ega bo‘ladi, bu yerda $C(\mathbb{T}^d)$ orqali $\mathbb{T}^d$ da aniqlangan uzluksiz funksiyalarning Banax fazosi belgilangan.

Binobarin, $\widehat{\mathcal{H}_1} \oplus \widehat{\mathcal{H}_2}$ fazoning

$$\widehat{\mathcal{H}_1} \oplus \widehat{\mathcal{H}_2} = \int_{\mathbb{T}^d} \oplus\ (\mathcal{H}_0 \oplus \mathcal{H}_1)dk$$

kabi to‘g‘ri integralga yoyilmasidan ushbu $\mathcal{A}_{\mu,\lambda}^{\text{ch}}$ operatorli matritsaning

$$\mathcal{A}_{\mu,\lambda}^{\text{ch}} = \int_{\mathbb{T}^d} \oplus\, \mathcal{A}_{\mu,\lambda}(k)dk \tag{6}$$

ko‘rinishdagi to‘g‘ri integral yoyilmasi kelib chiqadi. $\mathcal{A}_{\mu,\lambda}^{\text{ch}}$ operatorning (6) dagi yoyilmasi va yoyiluvchan operatorlarning ([15], XIII.86-teorema) spektri haqidagi teoremadan

$$\sigma(\mathcal{A}_{\mu,\lambda}^{\text{ch}}) = \bigcup_{p \in \mathbb{T}^d} \sigma(\mathcal{A}_{\mu,\lambda}(k))$$

ekanligi kelib chiqadi.

Oxirgi munosabatdan

$$\sigma(\mathcal{A}_{\mu,\lambda}(k)) = \sigma_{\text{disc}}(\mathcal{A}_{\mu,\lambda}(k)) \bigcup [m(k); M(k)], \qquad k \in \mathbb{T}^d$$

va

$$\bigcup_{k\in\mathbb{T}^d}[m(k);M(k)]=[m;M]$$

tengliklarni hisobga olib, 1-teoremaning isbotiga kelamiz. Isbot qilingan 1-teorema $\mathcal{A}_{\mu,\lambda}$ operatorli matritsa muhim spektrining joylashuv oʻrnini tavsiflovchi quyidagi teoremani isbot qilishda muhim ahamiyatga ega.

2-teorema. $\mathcal{A}_{\mu,\lambda}$ operatorli matritsaning muhim spektri $\sigma_{\text{ess}}(\mathcal{A}_{\mu,\lambda})$ bilan $\mathcal{A}_{\mu,\lambda}^{\text{ch}}$ operatorli matritsaning spektri $\sigma(\mathcal{A}_{\mu,\lambda}^{\text{ch}})$ ustma-ust tushadi, ya'ni $\sigma_{\text{ess}}(\mathcal{A}_{\mu,\lambda})=\sigma(\mathcal{A}_{\mu,\lambda}^{\text{ch}})$.

2-ta'rif. $\sigma_{\text{two}}(\mathcal{A}_{\mu,\lambda}^{\text{ch}})$ va $\sigma_{\text{three}}(\mathcal{A}_{\mu,\lambda}^{\text{ch}})$ toʻplamlar mos ravishda $\mathcal{A}_{\mu,\lambda}$ operatorli matritsa muhim spektrining "ikki zarrali" va "uch zarrali" tarmoqlari deyiladi.

Aniqlanishiga koʻra $\mathcal{A}_{\mu,\lambda}^{\text{ch}}$ kanal operator $\mathcal{A}_{\mu,\lambda}$ operatorli matritsaga nisbatan sodda koʻrinishga ega, oʻrganish uchun qulay, sof muhim spektrga ega, shu sababli 1-2-teoremalar $\mathcal{A}_{\mu,\lambda}$ operatorli matritsa muhim spektrini tadqiq qilishda muhim ahamiyatga ega.

Xulosa: Mazkur maqolada panjaradagi soni saqlanmaydigan va uchtadan oshmaydigan zarrachalar sistemasiga mos keluvchi hamda uchinchi tartibli operatorli matritsa koʻrinishdagi $\mathcal{A}_{\mu,\lambda},\ \mu,\lambda>0$ operator tadqiq qilindi. Oʻrganilayotgan $\mathcal{A}_{\mu,\lambda},\ \mu,\lambda>0$ operatorli matritsa muhim spektrining joylashuv oʻrni va tuzilishini, ya'ni muhim spektrning ikki zarrachali va uch zarrachali tarmoqlarini ajratishda, ularni tashkil qiluvchi kesmalar sonini aniqlashda, bu kesmalarning joylashuv oʻrnini koʻrsatishda, ya'ni tuzilishini aniqlashda asosiy vazifani bajaruvchi kanal operator topildi. Kanal operatorning spektri sof muhim spektr boʻlib, ya'ni diskret spektri boʻsh toʻplam boʻlib, mos umumlashgan Fridrixs modelining spektri orqali yoyiluvchi operatorlarning spektri haqidagi teoremani qoʻllab tavsiflandi.

ADABIYOTLAR :

1. *Mattis D. The few-body problem on a lattice. Rev. Modern Phys., 58 (1986), pp. 361- 379.*
2. *Фридрихс К.О. Возмущение спектра операторов в гильбертовом пространстве. М.: Мир, 1969.*
3. *Малышев В.А., Минлос Р.А. Кластерные операторы. Труды семинара им. И.Г. Петровского. (1983), вып. 9, стр. 63-80.*
4. *Minlos R.A., Spohn H. The three-body problem in radioactive decay: the case of one atom and at most two photons. Topics in Statistical and Theoretical Physics. Amer. Math. Soc. Transl. (2). Providence (R.I.): Amer. Math. Soc., 177 (1996) pp. 159-193.*
5. *Жуков Ю.В., Минлос Р.А. Спектр и рассеяние в модели "Спин-бозон" с не более чем тремя фотонами. Теор. и матем. физика, 103 (1995), № 1, стр. 63-81.*
6. *Лакаев С.Н., Расулов Т.Х. Модель в теории возмущений существенного спектра многочастичных операторов. Матем. заметки, 73 (2003), вып. 4, стр. 556-564.*
7. *Albeverio S., Lakaev S.N., Rasulov T.H. On the Spectrum of an Hamiltonian in Fock Space. Discrete Spectrum Asymptotics. J. Stat. Phys. 127:2 (2007), P. 191–220.*
8. *Albeverio S., Lakaev S.N., Rasulov T.H. The Efimov effect for a model operator associated with the Hamiltonian of a non conserved number of particles. Methods Funct. Anal. Topology 13:1(2007),P.1–16.*
9. *Muminov M., Neidhardt H., Rasulov T. On the spectrum of the lattice spin-boson 6 Hamiltonian for any coupling: 1D case. J. Math. Phys., 56 (2015), 053507.*
10. *Расулов Т.Х. О ветвях существенного спектра решётчатой модели спин-бозона с не более чем двумя фотонами. Теор. и мат. физ., 186:2 (2016), С. 293–310.*
11. *Muminov Z., Ismail F., Eshkuvatov Z. On the number of eigenvalues of a model operator in fermionic Fock space. Journal of Physics: Conference Series. 435 (2013), 012036.*
12. *Muminov Z., Ismail F., Eshkuvatov Z., Rasulov J. On the Discrete Spectrum of a Model Operator in Fermionic Fock Space. Abstract and Applied Analysis. Vol. 2013, ID 875194, 12 pages.*
13. *Абдуллаев Ж.И., Лакаев С.Н. Асимптотика дискретного спектра разностного трёхчастичного оператора Шредингера на решётке. Теор. и мат. физ., 136:2, (2003), 231–245.*
14. *Albeverio S., Lakaev S.N., Muminov Z.I. Schroedinger Operators on Lattices. The Efimov Effect and Discrete Spectrum Asymptotics. Ann. Henri Poincare, 5, (2004), 743–772.*
15. *Рид М., Саймон Б. Методы современной математической физики. т. 4. Анализ операторов. М.: Мир. 1982.*