SCIENTIFIC-METHODICAL JOURNAL OF

«SCIENTIFIC PROGRESS»

ISSN: 2181-1601

2021, MARCH 15

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TASHKENT, UZBEKISTAN
2021, MARCH 15

ON A NON-LINEAR P-ADIC DYNAMICAL SYSTEM

Sayitova Mehinbonu Abduqayumovna
Master Student Department of Mathematics, Bukhara State University

Abstract. In this paper, we study p-adic dynamical systems generated by rational functions. We consider the function $f(x)=\frac{a}{x-2b}$ and study the dynamical systems generated by this function in C_p. We give fixed points, periodic points, basin of attraction and Siegel disk of each fixed points.

Keywords: Rational dynamical systems; fixed point; invariant set; Siegel disk; complex p-adic field.

It is known that the theory of p-adic numbers has numerous applications in many branches of mathematics, biology, physics and other sciences.

Consider the dynamical system associated with the function $f: C_p \to C_p$ defined by

$$f(x)=\frac{a}{x-2b},\quad a\neq 0, a,b\in C_p,$$

where $x\neq 2b$.

Our goal here is to investigate the behavior of trajectories of $f(x)=\frac{a}{x-2b}$ in the complex p-adic filed C_p.

Remark. The case $b=0$ is simple: in this case any point $x\in C_p\setminus\{-b\}$ is two periodic. That is $f(f(x))=x$. Indeed,

$$f(f(x))=\frac{a}{\frac{a}{x}}=a\frac{x}{a}=x.$$

Therefore, below we consider the case $b\neq 0$.

Since C_p is algebraic closed, this function (for $ab\neq 0$) has two fixed points:

$$f(x)=x \;\Rightarrow\; x^2-2bx-a=0 \;\Rightarrow\; x_1=b-\sqrt{b^2+a},\; x_2=b+\sqrt{b^2+a}.$$

The following proposition says that f may have periodic (except fixed points) iff $b=0$.

Proposition 1. If $b(b^2+a)\neq 0$ then $f^n(x)=x, n\geq 2$ does not have any solution (except solutions of $f(x)=x$).

Proposition 2. If $b(b^2+a)\neq 0$ then the set

$$P=\{x\in C_p : \exists n\in N\cup\{0\}, f^n(x)=2b\}$$

is the following

$$P=\{2b\}\cup\left\{2b-\frac{b_n}{d_n} : d_n\neq 0, n\geq 1\right\}.$$

References

1. S. Albeverio, U.A. Rozikov, I.A. Sattarov. p-adic (2, 1)-rational dynamical systems. Jour. Math. Anal. Appl. 398(2) (2013), 553–566.
2. S. Albeverio, B. Tirozzi, A.Yu. Khrennikov, S. de Shmedt, p-adic dynamical systems. Theoret. and Math. Phys. 114(3) (1998), 276–287.
3. V. Anashin, A. Khrennikov, Applied Algebraic Dynamics, de Gruyter Expositions in Mathematics vol 49, Walter de Gruyter (Berlin - New York), 2009.