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# UCHINCHI TARTIBLI OPERATORLI MATRITSA MUHIM SPEKTRINING TUZILISHI

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## REZYUME

Ushbu maqolada panjaradagi soni saqlanmaydigan va uchtadan oshmaydigan zarrachalar sistemasiga mos uchinchi tartibli operatorli matritsa Hilbert fazosidagi chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator sifatida qaralgan. Ikki o‘lchamli qo‘zg‘alishga ega chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma umumlashgan Fridrixs modellar oilasining spektral xossalaridan foydalanib, qaralayotgan uchinchi tartibli operatorli matritsaning muhim spektri tadqiq qilingan. Muhim spektrning ikki va uch zarrachali tarmoqlari ajratilgan. Ikki zarrachali tarmoqlarning uch zarrachali tarmoqga nisbatan joylashuv o‘rni o‘rganilgan.


***Kalit so‘zlar:*** Hilbert fazo, operatorli matritsa, umumlashgan Fridrixs modeli, Fredgolm determinanti, muhim spektr.

## 1. Kirish

Operatorli matritsalar muhim va diskret spektrlari bilan bog‘liq masalalar qattiq jismlar fizikasi [1], kvant maydon nazariyasi [2], statistik fizika [3], magnetogidrodinamika [4], kvant mexanikasi [5] va boshqa ko‘plab sohalarda uchrab turadi. Hozirgi kunda operatorli matritsalarning muhim spektri va uning xos qiymatlari sonini o‘rganishga doir ko‘plab tadqiqot ishlar olib borilmoqda. Bunda, umumlashgan Fridrixs modellari oilasi uchun bo‘sag‘aviy hodisalarni tahlil qilish, uchinchi tartibli operatorli matritsalar oilasi muhim spektrining tuzilishini aniqlash [6], xos qiymatlar sonining chekli yoki cheksiz bo‘lish shartlarini topishga alohida e’tibor berilmoqda [7], [8].

Operatorli matritsalar spektral xossalariga bag‘ishlangan ayrim maqolalar tahliliga to‘xtalamiz.

[9] maqolada uchinchi tartibli model operatorli matritsa uchun Yefimov hodisasining mavjud bo‘lishi (muhim spektrning quyi chegarsiga intiluvchi cheksiz sondagi xos qiymatlarning mavjudligi) isbotlangan, hamda bu xos qiymatlar soni uchun asimptotik formula topilgan. [10] maqolada uchinchi tartibli operatorli matritsalar uchun olingan natijalar yordamida panjaradagi ko‘pi bilan ikkita fotonli spin-bozon modelining spektri batafsil o‘rganilgan. [11] maqolada esa bu turdagi operatorli matritsalar uchun muhim spektrning ichida (muhim spektrning bo‘shlig‘ida, muhim spektrdan chapda) cheksiz sondagi xos qiymatlar mavjud bo‘lish shartlari topilgan. [12] maqolada esa soni saqlanmaydigan va uchtadan oshmaydigan zarrachalarning sistemasi bilan bog‘liq ikkinchi tartibli operatorli matritsa uchun ikki yoqlama Yefimov hodisasining mavjudligi isbotlangan.

[13] maqolada dispersiya funksiyasi chegaralangan holda, $\alpha > 0$ ta’sirlashish parametrining ixtiyoriy qiymatida $\mathbb{R}^d$ fazodagi ko‘pi bilan ikki fotonli spin-bozon modelining

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spektri tahlil qilingan. Bu holda muhim spektrning aniq ko‘rinishi tavsiflangan hamda muhim spektrning ikki zarrachali va uch zarrachali tarmoqlari ajratilgan. Bundan tashqari, muhim spektr bitta kesmadan yoki oltitadan ko‘p bo‘lmagan kesmalar birlashmasidan iborat bo‘lish shartlari topilgan.

Ushbu maqolada panjaradagi soni saqlanmaydigan va uchtadan oshmaydigan zarrachalar sistemasiga mos uchinchi tartibli operatorli matritsa Hilbert fazosidagi chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator sifatida o‘rganilgan. Uning muhim spektri tarmoqlarning joylashuv o‘rni ikki o‘lchamli qo‘zg‘alishga ega chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma umumlashgan Fridrixs modellar oilasining spektral xossalari orqali tadqiq qilingan.

## 2. Panjaradagi soni saqlanmaydigan va ikkitadan oshmaydigan zarrachalar sistemasiga mos umumlashgan Fridrixs modellari oilasi

Dastlab zarur tushunchalarni kiritamiz. $\mathbb{T} := (-\pi;\pi]$ orqali bir o‘lchamli torni, $\mathcal{H}_0 := \mathbb{C}$ orqali bir o‘lchamli kompleks fazoni, $\mathcal{H}_1 := L_2(\mathbb{T})$ orqali $\mathbb{T}$ torda aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatli) funksiyalarning Hilbert fazosini belgilaymiz.

$\mathcal{H}_0 \oplus \mathcal{H}_1$ Hilbert fazosida ta’sir qiluvchi va

$$h_\mu(x) := \begin{pmatrix} h_{00}(x) & \frac{\mu}{\sqrt{2}} h_{01} \\ \frac{\mu}{\sqrt{2}} h_{01}^* & h_{11}(x) \end{pmatrix}$$

tenglik orqali aniqlanuvchi umumlashgan Fridrixs modellari oilasi deb ataluvchi ikkinchi tartibli operatorli matritsani qaraymiz. Bu yerda $\mu > 0$ ta’sirlashish parametri bo‘lib,

$$h_{00}(x) f_0 = f_0, \quad h_{01} f_1 = \int_{\mathbb{T}} f_1(t) dt,$$

$$(h_{11}(x) f_1)(y) = \omega(x,y) f_1(y) \quad f_i \in \mathcal{H}_i, \quad i = 0,1,$$

$$\omega(x,y) := 3 - \cos x - \cos y - \cos(x+y).$$

$h_\mu(x)$ operatorli matritsa $\mathcal{H}_0 \oplus \mathcal{H}_1$ Hilbert fazosida chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator ekanligini oson ko‘rsatish mumkin.

$h_\mu(x)$ operatorli matritsaning muhim spektrini aniqlash maqsadida $\mathcal{H}_0 \oplus \mathcal{H}_1$ Hilbert fazosida

$$h_0(x) := \begin{pmatrix} 0 & 0 \\ 0 & h_{11}(x) \end{pmatrix}$$

tenglik orqali aniqlangan operatorli matritsani (qo‘zg‘almas operator) qaraymiz. Ko‘rinib turibdiki, $h_0(x)$ operatorli matritsaning $h_\mu(x) - h_0(x)$ qo‘zg‘alish operatori 2 o‘lchamli operatorli matritsa bo‘ladi. Chekli o‘lchamli qo‘zg‘alishlarda muhim spektrning o‘zgarmasligi haqidagi Veyl teoremasiga ko‘ra $h_\mu(x)$ operatorli matritsaning muhim spektri $h_0(x)$ operatorli matritsaning muhim spektri bilan ustma-ust tushadi. Aniqlanishiga ko‘ra

$$\sigma_{\text{ess}}(h_0(x)) = [E_1(x); E_2(x)],$$

bu yerda $E_1(x)$ va $E_2(x)$ sonlari

$$E_1(x) := \min_{y\in\mathbb{T}} \omega(x,y) = 3 - \cos x - \sqrt{2+2\cos x},$$

$$E_2(x) := \max_{y\in\mathbb{T}} \omega(x,y) = 3 - \cos x + \sqrt{2+2\cos x}$$

tengliklar yordamida aniqlangan. Demak, $h_\mu(x)$ operatorli matritsaning muhim spektri uchun

$$\sigma_{\text{ess}}(h_\mu(x)) = [E_1(x); E_2(x)]$$

tenglik o‘rinlidir. Har bir tayinlangan $\mu > 0$ soni va $x \in \mathbb{T}$ uchun $\mathbb{C} \setminus [E_1(x); E_2(x)]$ sohada analitik bo‘lib,

$$\Delta_\mu(x,z) := \begin{cases} 1 - z - \frac{\pi\mu^2}{\sqrt{(3-\cos x - z)^2 - 4\cos^2\frac{x}{2}}}, & z < E_1(x), \\ 1 - z + \frac{\pi\mu^2}{\sqrt{(3-\cos x - z)^2 - 4\cos^2\frac{x}{2}}}, & z > E_2(x) \end{cases} \tag{1}$$

kabi aniqlangan funksiyani qaraymiz. Odatda, $\Delta_\mu(x,\cdot)$ funksiyaga $h_\mu(x)$ operatorli matritsaga mos Fredgolm determinanti deyiladi.

**1-lemma.** Har bir $x \in \mathbb{T}$ uchun $z \in \mathbb{C} \setminus [E_1(x); E_2(x)]$ soni $h_\mu(x)$ operatorli matritsaning xos qiymati bo‘lishi uchun $\Delta_\mu(x,z) = 0$ tenglikning bajarilishi zarur va yetarlidir.

1-lemmaning isboti [14] ishda keltirilgan.

1-lemmadan $h_\mu(x)$ operatorli matritsaning diskret spektri uchun

$$\sigma_{\text{disc}}(h_\mu(x)) = \{z \in \mathbb{C} \setminus [E_1(x); E_2(x)] : \Delta_\mu(x,z) = 0\}$$

tenglikni hosil qilamiz.

**2-lemma.** Har bir $\mu > 0$ soni va $x \in \mathbb{T}$ uchun $h_\mu(x)$ operatorli matritsa $E_1(x)$ dan chapda va $E_2(x)$ o‘ngda yotuvchi bittadan oddiy xos qiymatga ega.

**Isboti.** Aniqlanishiga ko‘ra har bir $\mu > 0$ soni va $x \in \mathbb{T}$ uchun $\Delta_\mu(x,\cdot)$ funksiya $(-\infty; E_1(x))$ oraliqda uzluksiz va monoton kamayuvchi bo‘lib, ushbu

$$\lim_{z\to-\infty} \Delta_\mu(x,z) = +\infty, \tag{2}$$

$$\lim_{z\to E_1(x)-0} \Delta_\mu(x,z) = -\infty \tag{3}$$

tengliklar o‘rinli bo‘ladi. (2), (3) tengliklar va $\Delta_\mu(x,\cdot)$ funksiya $(-\infty; E_1(x))$ oraliqda uzluksiz, monoton ekanligiga ko‘ra shunday yagona $E_\mu^{(1)}(x) \in (-\infty; E_1(x))$ soni topilib, $\Delta_\mu(x, E_\mu^{(1)}(x)) = 0$ tenglik bajariladi. 1-lemmaga ko‘ra $E_\mu^{(1)}(x) < E_1(x)$ soni $h_\mu(x)$ operatorli matritsa uchun xos qiymat bo‘ladi.

Xuddi shunday, har bir $\mu > 0$ soni va $x \in \mathbb{T}$ uchun $\Delta_\mu(x,\cdot)$ funksiya $(E_2(x); +\infty)$ oraliqda uzluksiz va monoton bo‘lib, ushbu

$$\lim_{z\to E_2(x)+0} \Delta_\mu(x,z) = +\infty, \tag{4}$$

$$\lim_{z\to+\infty} \Delta_\mu(x,z) = -\infty \tag{5}$$

tengliklar o‘rinli bo‘ladi. $\Delta_\mu(x,\cdot)$ uzluksiz funksiyaning $(E_2(x);+\infty)$ oraliqda monotonligiga va (4), (5) tengliklarga ko‘ra shunday yagona $E_\mu^{(2)}(x)\in(E_2(x);+\infty)$ soni topilib, $\Delta_\mu(x,E_\mu^{(2)}(x))=0$ munosabat bajariladi. 1-lemmaga ko‘ra $E_\mu^{(2)}(x)>E_2(x)$ soni $h_\mu(x)$ operatorli matritsa uchun xos qiymat bo‘ladi.

Qulaylik uchun $h_\mu(x)$ operatorli matritsaning $E_1(x)$ dan chapda va $E_2(x)$ dan o‘ngda yotuvchi xos qiymatlarini mos ravishda $E_\mu^{(1)}(x)$ va $E_\mu^{(2)}(x)$ orqali belgilaymiz.

## 3. Uchinchi tartibli operatorli matritsa muhim spektrining tuzilishi

$L_2^{\mathrm{s}}(\mathbb{T}^2)$ orqali $\mathbb{T}^2$ da aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatlarni qabul qiluvchi) simmetrik funksiyalarning Hilbert fazosini hamda $\mathcal{H}$ orqali $\mathcal{H}_0:=\mathbb{C}$, $\mathcal{H}_1:=L_2(\mathbb{T})$ va $\mathcal{H}_2=L_2^{\mathrm{s}}(\mathbb{T}^2)$ fazolarning to‘g‘ri yig‘indisini belgilaymiz, ya’ni $\mathcal{H}=\mathcal{H}_0\oplus\mathcal{H}_1\oplus\mathcal{H}_2$.

$\mathcal{H}$ Hilbert fazosida ta’sir qiluvchi quyidagi

$$\mathcal{A}_\mu:=\begin{pmatrix}\mathcal{A}_{00} & \mathcal{A}_{01} & 0\\ \mathcal{A}_{01}^* & \mathcal{A}_{11} & \mu\mathcal{A}_{12}\\ 0 & \mu\mathcal{A}_{12}^* & \mathcal{A}_{22}\end{pmatrix},\quad \mu>0 \tag{6}$$

uchinchi tartibli operatorli matritsani qaraymiz. Bu yerda matritsaviy elementlar

$$\mathcal{A}_{00}f_0=af_0,\quad \mathcal{A}_{01}f_1=\int_{\mathbb{T}}v(t)f_1(t)dt,\quad (\mathcal{A}_{11}f_1)(x)=f_1(x),$$

$$(\mathcal{A}_{12}f_2)(x)=\int_{\mathbb{T}}f_2(x,t)dt,\quad (\mathcal{A}_{22}f_2)(x,y)=\omega(x,y)f_2(x,y)$$

kabi aniqlanib, $a\in\mathbb{R}$ va $v(\cdot)$ funksiya $\mathbb{T}$ torda aniqlangan haqiqiy qiymatli uzluksiz funksiya. $\mathcal{A}_{01}^*$ va $\mathcal{A}_{12}^*$ operatorlar mos ravishda $\mathcal{A}_{01}$ va $\mathcal{A}_{12}$ operatorlarga qo‘shma operatorlar bo‘lib,

$$\mathcal{A}_{01}^*:\mathcal{H}_0\to\mathcal{H}_1,\quad (\mathcal{A}_{01}^*f_0)(x)=v(x)f_0,\quad f_0\in\mathcal{H}_0,$$

$$\mathcal{A}_{12}^*:\mathcal{H}_1\to\mathcal{H}_2,\quad (\mathcal{A}_{12}^*f_1)(x,y)=\frac{f_1(x)+f_1(y)}{2},\quad f_1\in\mathcal{H}_1.$$

(6) tenglik yordamida aniqlangan $\mathcal{A}_\mu$ operatorli matritsa $\mathcal{H}$ Hilbert fazosidagi chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator bo‘ladi.

**1-teorema.** $\mathcal{A}_\mu$ operatorli matritsaning muhim spektri uchun

$$\sigma_{\mathrm{ess}}(\mathcal{A}_\mu)=\operatorname{Im}E_\mu^{(1)}(x)\cup[0;\frac{9}{2}]\cup\operatorname{Im}E_\mu^{(2)}(x)$$

tenglik o‘rinli.

1-teorema Veyl mezoni, Faddeyev tenglamasining xarakteristik xossasi va Fredgolm teoremasidan foydalanib isbotlanadi.

Endi ushbu maqoladagi asosiy teoremani bayon qilamiz.

**2-teorema.** *a*) Agar $\mu\in(0;\frac{2}{\sqrt{\pi}}]$ bo‘lsa, u holda $\mathcal{A}_\mu$ operatorli matritsaning muhim spektri uchun

$$\sigma_{\mathrm{ess}}(\mathcal{A}_\mu)=[\min E_\mu^{(1)}(x);\max E_\mu^{(2)}(x)]$$

tenglik o‘rinli;

$b)$ agar $\mu \in (\frac{2}{\sqrt{\pi}}; \frac{\sqrt{21}}{2\sqrt{\pi}}]$ bo‘lsa, u holda $\mathcal{A}_\mu$ operatorli matritsaning muhim spektri uchun

$$\sigma_{\text{ess}}(\mathcal{A}_\mu) = \operatorname{Im} E_\mu^{(1)}(x) \cup [0; \max E_\mu^{(2)}(x)],$$

tenglik o‘rinli bo‘lib, $\max\limits_{x\in\mathbb{T}} E_\mu^{(1)}(x) < 0$ bo‘ladi;

$c)$ agar $\mu > \frac{\sqrt{21}}{2\sqrt{\pi}}$ bo‘lsa, u holda $\mathcal{A}_\mu$ operatorli matritsaning muhim spektri uchun

$$\sigma_{\text{ess}}(\mathcal{A}_\mu) = \operatorname{Im} E_\mu^{(1)}(x) \cup [0; \frac{9}{2}] \cup \operatorname{Im} E_\mu^{(2)}(x)$$

tenglik o‘rinli bo‘lib, $\max\limits_{x\in\mathbb{T}} E_\mu^{(1)}(x) < 0$ va $\frac{9}{2} < \min\limits_{x\in\mathbb{T}} E_\mu^{(2)}(x)$ munosabatlar o‘rinli bo‘ladi.

**Isboti.** 1-teoremaga ko‘ra $\mathcal{A}_\mu$ operatorli matritsaning muhim spektri

$$\sigma_{\text{ess}}(\mathcal{A}_\mu) = \operatorname{Im} E_\mu^{(1)}(x) \cup [0; \frac{9}{2}] \cup \operatorname{Im} E_\mu^{(2)}(x)$$

to‘plamdan iborat bo‘ladi. Aniqlanishiga ko‘ra $\Delta_\mu(\cdot, z)$ funksiya $\mathbb{T}$ torda uzluksizdir. Shu sababli $E_\mu^{(i)} : \mathbb{T} \to E_\mu^{(i)}(x)$, $i = 1, 2$ akslantirish haqiqiy qiymatli uzluksiz funksiyadir. Demak, $\operatorname{Im}E_\mu^{(i)}$ to‘plam yopiq va bir bog‘lamli bo‘ladi, ya’ni $\operatorname{Im}E_\mu^{(i)} = [\min\limits_{x\in\mathbb{T}} E_\mu^{(i)}(x); \max\limits_{x\in\mathbb{T}} E_\mu^{(i)}(x)]$ bo‘ladi.

$a)$ Faraz qilaylik, $\mu \in (0; \frac{2}{\sqrt{\pi}}]$ bo‘lsin. U holda

$$\max_{x\in\mathbb{T}} \Delta_\mu(x; 0) \geq 0, \tag{7}$$

$$\min_{x\in\mathbb{T}} \Delta_\mu(x; \frac{9}{2}) \leq 0 \tag{8}$$

tengsizliklar o‘rinli bo‘ladi. Agar $\Delta_\mu(x; \cdot)$ funksiyani $(-\infty; 0)$ va $(\frac{9}{2}; +\infty)$ oraliqlarda monoton va uzluksizligini hamda (2), (5), (7), (8) munosabatlarni inobatga olsak, u holda $\max\limits_{x\in\mathbb{T}} E_\mu^{(1)}(x) \geq 0$ va $\min\limits_{x\in\mathbb{T}} E_\mu^{(2)}(x) \leq \frac{9}{2}$ tengsizliklar o‘rinli ekanligi kelib chiqadi.

Ma’lumki,

$$\inf_{x\in\mathbb{T}} \Delta_\mu(x; 0) = -\infty \tag{9}$$

va

$$\sup_{x\in\mathbb{T}} \Delta_\mu(x; \frac{9}{2}) = \infty \tag{10}$$

munosabatlar o‘rinli. Agar $\Delta_\mu(x; \cdot)$ funksiyaning $(-\infty; 0)$ va $(\frac{9}{2}; +\infty)$ oraliqlarda monoton va uzluksizligini hamda (2), (5), (9), (10) tengliklarni inobatga olsak, u holda $\min\limits_{x\in\mathbb{T}} E_\mu^{(1)}(x) \leq 0$, $\max\limits_{x\in\mathbb{T}} E_\mu^{(2)}(x) \geq \frac{9}{2}$ tengsizliklar o‘rinli bo‘ladi. Demak, $\mathcal{A}_\mu$ operatorli matritsaning muhim spektri uchun

$$\sigma_{\text{ess}}(\mathcal{A}_\mu) = [\min_{x\in\mathbb{T}} E_\mu^{(1)}(x); \max_{x\in\mathbb{T}} E_\mu^{(2)}(x)]$$

tenglik o‘rinli bo‘ladi.

*b*) Faraz qilaylik, $\mu \in (\frac{2}{\sqrt{\pi}}; \frac{\sqrt{21}}{2\sqrt{\pi}}]$ bo‘lsin. U holda $\max\limits_{x\in\mathbb{T}} \Delta_\mu(x;0) < 0$ va (8) tengsizliklar o‘rinli bo‘ladi. $\Delta_\mu(x;\cdot)$ funksiyaning $(-\infty;0)$ va $(\frac{9}{2};+\infty)$ oraliqlarda monoton va uzluksizligini hamda $\max\limits_{x\in\mathbb{T}} \Delta_\mu(x;0) < 0$, shuningdek (2), (5), (8) munosabatlarni inobatga olsak, u holda $\max\limits_{x\in\mathbb{T}} E_\mu^{(1)}(x) < 0$, $\min\limits_{x\in\mathbb{T}} E_\mu^{(2)}(x) \leq \frac{9}{2}$ tengsizliklar o‘rinli ekanligi kelib chiqadi. $\max E_\mu^{(2)}(x) \geq \frac{9}{2}$ tengsizlik bajarilishi *a*) tasdiqda isbotlangan. Demak, $\mathcal{A}_\mu$ operatorli matritsaning muhim spektri uchun

$$\sigma_{\text{ess}}(\mathcal{A}_\mu) = \operatorname{Im} E_\mu^{(1)}(x) \cup [0; \max_{x\in\mathbb{T}} E_\mu^{(2)}(x)],$$

tenglik o‘rinli bo‘ladi.

*c*) Faraz qilaylik, $\mu > \frac{\sqrt{21}}{2\sqrt{\pi}}$ bo‘lsin. U holda $\max\limits_{x\in\mathbb{T}} \Delta_\mu(x;0) < 0$ va $\min\limits_{x\in\mathbb{T}} \Delta_\mu(x;\frac{9}{2}) > 0$ tengsizliklar o‘rinli bo‘ladi. $\Delta_\mu(x;\cdot)$ funksiyaning $(-\infty;0)$ va $(\frac{9}{2};+\infty)$ oraliqlarda monoton, uzluksizligini hamda $\max\limits_{x\in\mathbb{T}} \Delta_\mu(x;0) < 0$, $\min\limits_{x\in\mathbb{T}} \Delta_\mu(x;\frac{9}{2}) > 0$ va (2), (5) munosabatlarni inobatga olsak, u holda $\max\limits_{x\in\mathbb{T}} E_\mu^{(1)}(x) < 0$, $\min\limits_{x\in\mathbb{T}} E_\mu^{(2)}(x) > \frac{9}{2}$ tengsizliklar o‘rinli ekanligi kelib chiqadi. Demak, $\mathcal{A}_\mu$ operatorli matritsaning muhim spektri uchun

$$\sigma_{\text{ess}}(\mathcal{A}_\mu) = \operatorname{Im} E_\mu^{(1)}(x) \cup [0; \frac{9}{2}] \cup \operatorname{Im} E_\mu^{(2)}(x)$$

tenglik o‘rinli bo‘ladi.

**Xulosa.** Ushbu maqolada qattiq jismlar fizikasi, kvant maydon nazariyasi va boshqa ko‘plab sohalarda uchraydigan panjaradagi soni saqlanmaydigan va uchtadan oshmaydigan zarrachalar sistemasi Hamiltonianiga mos $\mathcal{A}_\mu$ operatorli matritsa qaralgan, bu yerda $\mu > 0$ - ta’sirlashish parametri. $\mathcal{A}_\mu$ operatorli matritsa muhim spektrining joylashuv o‘rni ikki o‘lchamli qo‘zg‘alishga ega umumlashgan Fridrixs modellar oilasini spektral xossalaridan foydalanib o‘rganilgan.

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## РЕЗЮМЕ

В данной статье операторная матрица третьего порядка, соответствующий системы с несохраняющимся и не более трех частиц на решетке, рассматривается как линейный, ограниченный и самосопряженный оператор в гильбертовом пространстве. Используя спектральные свойства линейной, ограниченной и самосопряженной семества обобщенных семейства моделей Фридриха с двумерным возмущением, исследован существенный спектр рассматриваемой операторной матрицы требьего порядка. Выделены двухчастичные и трехчастичные ветви существенного спектра. Изучено расположение двухчастичных ветвей относительно трехчастичной ветви.

***Ключевые слова:*** гильбертово пространство, операторная матрица, обобщенная модель Фридрихса, определитель Фредгольма, существенный спектр.

## RESUME

In this paper, operator matrix of order three corresponding to the system with non-conserved and no more than three particles on the lattice is considered as a linear, bounded and self-adjoint operator in Hilbert space. Using the spectral properties of the linear, bounded and self-adjoint a family of generalized Friedrich models with two-dimensional perturbation, the essential spectrum of the considered operator matrix of order was investigated. Two-particle and three-particle branches