O'ZBEKISTON RESPUBLIKASI
OLIY VA O'RTA MAXSUS
TA'LIM VAZIRLIGI

BUXORO
DAVLAT
1930 UNIVERSITETI

O'ZBEKISTON RESPUBLIKASI
INNOVATSION
RIVOJLANISH VAZIRLIGI

IQTIDORLI TALABALAR, MAGISTRANTLAR, TAYANCH
DOKTORANTLAR VA DOKTORANTLARNING

TAFAKKUR VA TALQIN

MAVZUSIDARESPUBLIKA
MIQYOSIDAGI ILMIY-AMALIY
ANJUMAN TO'PLAMI

Бухоро-2021

O‘ZBEKISTON RESPUBLIKASI OY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

BUXORO DAVLAT UNIVERSITETI

MAGISTRATURA BO’LIMI

IQTIDORLI TALABALAR, MAGISTRANTLAR, TAYANCH DOKTORANTLAR VA DOKTORANTLARNING

TAFAKKUR VA TALQIN

mavzusida

Respublika miqyosidagi ilmiy-amaliy anjuman to’plami

2021 yil, 27-may

1.S. Albeverio, U.A. Rozikov, I.A. Sattarov. p-adic (2, 1)-rational dynamical systems. Jour. Math. Anal. Appl. 398(2) (2013), 553–566.

2. S. Albeverio, B. Tirozzi, A.Yu. Khrennikov, S. de Shmedt, p-adic dynamical systems. Theoret. and Math. Phys. 114(3) (1998), 276–287.

3. U.A. Rozikov, I.A. Sattarov, S. Yam, p-adic dynamical systems of the function

4. $f(x)=\frac{ax}{x^2-a}$ p-Adic Numbers Ultrametric Anal. Appl. 11(1) (2019), 77–87.

PANJARADAGI UCH ZARRACHALI MODEL OPERATORGA MOS KANAL OPERATORLAR

G.H. Umirqulova

BuxDU magistranti

Annotatsiya. Bir o'lchamli panjarada lokal bo'lmagan potensialga ega uchta zarrachalar sistemasiga mos model operator qaraladi. Unga mos keluvchi ikkita kanal operatorlar aniqlanib, bu operatorlarning spektrlari tavsiflanadi.

Kalit so'zlar: panjara, model operator, lokal bo'lmagan potensial, kanal operator, spektr.

Panjaradagi uchta zarrachalar sistemasiga mos model operatorlarning (Gamiltonianlarning) muhim spektrini o'rganish masalasi chiziqli operatorlar spektral nazariyasining dolzarb muammolaridan biri hisoblanadi. Bunday model operatorlarning muhim spektrini tadqiq qilishda nisbatan sodda ko'rinishga ega "kanal operatorlar" deb ataluvchi operatorlarni aniqlash hamda ular spektrlari orasidagi bog'lanishni topish muhim ahamiyatga ega. Shu sababli mazkur maqolada kanal operatorlar topilgan. To'g'ri integralga yoyish usulidan foydalanib kanal operatorlarning spektral xossalarini o'rganish masalasi Fridrixs modellari oilasining spektral xossalarini o'rganish masalasiga keltirilgan. Kanal operatorlarning spektri to'liq tavsiflangan.

T^1 - bir o'lchamli tor va $T^2 = T^1 \times T^1$ - dekart ko'paytma bo'lsin. $L_2^s(T^2)$ orqali T^2 da aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatlarni qabul qiluvchi) simmetrik funksiyalarning Hilbert fazosini belgilaymiz.

$L_2^s(T^2)$ Hilbert fazosida

$$H_{\mu,\lambda} := H_0 - \mu(V_1 + V_2) - \lambda V_3 \tag{1}$$

ko'rinishda aniqlangan operatorni qaraymiz. Bu yerda H_0 - qo'zg'almas operatori bo'lib, ko'paytirish operatoridir:

$$(H_0 f)(x, y) = u(x, y) f(x, y);$$

V_α operatorlari esa quyidagicha ta'sir qiluvchi operatorlar:

$$(V_1 f)(x, y) = v(y)\int_{T^1} v(t) f(x, t)dt;$$

$$(V_2 f)(x, y) = v(x)\int_{T^1} v(t) f(t, y)dt;$$

$$(V_3 f)(x, y) = \int_{T^1} f(t, x + y - t)dt.$$

Bu yerda $\mu, \lambda -$ ta'sirlashish parametrlari deb ataluvchi musbat sonlar, $v(\cdot)$ $-T^1$ da aniqlanuvchi haqiqiy qiymatli uzluksiz funksiya va $u(\cdot,\cdot)$ $-T^2$da aniqlangan haqiqiy qiymatli simmetrik uzluksiz funksiya.

(1) formula bilan aniqlangan $H_{\mu,\lambda}$ operator chiziqli, chegaralangan va oʻz-oʻziga qoʻshma operator boʻladi. Bu tasdiq funksional analizdagi mos ta'riflar yordamida tekshiriladi.

$H_{\mu,\lambda}$ operatorning muhim spektrini aniqlash maqsadida $L_2(T^2)$ Hilbert fazosida

$$H_\mu^{(1)} = H_0 - \mu V_1;$$

$$H_\lambda^{(2)} = H_0 - \lambda V_3;$$

kabi aniqlangan kanal operatorlar deb ataluvchi operatorlarni qaraymiz.

Bunda $H_\mu^{(1)}$ va $H_\lambda^{(2)}$ operatorlar $L_2(T^2)$ Hilbert fazosida ta'sir qiluvchi chiziqli, chegaralangan va o'z- o'ziga qo'shma operatorlardir.

Aniqlanishiga ko'ra $H_\mu^{(1)}$ operator $L_2(T^2)$ Hilbert fazosidagi

$$(U_1 g)(x,y) = u_1(x) g(x,y), \ \ g \in L_2(T^2)$$

ko'rinishdagi ko'paytirish operatori bilan, xuddi shuningdek $H_\lambda^{(2)}$ operator $L_2(T^2)$ Hilbert fazosidagi

$$(U_2 g)(x,y) = u_2(x+y) g(x,y), \ \ g \in L_2(T^2),$$

ko'paytirish operatori bilan o'rin almashinish xossasiga ega. Shu sababli, $L_2(T^2)$ fazoning

$$L_2(T^2) = \int_{k \in T} \oplus L_2(T) dk \tag{2}$$

to'g'ri integralga yoyilmasidan $H_\mu^{(1)}$ va $H_\lambda^{(2)}$ operatorlar uchun

$$H_\mu^{(1)} = \int_{k \in T} \oplus h_\mu^{(1)}(k) dk, \ \ H_\lambda^{(2)} = \int_{k \in T} \oplus h_\lambda^{(2)}(k) dk \tag{3}$$

to'g'ri integralga yoyilmalar kelib chiqadi.

Bunda $h_\mu^{(1)}(k), \ h_\lambda^{(2)}(k), \ k \in T$ Fridrixs modellari oilalari $L_2(T)$ Hilbert fazosida

$$h_\mu^{(1)}(k) := h_0^{(1)}(k) - \mu v_1,$$

$$h_\lambda^{(2)}(k) := h_0^{(2)}(k) - \lambda v_2,$$

formulalar yordamida ta'sir qiladi. Oxirgi formulada

$$\left(h_0^{(1)}(k) f\right)(x) = u(k,x) f(x),$$

$$\left(h_0^{(2)}(k) f\right)(x) = u(x, x-k) f(x),$$

$$(v_1 f)(x) = v(x) \int_T v(t) f(t) dt, \ (v_2 f)(x) = \int_T f(t) dt.$$

$h_0^{(\alpha)}(k), \alpha = 1,2$ qo'zg'almas operatorning v_α, $\alpha = 1,2$ qo'zg'alish operatori bir o'lchamli chiziqli, chegaralangan va o'z- o'ziga qo'shma operator bo'ladi.

Shu sababli, chekli o'lchamli qo'zg'alishlarda muhim spektrning qo'zg'almasligi haqidagi Veyl teoremasiga ko'ra $h_\mu^{(1)}(k)$ operatorning muhim spektri $h_0^{(1)}(k)$ operatorning muhim spektri bilan, xuddi shuningdek $h_\lambda^{(2)}(k)$ operatorning muhim spektri $h_0^{(2)}(k)$ operatorning muhim spektri bilan ustma – ust tushadi.

Har bir $\alpha = 1{,}2$ soni uchun $h_0^{(\alpha)}(k)$ operatorning aniqlanishiga ko'ra

$$\sigma_{ess}\left(h_0^{(\alpha)}(k)\right) = [m_\alpha(k); M_\alpha(k)].$$

Bu yerda $m_\alpha(k)$ va $M_\alpha(k)$ sonlar quyidagicha aniqlangan:

$$m_1(k) := \min_{x\in T} u(k,x), \quad M_1(k) := \max_{x\in T} u(k,x),$$

$$m_2(k) := \min_{x\in T} u(x,k-x), \quad M_2(k) := \max_{x\in T} u(x,k-x).$$

Bundan esa o'z navbatida

$$\sigma_{ess}\left(h_\mu^{(1)}(k)\right) = [m_1(k); M_1(k)]$$

va

$$\sigma_{ess}\left(h_\lambda^{(2)}(k)\right) = [m_2(k); M_2(k)]$$

ekanligi kelib chiqadi.

Faraz qilaylik, har bir fiksirlangan $x \in T$ uchun $u(x,y) = 3 - \cos x - \cos y - \cos(x+y)$

bo'lsin. Bu holda $k_0 = \pi$ uchun $\sigma_{ess}\left(h_0^{(1)}(k_0)\right) = \{4\}$ tenglik o'rinli bo'ladi.

Har bir fiksirlangan $\mu, \lambda > 0$ soni va $k \in T$ uchun $C \setminus [m_\alpha(k); M_\alpha(k)]$ sohada analitik bo'lgan

$$\Delta_\mu^{(1)}(k;z) = 1 - \mu \int_T \frac{v^2(t)}{u(k,t) - z} dt;$$

$$\Delta_\lambda^{(2)}(k;z) = 1 - \lambda \int_T \frac{dt}{u(t,k-t) - z};$$

funksiyalarni kiritamiz

Odatda $\Delta_\mu^{(1)}(k;\cdot)$ va $\Delta_\lambda^{(2)}(k;\cdot)$ funksiyalarga $h_\mu^{(1)}(k)$ va $h_\lambda^{(2)}(k)$ operatorlarga mos Fredgolm determinantlari deyiladi.

1-lemma. Har bir fiksirlangan $k \in T$ va $\mu > 0$ ($\lambda > 0$) soni uchun $z_\alpha(k) \in C \setminus [m_\alpha(k); M_\alpha(k)]$ soni $h_\mu^{(1)}(k)$ $\left(h_\lambda^{(2)}(k)\right)$ operatorning xos qiymati bo‘lishi uchun $\Delta_\mu^{(1)}(k;\cdot) = 0$ $\left(\Delta_\lambda^{(2)}(k;\cdot) = 0\right)$ bo‘lishi zarur va yetarlidir.

Bu lemmadan $h_\mu^{(1)}(k)$ va $h_\lambda^{(2)}(k)$ operatorlarning diskret spektrlari uchun

$$\sigma_{disc}\left(h_\mu^{(1)}(k)\right) = \left\{z \in C \setminus [m_1(k); M_1(k)] : \Delta_\mu^{(1)}(k; z) = 0\right\},$$

$$\sigma_{disc}\left(h_\lambda^{(2)}(k)\right) = \left\{z \in C \setminus [m_2(k); M_2(k)] : \Delta_\lambda^{(2)}(k; z) = 0\right\}$$

tengliklar kelib chiqadi.

$H_\mu^{(1)}$ va $H_\lambda^{(2)}$ operatorlarning spektri $h_\mu^{(1)}(k)$ va $h_\lambda^{(2)}(k)$ operatorlarning spektri orqali quyidagicha tavsiflanadi.

1-teorema. Quyidagi tengliklar o‘rinlidir.

$$\sigma\left(H_\mu^{(1)}\right) = \bigcup_{k\in T} \sigma\left(h_\mu^{(1)}(k)\right) = \bigcup_{k\in T} \sigma_{disc}\left(h_\mu^{(1)}(k)\right) \cup [m; M];$$

$$\sigma\left(H_\lambda^{(2)}\right) = \bigcup_{k\in T} \sigma\left(h_\lambda^{(2)}(k)\right) = \bigcup_{k\in T} \sigma_{disc}\left(h_\lambda^{(2)}(k)\right) \cup [m; M],$$

bu yerda

$$m(k) := \min_{k,x\in T} u(k,x), \quad M(k) := \max_{k,x\in T} u(k,x).$$

Ko'rinib turibdiki, $H_\mu^{(1)}$ va $H_\lambda^{(2)}$ kanal operatorlar $H_{\mu,\lambda}$ operatorga nisbatan sodda ko'rinishga ega. Shuning uchun 1-teorema $H_{\mu,\lambda}$ operatorning muhim spektrini hamda uning ikki va uch zarrachali tarmoqlarini ajratishda [1,2] alohida ahamiyat kasb etadi.

Foydalanilgan adabiyotlar

1. Umirkulova G.H., Rasulov T.H. Characteristic property of the Faddeev equation for three-particle model operator on a one-dimensional lattice. European science. 51:2 (2020), Part II, pp. 19-22.
2. Умиркулова Г.Х. Оценки для граней существенного спектра модельного оператора трех частиц на решетке. Вестник науки и образования. 16-2 (94), 2020, С. 14-17.