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Chiziqli operatorning sonli tasviri haqida ayrim tasdiqlar va misollar

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Annotatsiya: Mazkur maqolada chiziqli operator uchun kiritilgan sonli tasvir tushunchasi haqida ayrim ma'lumotlar keltirilgan. Chiziqli operator sonli tasvirining asosiy xossalari va ularning isbotlari bayon qilingan. Asosiy xossalarning mazmuni misollar yordamida ochib berilgan. Fridrixs modelining sonli tasviri hamda spektrlari orasidagi bog'lanishlar o'rganilgan.

Kalit so'zlar: chiziqli operator, sonli tasvir, spektr, nuqtali spektr, qavariq to'plam, unitar ekvivalent operatorlar, birlik sfera, Fridrixs modeli, umumlashgan Fridrixs modeli.

Some assertions and examples about the numerical representation of a linear operator

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Abstract: This article provides some information about the concept of numeric representation introduced for a linear operator. The basic properties of the numerical representation of a linear operator and their proofs are described. The content of the main properties is explained using examples. The relationships between the numerical representations and spectra of the Friedrichs model are studied.

Keywords: linear operator, numerical image, spectrum, point spectrum, convex set, unitary equivalent operators, unit sphere, Friedrichs model, generalized Friedrichs model.

1. Sonli tasvir va uning asosiy xossalari

H orqali kompleks Gilbert fazoni belgilaymiz, $A: H \to H$ chiziqli operator bo'lib $D(A) \subset H$ uning aniqlanish sohasi bo'lsin. Ushbu

$$W(A) := \{(Ax, x): x \in D(A), \|x\| = 1\}$$

to'plamga A operatorning sonli tasviri deyiladi. $W(A)$ to'plamning aniqlanishiga ko'ra u kompleks tekislikning qism to'plami bo'ladi. $W(A)$

to‘plamning geometrik xossalari yordamida A operator haqida ayrim xulosalarni olish mumkin.

Gilbert fazosida aniqlangan chiziqli operatorning sonli tasvirini o‘rganish masalasi bunday operatorlar spektrining joylashuv o‘rnini o‘rganishdagi asosiy metodlardan biri hisoblanadi. Bu tushuncha birinchi marotaba [1] ishda kiritilgan bo‘lib unda matritsaning sonli tasviri o‘rganilayotgan matritsaning barcha xos qiymatlarini o‘zida saqlashi isbotlangan. [2] maqolada esa operatorning sonli tasviri yopiq to‘plam ekanligi ko‘rsatilgan. Ta’kidlab o‘tish joizki yuqorida sanab o‘tilgan natijalar nafaqat matritsalar uchun balki nisbatan umumiy bo‘lgan ixtiyoriy chiziqli chegaralangan operatorlar uchun ham o‘rinlidir. [3] maqolada ixtiyoriy chiziqli chegaralangan operatorning spektri bu operator sonli tasviri yopig‘ida saqlanishi isbotlangan. Keyinchalik bu tushuncha turli usullar bilan umumlashtirilgan.

H fazodagi birlik operatorni I orqali belgilaymiz. N, R va C lar orqali mos ravishda barcha natural, barcha haqiqiy va barcha kompleks sonlar to‘plamini belgilaymiz. O‘quvchi chiziqli operator tushunchasi haqida batafsilroq ma’lumotga ega bo‘lishi uchun uning asosiy xossalarini isboti bilan keltiramiz [4] va bu xossalardan misollar yechish davomida foydalanamiz.

1-xossa. Agar A chiziqli chegaralangan operator bo‘lsa, u holda

$$W(A) \subset \{\lambda \in C : |\lambda| \leq \|A\|\}$$

munosabat o‘rinli.

Isbot. Faraz qilaylik, $\lambda_0 \in W(A)$ ixtiyoriy element bo‘lsin. U holda shunday $x_0 \in H$ element topilib, $\|x_0\| = 1$ va $\lambda_0 = (Ax_0, x_0)$ tengliklar o‘rinlidir. Funksional analiz fanidan bizga yaxshi ma’lum bo‘lgan Koshi - Bunyakovskiy deb ataluvchi mashhur tengsizlikni qo‘llab

$$|(Ax_0, x_0)| \leq \|Ax_0\|\|x_0\| \qquad (1)$$

baholashni hosil qilamiz. Ma’lumki, agar A chiziqli chegaralangan operator bo‘lsa, u holda ixtiyoriy $x \in H$ element uchun

$$|Ax| \leq \|A\|\|x\| \qquad (2)$$

tensizlik o‘rinli bo‘ladi. $\|x_0\| = 1$ ekanligida (1) va (2) tengsizliklar yordamida

$$|\lambda_0| = |(Ax_0, x_0)| \leq \|A\|$$

munosabatlarga ega bo‘lamiz. Demak,

$$\lambda_0 \in \{\lambda \in C : |\lambda| \leq \|A\|\}$$

ekan. 1-xossa isboti λ_0 elementning ixtiyoriyligidan kelib chiqadi.

1-misol. $A: L_2[0;1] \to L_2[0;1]$, $(Ax)(t) = t\int_0^1 sx(s)ds$ operatorni qaraymiz.

$$t\int_0^1 sx(s)ds = \lambda f(t) \Longrightarrow \lambda = 0, \lambda = \frac{1}{3}$$

$$\|A\| = \frac{1}{3}, W(A) = \left[0; \frac{1}{3}\right] \subset \left\{\lambda: \quad |\lambda| \leq \frac{1}{3}\right\}$$

$$W(A) = \left[0; \frac{1}{3}\right].$$

2-xossa. Agar A chiziqli chegaralangan operator bo‘lsa, A^* qo‘shma operatorning sonli tasviri uchun

$$W(A^*) = \left\{\bar{\lambda}: \lambda \in W(A)\right\}$$

tenglik o‘rinlidir.

Isbot. Avvalo

$$W(A^*) \subset \left\{\bar{\lambda}: \lambda \in W(A)\right\} \qquad (3)$$

munosabatni isbotlaymiz. Faraz qilaylik $\lambda_0 \in W(A^*)$ ixtiyoriy element bo‘lsin. U holda shunday $x_0 \in D(A)$ element topilib, $\|x_0\| = 1$ va $\lambda_0 \in (A^*x_0, x_0)$ tengliklar o‘rinli bo‘ladi. Hilbert fazosidagi o‘z - o‘ziga qo‘shma operator ta’rifidan

$$\lambda_0 \in (A^*x_0, x_0) = (x_0, Ax_0) = \overline{(Ax_0, x_0)}$$

ya’ni

$$\bar{\lambda} \in (Ax_0, x_0)$$

ekanligi kelib chiqadi. Shunday qilib, agar $\lambda_0 \in W(A^*)$ bo‘lsa u holda $\overline{\lambda_0} \in W(A^*)$ o‘rinli ekan. λ_0 ning ixtiyoriyligidan esa (3) munosabatni hosil qilamiz.

Xuddi shunga o‘xshash

$$\left\{\bar{\lambda}: \lambda \in W(A)\right\} \subset W(A^*) \quad (4)$$

munosabatni isbotlash mumkin.

(3) va (4) munosabatlardan 2-xossa isboti kelib chiqadi.

3-xossa. $I -$birlik operator sonli tasviri uchun

$$W(I) = \{1\}$$

tenglik o‘rinlidir. Agar α va β istalgan kompleks sonlar bo‘lsa, u holda

$$W(\alpha A + \beta) = \alpha W(A) + \beta \qquad (5)$$

tenglik o‘rinli bo‘ladi.

Isbot. Birlik operator ta’rifiga ko‘ra ixtiyoriy $x \in H$ element uchun $Ix = x$ bo‘ladi. Shu sababli $\|x\| = 1$ bo‘ladigan barcha $x \in H$ lar uchun

$$(Ix, x) = (x, x) = \|x\|^2 = 1$$

o‘rinli, ya’ni

$$W(I) = \{1\}$$

tenglik o‘rinli.

Endi (5)-tenglikni isbotlaymiz. Istalgan $x \in D(A), \|x\| = 1$ element uchun

$$\big((\alpha A + \beta)x, x\big) = (\alpha Ax, x) + \beta(x, x) = \alpha(Ax, x) + \beta$$

tenglik o‘rinli. Bundan esa o‘z navbatida (5) tenglik kelib chiqadi.

4-xossa. O‘z - o‘ziga qo‘shma A operator uchun $W(A) \subset R$ munosabat o‘rinli.

Isbot. Faraz qilaylik, $\lambda_0 \in W(A)$- ixtiyoriy element bo‘lsin. U holda shunday $x_0 \in D(A)$ element topilib, $\|x_0\| = 1$ va $\lambda_0 \in (Ax_0, x_0)$ tengliklar o‘rinli bo‘ladi. A o‘z - o‘ziga qo‘shmaligidan

$$\lambda_0 = (Ax_0, x_0) = (x_0, Ax_0) = \overline{(Ax_0, x_0)} = \overline{\lambda_0}$$

kelib chiqadi. Bu esa 4-xossani isbotlaydi.

5-xossa. Agar $H -$ chekli o‘lchamli fazo bo‘lsa, u holda $W(A)$ kompakt to‘plam bo‘ladi.

Isbot. Yaxshi ma’lumki, chekli o‘lchamli fazoda to‘plam kompakt bo‘lishi uchun uning chegaralangan va yopiq bo‘lishi zarur va yetarlidir. 1-xossaga ko‘ra $W(A)$ to‘plam markazi koordinata boshida va radiusi $\|A\|$ ga teng doirada yotadi, ya’ni $W(A)$ chegaralangan to‘plam bo‘ladi. Endi $W(A)$ to‘plamning yopiq bo‘lishini isbotlaymiz. Teskarisini faraz qilamiz, ya’ni $W(A)$ yopiq to‘plam bo‘lmasin. U holda $C\backslash W(A)$ to‘plamda $W(A)$ to‘plamning kamida bitta limitik nuqtasi yotadi. Uni λ_0 kabi belgilaymiz. U holda shunday

$$\{\lambda_n\} \subset W(A)$$

ketma - ketlik topilib,

$$\lambda_n \to \lambda_0, n \to \infty$$

bo‘ladi. Shu sababli shunday

$$\{x_n\} \subset H, \|x_n\| = 1$$

ketma-ketlik topilib,

$$\lambda_n = (Ax_n, x_n), n \in N$$

bo‘ladi. A operatorning va skalyar ko‘paytmani uzluksizligiga hamda H dagi birlik sferani kompaktligiga ko‘ra shunday $x_0 \in H, \|x_0\| = 1$ element topilib,

$$\lambda_n = (Ax_n, x_n) = (Ax_0, x_0), n \to \infty$$

o‘rinli bo‘ladi. Limitning yagonaligiga ko‘ra

$$\lambda_0 = (Ax_0, x_0)$$

bo‘ladi, ya’ni $\lambda_0 \in W(A)$. Bu qarama - qarshilik 5-xossani isbotlaydi.

6-xossa. Agar $A, B: H \to H -$ unitar ekvivalent operatorlar bo‘lsa, u holda

$$W(A) = W(B)$$

tenglik o‘rinli bo‘ladi.

Isbot. Faraz qilaylik, A va B - unitar ekvivalent operatorlar bo‘lsin. U holda shunday $U: H \to H$ unitar operator topilib,

$$A = UBU^*$$

tenlik o‘rinli bo‘ladi. U holda

$$(Ax, x) = (UBU^*x, x) = (BU^*x, U^*x)$$

munosabatdan 6-xossa isboti kelib chiqadi. Bunda

$$\|U^*x\| = \|x\| = 1$$

7-xossa. A operatorning nuqtali spektri va sonli tasviri o‘rtasida

$$\sigma_p(A) \subset W(A)$$

munosabat o‘rinlidir.

Isbot. Faraz qilaylik, $\lambda_0 \in \sigma_p(A)$ixtiyoriy element bo‘lsin. U holda shunday $x_0 \in D(A)$ element topilib, $\|x_0\| = 1$ va $Ax_0 = \lambda_0 x_0$ tengliklar o‘rinli bo‘ladi. Shu sababli

$$(Ax_0, x_0) = (\lambda_0 x_0, x_0) = \lambda_0 (x_0, x_0) = \lambda_0 \|x_0\|^2 = \lambda_0$$

o‘rinli bo‘ladi. Demak, $\lambda_0 \in W(A)$. λ_0 nuqtaning ixtiyoriyligidan

$$\sigma_p(A) \subset W(A)$$

bo‘lishi kelib chiqadi. 7-xossa isbotlandi.

7-xossaga ko‘ra A operatorning barcha xos qiymatlari $W(A)$ da yotadi. Tabiiy savol paydo bo‘ladi: A operator spektrining qolgan qismi haqida nima deyish mumkin? Yaxshi ma’lumki, chegaralangan operator spektri yopiq bo‘ladi. 5-xossaga ko‘ra, agar H chekli o‘lchamli fazo bo‘lsa, u holda $W(A)$ yopiq bo‘ladi. Agar H cheksiz o‘lchamli bo‘lsa, u holda $W(A)$ umuman olganda yopiq bo‘lmasligi mumkin.

2-misol. $W(A)$ yopiq bo‘lmaydigan A operatorga misol keltiramiz. Quyidagi

$$A: l_2 \to l_2, A(x_1, x_2, \dots, x_n, \dots) = (x_1, \frac{1}{2}x_2, \dots, \frac{1}{n}x_n, \dots)$$

operatorni qaraymiz. Osongina tekshirish mumkinki

$$\sigma(A) = \left\{0,1, \frac{1}{2}, \dots, \frac{1}{n}, \dots\right\}$$

o‘rinlidir. $W(A) = (0,1]$ ekanligidan ko‘rsatamiz. A o‘z - o‘ziga qo‘shma operator bo‘lganligi uchun 4-xossaga ko‘ra $W(A) \subset R$ bo‘ladi. Ko‘rinib turibdiki,

$$\inf_{\|x\|=1} (Ax, x) = 0, \sup_{\|x\|=1} (Ax, x) = \|A\| = 1$$

o‘rinli. Biroq birorta ham $x \in l_2, \|x\| = 1$ element uchun (Ax, x) kvadratik forma 0 qiymatni qabul qilmaydi. Demak,

$$W(A) = (0,1].$$

Shunday qilib,

$$\sigma(A) \subset W(A)$$

hamisha ham o‘rinli bo‘lmaydi.

2. Sonli tasvirni hisoblashga doir misollar.

3-misol. Faraz qilaylik, H -kompleks Gilbert fazo va $\lambda \in C$ biror fiksirlangan kompleks son bo‘lsin. U holda

$$A_1: H \to H, A_1 x = \lambda x$$

Operator sonli tasviri uchun

$$W(A_1) = \{\lambda\}$$

tenglik o‘rinli bo‘ladi. Haqiqatan ham $x \in H, \|x\| = 1$ bo‘lsa, u holda

$$(A_1 x, x) = \lambda(x, x) = \lambda \|x\|^2 = \lambda$$

bo‘ladi, ya’ni

$$W(A_1) = \{\lambda\}$$

4-misol. Ushbu

$$A_2: R^2 \to R^2, A_2(x_1, x_2) = (ax_1, bx_2)$$

operator sonli tasvirini hisoblang, bunda a va b lar ixtiyoriy haqiqiy sonlar.

Yechish. $x \in (x_1, x_2) \in R^2$ elementni olamiz,

$$\|x\|^2 = 1, x_1^2 + x_2^2 = 1.$$

U holda

$$A_2(x, x) = ax_1^2 + \ bx_2^2.$$

Agar $x_1^2 = t$ kabi belgilash olamiz, u holda $x_2^2 = 1 - t$ bunda $t \in [0,1]$. Shu sababli

$$A_2(x, x) = at + \ b(1 - t) = b + (a - b)t$$

$t \in [0,1]$ bo‘lganligi uchun

$$W(A_2) = [\min\{a, b\}, \max\{\mathrm{a}, \mathrm{b}\}]$$

o‘rinli.

5-misol. Ushbu

$$A_3: C^2 \to C^2, A_3(z_1, z_2) = (\ z_2, 0)$$

operatorning sonli tasvirini hisoblang.

Yechish. Ixtiyoriy $z = (z_1, z_2) \in C$ elementni olamiz, koordinatalari $|z_1|^2 + |z_2|^2 = 1$ shartni qanoatlantirsin. $z_k = \varphi_k, k = 1{,}2$ belgilash kiritamiz. U holda

$$z_k = |z_k|e^{i\varphi}, k = 1{,}2.$$

Endi (Az, z) kvadratik formani $z \in C^2, \|z\| = 1$ elementlar uchun qaraymiz:

$$(A_3 z, z) = \overline{z_1} z_2 = |z_1|e^{-i\varphi_1}|z_2|e^{i\varphi_2} = |z_1||z_2|e^{-i(\varphi_1 - \varphi_2)}.$$

Bu yerda $\varphi_2 - \varphi_1 \in [0; 2\pi)$. Agar $|z_1| = t$ deb belilash olsak, u holda $|z_2| = \sqrt{1 - t^2}$ va $0 \leq t \leq 1$ bo‘ladi. Shu sababli

$$(A_3 z, z) = e^{i(\varphi_1 - \varphi_2)} t\sqrt{1 - t}.$$

Ko‘rinib turibdiki, $\varphi_2 - \varphi_1$ burchak 0 dan 2π gacha o‘zgarmasa $(A_3 z, z)$ kvadratik forma markazi koordinata boshida radiusi $t\sqrt{1 - t^2}$ bo‘lgan aylanani tasvirlaydi. U holda bunday aylanalarni $t \in [0,1]$ bo‘yicha birlashmasi $W(A_3)$ to‘plamni beradi.

$$\max_{0 \leq t \leq 1} t\sqrt{1 - t^2} = \frac{1}{2}$$

ekanligini inobatga olgan holda $W(A_3)$ to‘plam markazi koordinata boshida va radiusi $\frac{1}{2}$ ga teng doirani tavsiflashi kelib chiqadi, ya’ni

$$W(A_3) = \left\{\lambda \in C: |\lambda| \leq \frac{1}{2}\right\}.$$

6-misol. Ushbu

$$A_4: C^2 \to C^2, A_4(z_1, z_2) = (\ z_2, z_1)$$

operator sonli tasviri 5-misoldagi kabi hisoblanib,

$$W(A_4) = \{\lambda \in C: |\lambda| \leq 1\}$$

kabi bo‘lishini ko‘rsatish mumkin.

7-misol. O‘ngga siljitish operatori

$$A_5 : l_2 \to l_2, A_5(x_1, x_2, \dots, x_n, \dots) = (x_2, x_3, \dots, x_{n+1}, \dots)$$

ning sonli tasviri uchun

$$W(A_5) = \{\lambda \in C : |\lambda| \leq 1\}$$

tenglik o‘rinli bo‘lishini ko‘rsating.

Ko‘rinib turibdiki, har bir $\lambda \in U$ uchun $x_\lambda \coloneqq (1, \lambda, \lambda^2, \dots)$ vektor l_2 fazoga tegishli va $A_5 x_\lambda = \lambda x_\lambda$ tenglik o‘rinli, ya’ni har bir $\lambda \in U$ soni A_5 operator uchun xos qiymati bo‘ladi va xos vektor x_λ ga teng bo‘ladi. 5- xossaga ko‘ra $U \subset W(A_5)$ bo‘ladi. $A_5 = 1$ bo‘lganligi uchun 1-xossadan $W(A_5) \subset \bar{U}$ bo‘lishi kelib chiqadi. Shu sababli birlik aylananig birorta ham nuqtasi $W(A_5)$ ga tegishli emasligini ko‘rsatish yetarli. Teskarisini faraz qilamiz, ya’ni moduli 1 ga teng biror λ kompleks son $W(A_5)$ ga yotsin. U holda shunday $x \in H$ element topilib, $\|x\| = 1$ va $\lambda = (A_5 x, x)$ bo‘ladi. $\|A_5\| = 1$ bo‘lgani uchun

$$1 = |\lambda| = |(A_5 x, x)| \leq \|A_5 x\| \|x\| \leq \|x\|^2 = 1$$

o‘rinli. Bunda biz Koshi - Bunyakovskiy tengsizligidan foydalandik. Bunda $A_5 x = \lambda x$ tenglamani faqat $x = x_\lambda$ qanoatlantiradi. Ikkinchi tomondan, $|\lambda| = 1$, shu sababli $x = x_\lambda$. Bu esa $\lambda \notin W(A_5)$ bo‘lishini ko‘rsatadi.

H Gilbert fazosidagi chegaralangan o‘z - o‘ziga qo‘shma B operator uchun

$$\sigma(B) \subset W(B) \text{ yoki } W(B) \subset \sigma(B)$$

bo‘lishini aytish qiyin. Ushbu

$$B : l_2 \to l_2, Bx = \left(x_1, \frac{x_2}{2}, \dots, \frac{x_n}{n}, \dots\right), x = (x_1, x_2, \dots, x_n, \dots) \in l_2$$

operatorni qaraymiz. U holda

$$\sigma(B) = \overline{\left\{1, \tfrac{1}{2}, \dots, \tfrac{1}{n}, \dots\right\}} = \left\{0, 1, \tfrac{1}{2}, \dots, \tfrac{1}{n}, \dots\right\}, W(B) = (0,1]$$

bo‘lishini oson tekshirish mumkin. $0 \notin W(B)$ tasdiqning isbotiga to‘xtalamiz. Teskarisini faraz qilamiz. Faraz qilaylik, $0 \in W(B)$ bo‘lsin. U holda shunday $x = (x_1, x_2, \dots, x_n, \dots) \in l_2$ topilb, $\|x\| = 1$ va $(Bx, x) = 0$ bo‘ladi. Shu sababli

$$(Bx, x) = \textstyle\sum_{n=1}^{\infty} \frac{1}{n} |x_n|^2 = 0$$

o‘rinli. Bundan, $x \equiv 0$ kelib chiqadi. Bu esa $\|x\| = 1$ ekanligiga zid. Demak, $0 \notin W(B)$. Shu sababli bu holda

$$\sigma(B) \not\subset W(B), W(B) \not\subset \sigma(B)$$

bo‘ladi.

8-misol. Quyidagi matritsaning sonli tasvirini hisoblang. $A_3 := \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$.

A_3 matritsaning xos sonlarini topamiz. Buning uchun

$$\det(A_{3-}\lambda) = det \begin{pmatrix} 2-\lambda & 1 \\ 1 & 3-\lambda \end{pmatrix} = 0$$

xarakteristik tenglamani qaraymiz.

Hosil bo‘lgan tenglama yechimlarini topamiz:

$$(2-\lambda)(3-\lambda)-1=\lambda^2-5\lambda+5=0 \implies$$

$$\lambda_1=\frac{5-\sqrt{5}}{2};$$

$$\lambda_2=\frac{5+\sqrt{5}}{2};$$

$$W(A_3)=[\lambda_1,\lambda_2]=\left[\frac{5-\sqrt{5}}{2},\frac{5+\sqrt{5}}{2}\right].$$

3. Fridrixs modelining sonli tasviri.

$L_2[-\pi,\pi]$ Hilbert fazosida

$$(Af)(x)=(1-\cos x)f(x)-\sin x\int_{-\pi}^{\pi}\sin t f(t)dt$$

formula yordamida ta’sir qiluvchi va Fridrixs modeli deb ataluvchi operatorni qaraymiz.

Aniqlanishiga ko‘ra A Fridrixs modeli chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator bo‘ladi. Bunda A Fridrixs modelining chiziqli operator ekanligini ko‘rsatish uchun ixtiyoriy α,β kompleks sonlari va $f,g\in L_2[-\pi,\pi]$ elementlar uchun $A(\alpha f+\beta g)=\alpha Af+\beta Ag$ tenglik tekshiriladi. A Fridrixs modelining chegaralangan operator ekanligini ko‘rsatish uchun shunday $M>0$ soni topilib ixtiyoriy $f\in L_2[-\pi,\pi]$ element uchun $\|Af\|\le M\|f\|$ tengsizlik bajarilishini ko‘rsatish yetarli. Xuddi shuningdek, A Fridrixs modelining o‘z-o‘ziga qo‘shma ekanligini ko‘rsatish uchun ixtiyoriy $f,g\in L_2[-\pi,\pi]$ elementlar uchun $(Af,g)=(f,Ag)$ tenglik o‘rinli ekanligini tekshiriladi.

Operatorlar nazariyasidan bizga yaxshi ma’lum bo‘lgan chekli o‘lchamli qo‘zg‘alishlarda muhim spektrning o‘zgarmasligi haqidagi mashhur Veyl teoremasiga ko‘ra A Fridrixs modelining muhim spektri $1-\cos x$ funksiyaning qiymatlar sohasiga, ya’ni $[0;2]$ kesmaga teng bo‘ladi:

$$\sigma_{ess}(A)=[0;2].$$

Endi A Fridrixs modelining diskret spektrini o‘rganish maqsadida xos qiymatga nisbatan $Af=zf$ tenglamani qaraymiz va uni

$$(1-\cos x)f(x)-\sin x\int_{-\pi}^{\pi}\sin t f(t)dt=zf(x) \qquad (6)$$

ko‘rinishda yozib olamiz. (6) tenglamada

$$a=\int_{-\pi}^{\pi}\sin t f(t)dt \qquad (7)$$

kabi belgilash olamiz. $z \notin [0;2]$ ekanligidan barcha $x \in [-\pi;\pi]$ lar uchun $1-\cos x - z \neq 0$ munosabat bajalishi kelib chiqadi. Shu sababli (6) tenglamadan $f(x)$ ni

$$f(x) = \frac{a\sin x}{1-\cos x - z} \qquad (8)$$

ko‘rinishda topamiz. Topilgan ifodani (7) belgilashga qo‘yamiz:

$$a = \int_{-\pi}^{\pi} \sin t \frac{a\sin t}{1-\cos t - z} dt$$

yoki

$$a\left(1 - \int_{-\pi}^{\pi} \frac{\sin^2 t}{1-\cos t - z} dt\right) = 0.$$

Agar oxirgi tenglikda $a = 0$ bo‘lsa, u holda (8) tenglikga ko‘ra $f(x) = 0$ bo‘ladi. Bu esa $f(x)$ funksiyaning xos funksiya ekanligiga zid. Demak $a \neq 0$ va shu sababli

$$\Delta(z) := 1 - \int_{-\pi}^{\pi} \frac{\sin^2 t}{1-\cos t - z} dt = 0.$$

Shunday qilib quyidagi tasdiq o‘rinli:

$z \notin [0;2]$ soni A Fridrixs modelining xos qiymati bo‘lishi uchun $\Delta(z) = 0$ bo‘lishi zarur va yetarlidir.

Bundan esa A Fridrixs modelining diskret spektri uchun

$$\sigma_{disc}(A) = \{z \notin [0;2] : \Delta(z) = 0\}$$

tenglik kelib chiqadi.

Sodda hisoblashlarni bajaramiz:

$$\Delta(0) = 1 - \int_{-\pi}^{\pi} \frac{\sin^2 t}{1-\cos t} dt = 1 - \int_{-\pi}^{\pi} \frac{1-\cos^2 t}{1-\cos t} dt = 1 - \int_{-\pi}^{\pi} \frac{(1-\cos t)(1+\cos t)}{1-\cos t} dt$$

$$= 1 - \int_{-\pi}^{\pi} (1+\cos t) dt = 1 - 2\pi < 0.$$

Ikkinchi tomondan

$$\lim_{z\to\infty} \Delta(z) = 1.$$

$\Delta(\cdot)$ funksiyaning $(-\infty;0)$ uzluksiz va monoton kamayuvchi ekanligini hamda yuqorida keltirilgan ma’lumotlarni inobatga olsak, shunday yagona $z_0 \in (-\infty;0)$ soni topilib $\Delta(z_0) = 0$ bo‘lishi kelib chiqadi. Bu esa o‘z navbatida topilgan $z_0 \in (-\infty;0)$ soni A Fridrixs modelining xos qiymati ekanligini bildiradi. $\Delta(\cdot)$ funksiyaning aniqlanishiga ko‘ra ixtiyoriy $z > 2$ soni uchun $\Delta(z) > 0$ tengsizlik bajariladi. Ya’ni A Fridrixs modeli $(2;+\infty)$ oraliqda yotuvchi xos qiymatlarga ega bo‘lmaydi. Demak,

$$\sigma_{disc}(A) = \{z \notin [0;2] : \Delta(z) = 0\} = \{z_0\},\ z_0 < 0.$$

Bundan esa A Fridrixs modelining sonli tasviri uchun

$$W(A) = [z_0;2)$$

tenglikni hosil qilamiz.

Xuddi shunga o‘xshash mulohazalardan foydalanib umumlashgan Fridrixs modelining sonli tasvirini tahlil qilish mumkin.

Ta’kidlash joizki, [5-21] maqolalarda turli tabiatga Fridrixs modellarining ba’zi spektral xossalari o‘rganilgan. Odatda bunday xossalar panjaradagi uch zarrachali model operatorlarning muhim va diskret spektrlarini tadqiq qilishda muhim ahamiyat kasb etadi. [22-30] ishlarda esa umumlashgan Fridrixs modelining ayrim spektral xossalari tahlil qilingan. Ular panjaradagi soni saqlanmaydigan va uchtadan oshmaydigan zarrachalar sistemaga mos operatorli matrisalar muhim spektrini tavsiflashda ham xos qiymatlar sonining chekli yoki cheksizligini aniqlashda keng tadbiqga egadir.

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