

The background features a complex, layered design. At the top, there are faint, light blue gears and arrows. A central focus is a large gear with a bright orange and yellow gradient inside, surrounded by several overlapping circles. Below this, there's a cluster of blue squares of varying shades, some with white circles inside. The entire composition is set against a backdrop of light blue and grey geometric patterns, including lines, circles, and arrows, suggesting a technical or scientific theme.

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## Fridriks modellari tenzor yig'indisining spektri haqida

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**Annotatsiya:** Ushbu maqolada panjaradagi uchta zarrachalar sistemasiga mos model operator Gilbert fazosidagi chiziqli chegaralangan va o'z-o'ziga qo'shma operator sifatida qaralgan. Bu operator bir o'lchamli qo'zg'alishga ega Fridriks modelining tenzor yig'indisi sifatida tasvirlangan. Fridriks modeli muhim spektrdan chapda yotuvchi yagona oddiy xos qiymatga ega ekanligi ko'rsatilgan. Operatorlar tenzor yig'indisining spektri haqidagi teoremdan foydalanib dastlabki operatorning muhim va diskret spektrlari tadqiq qilingan. Bu model operator ham yagona soda xos qiymatga ega ekanligi isbotlangan.

**Kalit so'zlar:** model operator, Fridriks modeli, tenzor yig'indi, muhim spektr, diskret spektr, xos qiymat.

## About the spectrum of the tensor set of Friedrichs models

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**Abstract:** In this paper, a model operator corresponding to a three-particle system in a grid is considered to be a linear boundary in Gilbert space and a self-contained operator. This operator is described as a tensor sum of the Friedrichs model with one-dimensional motion. The Friedrichs model is shown to have a single simple characteristic that lies to the left of the important spectrum. Important and discrete spectra of the original operator were studied using the theorem on the spectrum of the sum of tensors of operators. This model operator has also been proven to have a unique soda specific value.

**Keywords:** model operator, Friedrichs model, tensor sum, significant spectrum, discrete spectrum, eigenvalue.

Zamonaviy matematik fizika masalalarida, xususan, statistik fizika, qattiq jismlar fizikasi, kvant maydon nazariyasi va yana ko'plab sohalarda panjaradagi uchta

zarrachalar sistemasiga mos model operatorlarning spektral xossalarini o‘rganish bilan bog‘liq masalalar uchrab turadi. Mazkur maqola shu turdagi model operatorlardan birining muhim va diskret spektrini o‘rganish masalasiga bag‘ishlangan.

$L_2^S[-\pi; \pi]^2$  orqali  $[-\pi; \pi]^2$  to‘plamda aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatlarni qabul qiluvchi) simmetrik funksiyalarning Gilbert fazosini belgilaymiz. Bu fazodan olingan  $f$  va  $g$  elementlar uchun

$$(f, g) = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) \overline{g(x, y)} dx dy;$$

$$\|f\| = \left( \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |f(x, y)|^2 dx dy \right)^{\frac{1}{2}}$$

tengliklar o‘rinlidir.

Ushbu maqolada  $L_2^S[-\pi; \pi]^2$  Gilbert fazosida

$$H = H_0 - V_1 - V_2 \quad (1)$$

kabi aniqlanuvchi  $H$  model operatorni qaraymiz. Bunda  $H_0$  ko‘paytirish operatori:

$$(H_0 f)(x, y) = (2 - \cos x - \cos y) f(x, y),$$

$V_1$  va  $V_2$  operatorlar esa quyidagicha ta’sir qiluvchi xususiy integralli operatorlar:

$$(V_1 f)(x, y) = \sin y \int_{-\pi}^{\pi} \sin t f(x, t) dt;$$

$$(V_2 f)(x, y) = \sin x \int_{-\pi}^{\pi} \sin t f(t, y) dt.$$

Dastlab  $H$  operatorni chiziqlilikga tekshiramiz. Buning uchun ixtiyoriy  $\alpha, \beta \in \mathbb{C}$  kompleks sonlari va ixtiyoriy  $f, g \in L_2^S[-\pi; \pi]^2$  elementlar uchun

$$H(\alpha f + \beta g) = \alpha Hf + \beta Hg$$

tenglik bajarilishini ko‘rsatamiz.

Haqiqatan ham,

$$H(\alpha f + \beta g)(x, y) = (2 - \cos x - \cos y)(\alpha f(x, y) + \beta g(x, y))$$

$$- \sin y \int_{-\pi}^{\pi} \sin t (\alpha f(x, t) + \beta g(x, t)) dt -$$

$$- \sin x \int_{-\pi}^{\pi} \sin t (\alpha f(t, y) + \beta g(t, y)) dt =$$

$$\alpha \left[ (2 - \cos x - \cos y) f(x, y) - \sin y \int_{-\pi}^{\pi} \sin t f(x, t) dt - \right.$$

$$\left. - \sin x \int_{-\pi}^{\pi} \sin t f(t, y) dt \right] +$$

$$+ \beta \left[ (2 - \cos x - \cos y) g(x, y) - \sin y \int_{-\pi}^{\pi} \sin t g(x, t) dt - \right.$$

$$-\sin x \int_{-\pi}^{\pi} \sin t g(t, y) dt \Big] = \alpha(Hf)(x, y) + \beta(Hg)(x, y).$$

Demak,  $H$  chiziqli operator ekan.

$H$  operatorning chegaralanganligini ko'rsatish uchun  $\|Hf\|^2$  ni qaraymiz va uni quyidagicha baholaymiz:

$$\begin{aligned} \|Hf\|^2 &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |(Hf)(x, y)|^2 dx dy = \\ &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left| (2 - \cos x - \cos y)f(x, y) - \sin y \int_{-\pi}^{\pi} \sin t f(x, t) dt - \right. \\ &\quad \left. - \sin x \int_{-\pi}^{\pi} \sin t f(t, y) dt \right|^2 dx dy \leq 3 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |2 - \cos x - \cos y|^2 \cdot \\ &\quad \cdot |f(x, y)|^2 dx dy + 3 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left| \sin y \int_{-\pi}^{\pi} \sin t f(x, t) dt \right|^2 dx dy + \\ &+ 3 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left| \sin x \int_{-\pi}^{\pi} \sin t f(t, y) dt \right|^2 dx dy \leq 48 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |f(x, y)|^2 dx dy + \\ &\quad + 3 \int_{-\pi}^{\pi} \sin^2 y dy \int_{-\pi}^{\pi} \sin^2 t dt \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |f(x, t)|^2 dx dt + \\ &\quad + 3 \int_{-\pi}^{\pi} \sin^2 x dx \int_{-\pi}^{\pi} \sin^2 t dt \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |f(t, y)|^2 dt dy = \\ &= (48 + 6\pi^2) \|f\|^2, \end{aligned}$$

ya'ni

$$\|Hf\| \leq \sqrt{48 + 6\pi^2} \|f\|.$$

Demak,  $H$  chegaralangan operator ekan.

Endi  $H$  operatorni o'z-o'ziga qo'shmalikga tekshiramiz:

$$\begin{aligned} (Hf, g) &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} (Hf)(x, y) \overline{g(x, y)} dx dy = \\ &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left[ (2 - \cos x - \cos y)f(x, y) - \sin y \int_{-\pi}^{\pi} \sin t f(x, t) dt - \right. \\ &\quad \left. - \sin x \int_{-\pi}^{\pi} \sin t f(t, y) dt \right] \overline{g(x, y)} dx dy = \\ &\quad \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} (2 - \cos x - \cos y) f(x, y) \overline{g(x, y)} dx dy - \\ &\quad - \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \sin y \sin t f(x, t) \overline{g(x, y)} dx dy dt - \\ &\quad - \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \sin x \sin t f(t, y) \overline{g(x, y)} dx dy dt = \end{aligned}$$

$$\begin{aligned}
 &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) \left[ \overline{(2 - \cos x - \cos y)g(x, y) -} \right. \\
 &\left. - \sin y \int_{-\pi}^{\pi} \sin t g(x, t) dt - \sin x \int_{-\pi}^{\pi} \sin t g(t, y) dt \right] dx dy = \\
 &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) \overline{(Hg)(x, y)} dx dy = (f, Hg).
 \end{aligned}$$

Shunday qilib,  $H^* = H$ , ya'ni  $H$  o'z-o'ziga qo'shma operator ekan.

(1) formula yordamida ta'sir qiluvchi  $H$  operator panjaradagi uchta zarrachalar sistemasiga mos keluvchi model operator hisoblanadi. Bunda  $H_0$  ko'paytirish operatoriga qo'zg'almas operator deyiladi va  $V_1$  hamda  $V_2$  operatorlarga esa lokal bo'lmagan potensial operatorlari deyiladi.

$H$  operatorning spektral xossalarini o'rganish maqsadida  $L_2[-\pi; \pi]$  Gilbert fazosida

$$h = h_0 - v \tag{2}$$

ko'rinishda ta'sir qiluvchi operatorni qaraymiz. Bunda  $h_0$  ko'paytirish operatori:

$$(h_0 g)(x) = (1 - \cos x)g(x),$$

$v$  esa integral operator:

$$(vg)(x) = \sin x \int_{-\pi}^{\pi} \sin t g(t) dt.$$

Yuqorida keltirilgan mulohazalarga o'xshash tekshirishlar yordamida (2) formula yordamida ta'sir qiluvchi  $h$  operatorning chiziqli, chegaralangan va o'z-o'ziga qo'shma ekanligini ko'rsatish mumkin.

$h$  operator zamonaviy matematik fizikada Fridriks modeli nomi bilan mashhur bo'lib, panjaradagi ikki zarrachali sistemaga mos model operator sifatida ham qarash mumkin.

Quyida Fridriks modeli haqida dastlabki tushunchalarni keltirib o'tamiz.

$L_2[-1;1]$  Gilbert fazosida

$$H_{\lambda} f(x) = xf(x) + \lambda \int_{-1}^1 K(x; y) f(y) dy$$

ko'rinishida ta'sir qiluvchi  $H_{\lambda}$  operatorni qaraymiz. Bu yerda  $\lambda$  haqiqiy musbat son,  $K(x; y)$  esa  $[-1;1]^2$  da aniqlangan haqiqiy qiymatli uzluksiz simmetrik funksiya, ya'ni  $K(x; y) = K(y; x)$  tenglik o'rinlidir. Bu operator birinchi marta K.Fridriks tomonidan uzluksiz spektr qo'zg'alishlari nazariyasi modeli sifatida [1] ishda qaralgan. Bu maqolada  $K(x; y)$  yadro o'zining o'zgaruvchilarining uzluksiz funksiyasi bo'lib, Gyolder shartini va

$$K(x; -1) = K(x; 1) = K(-1; y) = K(1; y) = 0, \quad x, y \in [-1; 1]$$

shartlarni qanoatlantirishi talab qilingan.

Fridriks tomonidan  $\lambda \in R$  parametrning yetarlicha kichik qiymatlarida  $H_\lambda$  va  $H_0$  operatorlar unitar ekvivalent ekanligi isbotlangan, ya'ni  $H_\lambda$  operator  $[-1;1]$  ga teng bo'lgan sodda Lebeg spektriga ega ekanligi ko'rsatilgan. 1948-yilda Fridriks o'zining [2] ishida o'z modelini quyidagicha umumlashtirish masalasini taklif qilgan: birinchidan,  $[-1;1]$  o'rniga haqiqiy sonlar o'qidagi ixtiyoriy chekli yoki cheksiz bo'lgan  $\ell$  intervalni qarash; ikkinchidan, qiymatlari biror abstrakt Gilbert fazosi bo'lgan  $f$  funksiyalarni qarash.  $\ell$  interval cheksiz bo'lgan holda  $K(x;y)$  yadro cheksizlikda kamayuvchi bo'lsin degan qo'shimcha shart kiritib Fridriks bu nisbatan umumiy holda  $H_\lambda$  va  $H_0$  operatorlarning unitar ekvivalent ekanligini isbotlagan.

Keyinchalik Fridriksning [1] va [2] ishlari O.A. Ladijenskiy, .D.Faddeyevlar tomonidan [3] maqolada va L.D.Faddeyev tomonidan [4] maqolada rivojlantirilgan.

Panjaradagi Fridriks modeli bilan bog'liq tadqiqotlar [5-19] ishlarda olib borilgan. Umumlashgan Fridriks modelining ayrim spektral xossalari [20-30] ishlarda o'rganilgan. Bu xossalar o'z navbatida panjaradagi soni saqlanmaydigan va uchtadan oshmaydigan zarrachalar sistemasiga mos 3-tartibli operatorli matrisalarning muhim va diskret spektrlarini tadqiq qilishda foydalanilgan.

Aniqlanishiga ko'ra  $v$  bir o'lchamli operator, ya'ni  $\dim Im v = 1$ . Haqiqatan ham,

$$Im v = \{g(x) = asinx: a \in C\}$$

tenglikdan  $\dim Im v = 1$  ekanligi kelib chiqadi. Chekli o'lchamli qo'zg'alishlarda muhim spektrning o'zgarmasligi haqidagi Veyl teoremasiga teoremasiga ko'ra  $h_0$  va  $h_0 - v$  operatorlarning muhim spektrlari ustma-ust tushadi.  $h_0$  operator  $1 - \cos x$  funksiyaga ko'paytirish operatori. Shu bois

$$\sigma_{ess}(h_0) = [0; 2].$$

Yuqorida keltirilgan mulohazalarga ko'ra

$$\sigma_{ess}(h) = [0; 2].$$

$h$  operatorning xos qiymatlarini o'rganish maqsadida  $C \setminus [0; 2]$  sohada regulyar bo'lgan

$$\Delta(z) = 1 - \int_{-\pi}^{\pi} \frac{\sin^2 t dt}{1 - \cos t - z}$$

funksiyani qaraymiz. Zamonaviy matematik fizikada  $\Delta(\cdot)$  funksiyaga  $h$  operatorga mos Fredholm determinanti deyiladi.

Quyidagi lemma  $h$  operator xos qiymatlari va  $\Delta(\cdot)$  funksiya nollari orasidagi bog'lanishni ifodalaydi.

1-lemma.  $z \in C \setminus [0; 2]$  soni  $h$  Fridriks modelining xos qiymati bo'lishi uchun  $\Delta(z) = 0$  bo'lishi zarur va yetarlidir.



Isboti. Zaruriyligi. Faraz qilaylik,  $z \in C \setminus [0; 2]$  soni  $h$  Fridrixs modelining xos qiymati,  $g \in L_2[-\pi; \pi]$  esa unga mos xos funksiya bo'lsin. U holda  $g(\cdot)$  xos funksiya  $hg = zg$  xos qiymatga nisbatan tenglamani qanoatlantiradi.

$$hg = zg \text{ tenglamani}$$

$$(1 - \cos x - z)g(x) - \sin x \int_{-\pi}^{\pi} \sin t g(t) dt = 0 \quad (3)$$

ko'rinishida yozib olamiz. Ushbu

$$a = \int_{-\pi}^{\pi} \sin t g(t) dt \quad (4)$$

belgilashni kiritamiz.  $z \in C \setminus [0; 2]$  ekanligidan barcha  $x \in [-\pi; \pi]$  lar uchun  $1 - \cos x - z \neq 0$  munosabat kelib chiqadi. Shu bois (3) tenglamadan  $g(x)$  xos funksiya uchun

$$g(x) = \frac{a \sin x}{1 - \cos x - z} \quad (5)$$

ifodaga ega bo'lamiz.  $g(x)$  xos funksiya uchun topilgan (5) ifodani (4) belgilashga qo'yamiz:

$$a = \int_{-\pi}^{\pi} \sin t \frac{a \sin t}{1 - \cos t - z} dt$$

yoki

$$a \left( 1 - \int_{-\pi}^{\pi} \frac{\sin^2 t dt}{1 - \cos t - z} \right) = 0$$

yoki

$$a \cdot \Delta(z) = 0.$$

Agar oxirgi tenglikda  $a = 0$  bo'lsa, u holda (5) tenglikga ko'ra  $g(x) = 0$  bo'ladi. Bu esa  $g(x)$  funksiyaning xos funksiya ekanligiga, ya'ni nolmas funksiya ekanligiga zid. Shu sababli  $\Delta(z) = 0$  bo'ladi. 1-lemmaning zaruriylik qismi bajarildi.

Yetarliligi. Faraz qilaylik, biror  $z_0 \in C \setminus [0; 2]$  soni uchun  $\Delta(z_0) = 0$  bo'lsin. U holda

$$g(x) = \frac{\sin x}{1 - \cos x - z_0}$$

funksiya  $hg = z_0g$  tenglamani qanoatlantiradi:

$$(hg)(x) - z_0g(x) = (1 - \cos x - z_0)g(x) - \sin x \int_{-\pi}^{\pi} \sin t g(t) dt =$$

$$(1 - \cos x - z_0) \frac{\sin x}{1 - \cos x - z_0} - \sin x \int_{-\pi}^{\pi} \frac{\sin^2 t dt}{1 - \cos t - z_0} =$$

$$\sin x \left( 1 - \int_{-\pi}^{\pi} \frac{\sin^2 t dt}{1 - \cos t - z_0} \right) = \sin x \cdot \Delta(z_0) = 0.$$

$g(x)$  funksiya  $[-\pi; \pi]$  da aniqlangan uzluksiz funksiya. Shu sababli  $g \in L_2[-\pi; \pi]$  munosabat o'rinli bo'ladi. 1-lemma to'liq isbotlandi.

Isbot qilingan 1-lemmadan  $h$  operatorning diskret spektri uchun quyidagi natija kelib chiqadi.

1-natija.  $h$  Fridriks modelining diskret spektri uchun

$$\sigma_{disc}(h) = \{z \in \mathbb{C} \setminus [0; 2] : \Delta(z) = 0\}$$

tenglik o‘rinlidir.

$h$  Fridriks modelining xos qiymatlarini o‘rganishda

$$I(z) = \int_{-\pi}^{\pi} \frac{\sin^2 t \, dt}{1 - \cos t - z}$$

integralning  $z = 0$  nuqtadagi qiymati muhim ahamiyat kasb etadi.

$z = 0$  nuqtadagi qiymatini hisoblaymiz:

$$\begin{aligned} I(0) &= \int_{-\pi}^{\pi} \frac{\sin^2 t \, dt}{1 - \cos t} = \int_{-\pi}^{\pi} \frac{1 - \cos^2 t}{1 - \cos t} \, dt = \\ &= \int_{-\pi}^{\pi} \frac{(1 - \cos t)(1 + \cos t)}{1 - \cos t} \, dt = \int_{-\pi}^{\pi} (1 + \cos t) \, dt = 2\pi. \end{aligned}$$

Integral belgisi ostida limitga o‘tish haqidagi Lebeg teoremasiga ko‘ra

$$\Delta(0) = \lim_{z \rightarrow -0} \Delta(z) = \lim_{z \rightarrow -0} (1 - I(z)) = 1 - I(0) = 1 - 2\pi < 0.$$

$\Delta(\cdot)$  funksiya  $(-\infty; 0)$  oraliqda uzluksiz, monoton kamayuvchi va  $\Delta(0) = 1 - 2\pi < 0$  hamda

$$\lim_{z \rightarrow -\infty} \Delta(z) = 1$$

ekanligini inobatga olsak, u holda shunday  $z_0 \in (-\infty; 0)$  soni topilib,  $\Delta(z_0) = 0$  bo‘ladi. Bunda  $z_0$  soni  $\Delta(\cdot)$  funksiyaning oddiy (bir karrali) noli bo‘ladi. 1-lemmaga ko‘ra  $z_0 \in (-\infty; 0)$  soni  $h$  Fridriks modelining oddiy (bir karrali) xos qiymati bo‘ladi.

$v$  operatorning musbat aniqlangan ekanligini ekanligini ko‘rsatamiz: ixtiyoriy  $f \in L_2[-\pi; \pi]$  uchun

$$\begin{aligned} (vf, f) &= \int_{-\pi}^{\pi} (vf)(x) \overline{f(x)} \, dx = \int_{-\pi}^{\pi} \left( \sin x \int_{-\pi}^{\pi} \sin t f(t) \, dt \right) \overline{f(x)} \, dx = \\ &= \int_{-\pi}^{\pi} \sin x f(x) \, dx \cdot \overline{\int_{-\pi}^{\pi} \sin t f(t) \, dt} = \left| \int_{-\pi}^{\pi} \sin x f(x) \, dx \right|^2 \geq 0 \end{aligned}$$

tengsizlik o‘rinli. Shu sababli  $v$  musbat aniqlangan operator.

Faraz qilaylik,  $z > 2$  bo‘lsin. U holda ixtiyoriy  $f \in L_2[-\pi; \pi]$  uchun

$$\begin{aligned} (hf, f) - z(f, f) &= ((h_0 - Iz)f, f) - (vf, f) = \\ &= \int_{-\pi}^{\pi} (1 - \cos x - z) |f(x)|^2 \, dx - (vf, f) < 0 \end{aligned}$$

tengsizlik o‘rinli. Bu esa o‘z navbatida  $h$  Fridriks modeli muhim spektrdan o‘ngda joylashgan xos qiymatlarga ega emasligini bildiradi. Shunday qilib,  $h$  Fridriks modelining spektri uchun

$$\sigma(h) = \{z_0\} \cup [0; 2], z_0 < 0$$

tenglik o‘rinlidir. Bu yerda  $z_0$  soni  $\Delta(\cdot)$  funksiyaning yagona manfiy noli.  $H$  va  $h$  operatorlarning aniqlanishiga ko‘ra

$$H = h \otimes I + I \otimes h \quad (6)$$

tasvir o‘rinlidir. Operatorlar tenzor yig‘indisining spektri haqida teorema ko‘ra

$$\sigma(H) = \sigma(h) + \sigma(h)$$

tenglik o‘rinlidir. Bu yerda  $A, B \subset R$  soni to‘plamlar uchun  $A + B$  to‘plam

$$A + B = \{a + b : a \in A, b \in B\}$$

kabi aniqlanadi.

Shu sababli

$$\sigma(H) = \sigma(h) + \sigma(h) = \{\{z_0\} \cup [0; 2]\} + \{\{z_0\} \cup [0; 2]\} = \{2z_0\} \cup [z_0; z_0 + 2] \cup [0; 4]$$

tenglik o‘rinlidir.

$H$  operatorning muhim va diskret spektrlari, haqidagi asosiy teorema yuqorida keltirilgan mulohazalardan xulosa sifatida kelib chiqadi:

Asosiy teorema:  $H$  operatorning muhim spektri uchun

$$\sigma_{ess}(H) = [z_0; z_0 + 2] \cup [0; 4]$$

tenglik o‘rinli, bu yerda  $z_0$  soni  $\Delta(\cdot)$  funksiyaning yagona manfiy noli. Bundan tashqari,  $2z_0$  soni  $H$  operatorning yagona oddiy (bir karrali) xos qiymati bo‘ladi, ya’ni

$$\sigma_{disc}(H) = \{2z_0\}.$$

Xulosa o‘rnida shuni aytish mumkinki, maqolada tadqiq qilingan  $H$  operatorni (6) ko‘rinishida tasvirlash mumkinligi  $H$  operator muhim va diskret spektrlarini aniq hisoblash imkonini bermoqda. Muhim spektr oralig‘ida bo‘shliq hosil bo‘lishi  $z_0$  xos qiymatning joylashgan oralig‘idan (aniq qiymatidan) bog‘liq. Uni Maple amaliy maketi yordamida hisoblash mumkin.

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