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Determination of coefficients of fractional differential equations with the Generalized Riemann-Liouville Time Derivative

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Let $T>0,\ \ l>0$ be fixed numbers and $\Omega_{lT}:=\{(x,t):0<x<l,0<t\le T\}$. Consider the time-fractional diffusion equation

$$D^{\alpha,\beta}_{0+,t}u(x,t)-u_{xx}+q(t)u(x,t)=p(t)f(x,t),\quad x\in(0,l),\ t\in(0,T], \tag{1}$$

the initial conditions of Cauchy type

$$I^{(2-\alpha)(1-\beta)}_{0+,t}u(x,t)\big|_{t=0}=\varphi_1(x),$$

$$\frac{\partial}{\partial t}\left(I^{(2-\alpha)(1-\beta)}_{0+,t}u\right)(x,t)\big|_{t=0}=\varphi_2(x),\ x\in[0,l], \tag{2}$$

the boundary conditions

$$u(0,t)=u(l,t)=0,\quad 0\le t\le T, \tag{3}$$

and the nonlocal additional condition

$$\int\limits_0^l w_i(x)u(x,t)dx=h_i(t),\ i=1,2,\ t\in[0,T]. \tag{4}$$

Here the generalized Riemann-Liouville (Hilfer) fractional differential operator $D^{\alpha,\beta}_{0+,t}$ of the order $1<\alpha<2$ and type $0\le\beta\le1$ (see, [1], pp. 112-118, [2], pp. 62-65).

Assume that throughout this article, given functions $\varphi_1,\ \varphi_2,\ f,\ w$ and h satisfy the following assumptions:

A1) $\varphi_i\in C^3[0,l],\ \varphi_i^{(4)}\in L_2[0,l],\ \varphi_i(0)=\varphi_i(l)=0,\ \varphi''{}_i(0)=\varphi''{}_i(l)=0,\ i=1,2;$

A2) $f(x,\cdot)\in C[0,T]$ *and for* $t\in[0,T], f(\cdot,t)\in C^3[0,l],\ f(\cdot,t)^{(4)}\in L_2[0,l],\ f(0,t)=f(l,t)=0,\ f_{xx}(0,t)=f_{xx}(l,t)=0;$

A3) $w(x)\in C^2[0,l]$ *and* $w(0)=w(l)=0$ *and* $w''(0)=w''(l)=0;$

A4) $\left(D^{\alpha,\beta}_{0+,t}h\right)(t)\in C[0,T],\ |h(t)|\ge h_0>0,\ h_0$ *is a given number,*

$$\int\limits_0^l w_i(x)\varphi_1(x)dx=I^{(2-\alpha)(1-\beta)}_{0+,t}h_i(t)\big|_{t=0+},$$

$$\int_0^l w_i(x)\varphi_2(x)dx = \frac{\partial}{\partial t}\left(I_{0+,t}^{(2-\alpha)(1-\beta)}h_i(t)\right)(t)\Big|_{t=0+},\ i=1,2.$$

We consider the weighted spaces of continuous functions [[3], pp. 4-5, 162-163].

$$C_\gamma^{2,\alpha,\beta}(\Omega) = \Big\{u(x,t):\ u(\cdot,t)\in C^2(0,1);\ t\in[0,T] \text{ and}$$

$$D_{0+,t}^{\alpha,\beta}u(x,\cdot)\in C_\gamma(0,T];\ x\in[0,1],\ 1<\alpha\le 2,\ 0\le\beta\le 1\Big\},$$

where $\overline{\Omega}_{lT} := \{(x,t): 0\le x\le l,\ 0\le t\le T\}$.

The papers [3] and [4] study inverse problems of finding time-dependent source terms, respectively, in time-fractional diffusion equation by using eigenfunction expansion of the non-self adjoint spectral problem along the generalized Fourier method. The main results of these studies comprise the existence and uniqueness theorems, as well as a stability estimate for the solution of the problem of determining the coefficient in a time-fractional diffusion and wave equation.

Using the above results, we obtain the following assertion.

Lemma. Let $p(t)$, $q(t)\in C[0,T]$, A1)-A2) are satisfied, then there exists a unique solution of the direct problem (1)-(3) $u(x,t)\in C_\gamma^{2,\alpha,\beta}(\overline{\Omega}_{lT})$.

Theorem Let A1)-A4) are satisfied. Then there exists a number $T^*\in(0,T)$, such that there exists a unique solution $p(t),\ q(t)\in C[0,T^*]$ of the inverse problem (1)-(4).

Keywords: Hilfer fractional differential equation, Riemann-Liouville fractional derivative, inverse problem, initial conditions, boundary conditions.

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Self-similar solutions for the membrane transverse vibration equation

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In our modern life, many problems of modern mathematics and theoretical physics lead to the investigation of hypergeometric functions of one and several variables, (for example) partial differential equations are obtainable with the help of such hypergeometric functions [1].In particular, the energy absorbed by some nonferromagnetic conductor sphere included in an internal magnetic field can be calculated with the help of such functions [2]. Hypergeometric functions of several variables are used in physical and quantum chemical applications as well [3]. Especially, many problems in gas dynamics lead to solutions of degenerate second-order partial differential equations which are then solvable in terms of multiple hypergeometric functions [5-6].

We consider and establish the solutions of the degenerating model equation in terms of the hypergeometric function ${}_2F_1\left(a,b;c;z\right)$.

When solving vibration problems, the model is obtained by calculating the transverse displacement $u\left(r,t\right)$ of a symmetrically deformed membrane. To process inhomogeneous waves $u\left(r,t\right)$ representing the frequency, the following equation is modeled and considered by J.Kastillo, C.Jiménez and R.Meléndez [4].

$$a^{-2}u_{tt}\left(r,t\right)=r^{2-2c}u_{rr}\left(r,t\right)+\left(1-2l\right)r^{1-2c}u_{r}\left(r,t\right)+\left(l^2-c^2\nu^2\right)r^{-2c}u\left(r,t\right) \tag{8}$$

$$\left(\nu,l,c=const>0\right),$$

where $a^2=\frac{T}{D}$, T membrana tension and D mass per unit area of the membrane.

We obtain the following special solutions of equation (8):

$$u_1\left(r,t\right)=r^{l-\nu c}t^{\nu}{}_2F_1\left(-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2},1-\nu,\frac{r^{2c}}{a^2c^2t^2}\right), \tag{9}$$

$$u_2\left(r,t\right)=r^{l+c\nu}\left(\frac{1}{a^2c^2t}\right)^{\nu}\delta{}_2F_1\left(-\frac{\nu}{2}-\frac{1}{2},-\frac{\nu}{2},1+\nu,\frac{r^{2c}}{a^2c^2t^2}\right), \tag{10}$$

where ${}_2F_1\left(a,b;c;z\right)$ is Gaussian hypergeometric function with two numerator parameters and one denominator parameter.

Keywords: Parabolic PDE of degenerate type; Self-made solution; Linearly independent solution, Generalized hypergeometric function, Integral representation.

2010 Mathematics Subject Classification: 35L80, 33C05, 35C06.

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Solvability of an inverse coefficient problem for a time-fractional diffusion equation with periodic boundary and integral overdetermination conditions

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We consider the initial-periodic boundary problem for the fractional diffusion equation

$$\partial_t^{\alpha}u-u_{xx}+a(t)u=f(x,t)g(t),\quad (x,t)\in D_T, \tag{1}$$

$$u(x,0)=\varphi(x),x\in[0,1], \tag{2}$$

$$u(0,t) = u(1,t), \quad u_x(0,t) = u_x(1,t), \varphi(0) = \varphi(1), \quad \varphi'(0) = \varphi'(1), t \in [0,T], \tag{3}$$

where ∂_t^α is the Caputo fractional derivative of order $0 < \alpha \leq 1$ in the time variable (see [1, pp. 90-94]), $a(t), g(t), t > 0$ are the source control terms, $f(x,t)$ is known source term, $\varphi(x)$ is the initial temperature, T is arbitrary positive number and $D_T := \{(x,t) : 0 < x < 1, 0 < t \leq T\}$).
.

The problem of determining a function $u(x,t), (x,t) \in D_T$, that satisfies (1)-(3) with known functions $a(t), g(t), f(x,t)$ and $\varphi(x)$ will be called the direct problem.

In the inverse problem, it is required to determine the coefficients $a(t), g(t), t > 0$, in (1) using over-determination conditions about the solution of the direct problem (1)-(3):

$$\int_0^1 \omega_i(x) u(x,t) dx = h_i(t), \; i = 1, 2, \quad x \in [0,1], \tag{4}$$

where $\omega_i(x), h_i(t), i = 1, 2$ are given functions.

By $C^{2,\alpha}(D_T)$ we denote the class of 2 times continuously differentiable with respect to x and α times continuously differentiable with respect to t in the domain D_T functions.

Definition 1. *The triple of functions $\{u(x,t), a(t), g(t)\}$ from the class $C^{2,\alpha}(D_T) \cap C^{1,0}(\overline{D}_T) \times C[0,T] \times C[0,T]$ is said to be a classical solution of problem (1)-(4), if the functions $u(x,t), a(t)$ and $g(t)$ satisfy the following conditions:*

(1) The function $u(x,t)$ and its derivatives $\partial_t^\alpha u(x,t), u_{xx}(x,t)$ are continuous in the domain D_T ;

(2) the function $a(t), g(t)$ is continuous on the interval $[0,T]$;

(3) equation (3) and conditions (2)-(4) are satisfied in the classical sense.

Throughout this article the functions $\varphi, \;\; f, \;\; \omega_i$ and $h_i \;\; (i := 1, 2)$ are assumed to satisfy the following conditions:

(*A*1) $\varphi(x) \in C^2(0,1)$; $\varphi^{(3)}(x) \in L_2(0,1)$; $\varphi(0) = \varphi(1)$; $\varphi'(0) = \varphi'(1)$; $\varphi''(0) = \varphi''(1)$; $\varphi^{(3)}(0) = \varphi^{(3)}(1)$;

(*A*2) $f(x,t) \in C(\overline{D}_T) \cap C^{2,1}(D_T)$; $f^{(3)}(x,t) \in L_2(D_T)$; $f(0,t) = f(1,t)$; $f'(0,t) = f'(1,t)$; $f''(0,t) = f''(1,t)$;

(*A*3) $h_i(t) \in AC[0,T]$; $\omega_i(x) \in C^2[0,1]$; $\omega_i^{(3)}(x,t) \in L_2[0,l]$; $\int_0^1 \omega_i(x)\varphi(x)dx = h_i(0)$; $\omega_i(0) = \omega_i(1)$; $\omega_i'(0) = \omega_i'(1)$; $\omega_i''(0) = \omega_i''(1), i = 1, 2$.

Lemma 1. *Let $\{g(t), a(t)\} \in C[0,T], (A1), (A2)$ are satisfied, then there exists a unique solution of the direct problem (1)-(3) $u(x,t) \in C^{2,\alpha}(D_T) \cap C^{1,0}(\overline{D}_T)$.*

The main result of this work is presented as follows:

Theorem 1. *Let (A1)-(A4) are satisfied. Then there exists a number $T^* \in (0,T)$, such that there exists a unique solution $a(t), \; g(t) \in C[0,T^*]$ of the inverse problem (1)-(4).*

For proving this theorem, inverse problem (1)-(4) reduces to the equivalent integral equations with respect unknown functions $u(x,t), a(t), g(t)$. For solving this equation the contracted mapping principle is applied. The local existence and uniqueness results are proven.

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