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functions are obtained, under which the inverse problem has unique solutions for a sufficiently small interval.

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INITIAL BOUNDARY VALUE PROBLEMS FOR TIME FRACTIONAL WAVE EQUATION WITH GENERALIZED FRACTIONAL DERIVATIVE

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In the domain $\Omega = \{(x,t): 0 \le x \le 1,\ 0 \le t \le T\}$, we consider the unique solvability of the inverse problem of determining a pair of functions $\{u(x,t),\ q(t)\}$ satisfying the equation

$$\left(D_{0+,t}^{\alpha,\beta}u\right)(x,t) - u_{xx} + q(t)u(x,t) = f(x,t) \qquad (1)$$

with the initial conditions of Cauchy type

$$I_{0+,t}^{(2-\alpha)(1-\beta)}u(x,t)|_{t=0} = \varphi_1(x),$$

$$\frac{\partial}{\partial t}\left(I_{0+,t}^{(2-\alpha)(1-\beta)}u\right)(x,t)|_{t=0} = \varphi_2(x), \qquad x \in [0,1], \qquad (2)$$

the boundary conditions

$$u(0,t) = u(1,t), \qquad u_x(1,t) = 0, \qquad 0 \le t \le T. \qquad (3)$$

Here the generalized Riemann-Liouville (Hilfer) fractional differential operator $D_{0+,t}^{\alpha,\beta}$ of the order $1 < \alpha < 2$ and type $0 \leq \beta \leq 1$ is defined as follows :

$$D_{0+,t}^{\alpha,\beta} u(\cdot,t) = \left(I_{0+,t}^{\beta(2-\alpha)} \frac{\partial^2}{\partial t^2} \left(I_{0+,t}^{(1-\beta)(2-\alpha)} u \right) \right)(\cdot,t),$$

$$I_{0+,t}^{\gamma} u(x,t) = \frac{1}{\Gamma(\gamma)} \int_0^t \frac{u(x,\tau)}{(t-\tau)^{1-\gamma}} d\tau, \ \ \gamma \in (0,1)$$

is the Riemann–Liouville fractional integral of the function $u(x,t)$ with respect to t , $\Gamma(\cdot)$ is the Euler's Gamma function. The function $f(x,t)$, $\varphi_1(x)$, $\varphi_2(x)$ are known functions.

In [1, pp. 112-118], by R. Hilfer was introduced a generalized form of the Riemann-Liouville fractional derivative of order α and a type $\beta \in [0,1]$, which coincides with the Riemann-Liouville fractional derivative at $\beta = 0$ and with Gerasimov-Caputo fractional derivative $\beta = 1$, and $\beta \in (0,1)$ interpolates these both fractional derivatives.

Assume that throughout this article, given functions φ_1, φ_2, f satisfy the following assumptions:

A1) $\{\varphi_1, \varphi_2\} \in C^3[0,1]$, $\left\{\varphi_1^{(4)}, \varphi_2^{(4)}\right\} \in L_2[0,1]$, $\varphi_1(0) = \varphi_1(1) = 0$, $\varphi_2(0) = \varphi_2(1) = 0$, $\varphi''_1(0) = \varphi''_1(1) = 0$, $\varphi''_2(0) = \varphi''_2(1) = 0$, *and* $\varphi_1^{(4)}(0) = \varphi_1^{(4)}(1) = 0$, $\varphi_2^{(4)}(0) = \varphi_2^{(4)}(1) = 0$,;

A2) $f(x,\cdot) \in C[0,T]$ and for $t \in [0,T]$, $f(\cdot,t) \in C^3[0,1]$, $f(\cdot,t)^{(4)} \in L_2[0,1]$, $f(0,t) = f(1,t) = 0$, $f_{xx}(0,t) = f_{xx}(1,t) = 0$ and $f_{xxxx}(0,t) = f_{xxxx}(1,t) = 0$;

Fractional Calculus is a new growing field. Fractional derivative is the generalization of the classical derivative of whole order. Fractional derivative had been used in physical events such as visco-elasticity, dynamical processes in self-similar structures, biosciences, signal processing, system control theory, electrochemistry, diffusion processes and etc [2]-[6].

Inverse problems for classical integro-differential wave propagation equations have been extensively studied. Nonlinear inverse coefficient problems with various types of sufficient determination conditions are often found in the literature (e.g., [7]-[12] and references therein).

We consider the weighted spaces of continuous functions .

$$C_\gamma[a,b] := \{f:(a,b] \to R: (x-a)^\gamma f(x) \in C[a,b],\ 0 \le \gamma < 1,\},$$

$$C_\gamma^{2,\alpha,\beta}(\Omega) = \{u(x,t):\ u(\cdot,t) \in C^2(0,1);\ t \in [0,T] \text{ and}$$

$$D_{0+,t}^{\alpha,\beta} u(x,\cdot) \in C_\gamma(0,T];\ x \in [0,1],\ 1 < \alpha \le 2,\ 0 \le \beta \le 1\}.$$

We obtain the following assertion.

Theorem 1. Let $q(t) \in C[0,T]$, A1), A2) are satisfied, then there exists a unique solution of the direct problem (1)-(3) , $u(x,t) \in C_\gamma^{2,\alpha}(\overline{\Omega})$.

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CARLEMAN'S FORMULA FOR THE GENERALIZED CAUCHY-RIEMANN SYSTEM IN A BOUNDED DOMAIN

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Let $x=(x_1,x_2,x_3)$ and $y=(y_1,y_2,y_3)$ be points of the real three dimensional Euclidean space R^3, Ω is a part of a ball $B=B(0,R)$ with center at the origin and radius $R>0$. Let S be a smooth closed surface in B which does not meet $x=0$ and divides B into two domains. Denote by Ω the closed domain that does not contain the origin. Its boundary $\partial\Omega$ consists of simply connected domain in R^3, with piecewise-smooth bondary consisting of S and a part of the sphere ∂B in R^3.

Suppose that vector function $U=(u_1,u_2,u_3)$ satisfied in D the system equations [1]

$$\sum_{k=1}^{3}\left(\frac{\partial u_k}{\partial x_k}+a_k u_k\right)=0,\quad \frac{\partial u_j}{\partial x_k}-\frac{\partial u_k}{\partial x_j}-b_k u_j+b_j u_k=0\quad (k,j=1,2,3). \quad (1)$$

Statement of the problem. Find a regular solution U of system (1) in the domain Ω using its Cauchy data on the surface S:

$$U(y)=f(y),\quad y\in S, \quad (2)$$

where $f(y)$ is a given continuous vector function on S.

Using results from [2], [3],[4] on solving the Cauchy problem, we construct the Carleman matrix for the Laplace and Helmholts equations in explicit form and, on its basis, the regularized solution of the Cauchy problem for system (1). By using the continuation formula we found necessary and sufficient for the extendibility of functions given an a part of a boundary to the domain as a solution of the system (1). We prove the