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The problem of determining the memory in two-dimensional system of integro-differential Maxwell's equations

Turdiev Kh. Kh. [1]

Ikki oʻlchovli integro differensial Maksvell tenglamalar sistemasi uchun xotirani aniqlash masalasi

Ushbu maqolada ikki oʻlchovli integro differensial Maksvell tenglamalar sistemasi kanonik koʻrinishga keltirilgan, elektromagnit maydon va diagonal shaklidagi xotira matritsasini aniqlash uchun toʻgʻri va teskari masalalari qoʻyilgan. Ushbu masala yopiq ikkinchi tur Volterra tipidagi integral tenglamalar sistemasiga keltirilgan hamda x_1 oʻzgaruvchiga nisbatan Fur'e almashtirishi qoʻllanilgan. Uzluksiz funksiyalar fazosida vazn norma orqali qisqartirib aks ettirish prinsipi qoʻllab toʻgʻri va teskari masala yechilgan. Shunday qilib, maqolada qoʻyilgan masala yechimi mavjudligi va yagonaligi haqidagi global teorema isbotlangan.
Kalit soʻzlar: Giperbolik sistema; ikki oʻlchovli Maksvell tenglamalar sistemasi; integral tenglamalar; integro-differentsial tenglamalar; qisqartirib aks ettirish prinsipi.

Задача определения памяти в двумерной системе интегро-дифференциальных уравнений Максвелла

В этой статье, для приведенной канонической двумерной системы интегро-дифференциальных уравнений Максвелла ставятся прямая и обратная задачи определения поля электромагнитного напряжения и диагональной матрицы памяти. Задачи заменяются замкнутой системой интегральных уравнений второго рода вольтерровского типа относительно Фурье образа по переменным x_1 решения прямой задачи и неизвестных обратной задачи. Далее к этой системе применяется метод сжимающих отображений в пространстве непрерывных функций с весовой нормой. Таким образом, доказываются глобальные теоремы существования и единственности решений поставленных задач.
Ключевые слова: Гиперболическая система; двумерная система уравнений Максвелла; интегральное уравнение; интегро-дифференциальное уравнение; принцип сжатых отображений.

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Keywords: Hyperbolic system; two-dimensional system of Maxwell's equations; integral equation; integro-differential equations; contraction mapping principle.

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Introduction

The propagation of different waves is described by hyperbolic systems of first-order equations. In media with aftereffect, such a phenomenon in totally depends on the previous state of the process. For example such property possesses the phenomenon of the propagation of electromagnetic waves in media with dispersion. Thus, in these media, a violation occurs of the unique dependence of D, B (the induction of the electric and magnetic fields respectively) on E and H (the intensities of the corresponding fields) at the same time. The more general kind of linear dependence between $D(x,t)$, $B(x,t)$ and the corresponding values of the functions $E(x,t)$, $H(x,t)$ at all previous time can be written in the form of the integral relations (see [1], pp. 357-376):

$$D(x,t)=\epsilon E+\int_0^t \overline{\varphi}(t-\tau)E(x,\tau)d\tau, \quad B(x,t)=\mu H+\int_0^t \overline{\psi}(t-\tau)H(x,\tau)d\tau. \tag{1}$$

Here $E=(E_1,E_2,E_3)$, $H=(H_1,H_2,H_3)$, $D=(D_1,D_2,D_3)$, $B=(B_1,B_2,B_3)$, $x=(x_1,x_2,x_3)$, $\overline{\varphi}(t)=diag(\varphi_1,\varphi_2,\varphi_3)$, $\overline{\psi}(t)=diag(\psi_1,\psi_2,\psi_3)$ are diagonal matrices that represent the memory.

In an anisotropic medium with dispersion, the system of Maxwell's equations has the form

$$\nabla\times H=\frac{\partial}{\partial t}D(x,t)+\sigma E+J,\ \nabla\times E=-\frac{\partial}{\partial t}B(x,t), \quad divB=0,\ divD=\rho. \tag{2}$$

The matrices ϵ and μ are assumed positive definite, symmetric, and depending only on x_3 and σ is a constant matrix. In the first equation of (2) $J=J(x,t)$ is a vector-function characterizing the external current density. The third equation in (2) is a consequence of the second equation and the condition

$$div\,(\hat{\mu}H)\big|_{t=0}=0. \tag{3}$$

This section examines the problem of defining functions $\varphi(t)$, $\psi(t)$. In this case ϵ, μ are assumed positive and dependent only on the coordinate x_3, $\sigma=\sigma(x_3)$.

Here after, we assume $\varphi(t)=\varphi_i(t)$, $\psi(t)=\psi_i(t)$, $i=1,2,3$. Taking into account the relations (1) from the system of (2), we get a two-dimensional system of Maxwell's equations (see [2], pp. 81-95), ([3], pp. 5-20), ([4], pp. 71-80)

$$\epsilon\frac{\partial E_2}{\partial t}-\frac{\partial H_1}{\partial x_3}+\frac{\partial H_3}{\partial x_1}+(\varphi_0+\sigma)E_2+\int_0^t \varphi'(t-\tau)E_2(x_1,x_3,\tau)d\tau+J_2=0,$$

$$\mu\frac{\partial H_1}{\partial t}-\frac{\partial E_2}{\partial x_3}+\psi_0H_1+\int_0^t \psi'(t-\tau)H_1(x_1,x_3,\tau)d\tau=0,$$

$$\mu\frac{\partial H_3}{\partial t}+\frac{\partial E_2}{\partial x_1}+\psi_0H_3+\int_0^t \psi'(t-\tau)H_3(x_1,x_3,\tau)d\tau=0, \tag{4}$$

where $\varphi_0=\varphi(0)$, $\psi_0=\psi(0)$.

Consider the problem of determining the function $\sigma=\sigma(x_3)$ in the half-space $x_3\geq 0$, assuming that ϵ, μ in this half-space are known. To simplify the presentation, we assume here that the coefficients ϵ, μ are piece-wise constant

$$\epsilon=\begin{cases}\epsilon^-, & x_3<0,\\ \epsilon^+, & x_3>0,\end{cases} \qquad \mu=\begin{cases}\mu^-, & x_3<0,\\ \mu^+, & x_3>0,\end{cases}$$

and if $x_3<0$ then $\sigma=0$. The assumption of the constans ϵ, μ in the domain $x_3<0$ is actually not essential, but from a technical point of view, it makes significantly facilitate many calculations.

In the future, in (4), we introduce the new functions

$$U_1=\frac{1}{\sqrt{2}}\left(\sqrt{\epsilon}E_2+\sqrt{\mu}H_1\right),\ U_2=\frac{1}{\sqrt{2}}\left(\sqrt{\epsilon}E_2-\sqrt{\mu}H_1\right),\ U_3=\sqrt{\mu}H_3.$$

Then the system of equations (4) is written in the following compact form:

$$\left(A_0\frac{\partial}{\partial t}+A_1\frac{\partial}{\partial x_1}+A_2\frac{\partial}{\partial x_3}+A_3\right)U=\int_0^t \widetilde{K}(x_3,\tau)U(x,t-\tau)d\tau+\widehat{J}(x_1,x_3,t). \tag{5}$$

Here $U=(U_1,U_2,U_3)^*$is the column vector,

$$A_0=\begin{pmatrix}\sqrt{\frac{\epsilon}{2}} & \sqrt{\frac{\epsilon}{2}} & 0\\ \sqrt{\frac{\mu}{2}} & -\sqrt{\frac{\mu}{2}} & 0\\ 0 & 0 & \sqrt{\mu}\end{pmatrix},\ A_1=\begin{pmatrix}0 & 0 & \frac{1}{\sqrt{\mu}}\\ 0 & 0 & 0\\ \frac{1}{\sqrt{2\epsilon}} & \frac{1}{\sqrt{2\epsilon}} & 0\end{pmatrix},\ A_2=\begin{pmatrix}-\frac{1}{\sqrt{2\mu}} & \frac{1}{\sqrt{2\mu}} & 0\\ -\frac{1}{\sqrt{2\epsilon}} & -\frac{1}{\sqrt{2\epsilon}} & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$A_3 = \begin{pmatrix} \frac{\varphi_0+\sigma}{\sqrt{2\epsilon}} + \frac{\mu'}{2\sqrt{2}\mu^{\frac{3}{2}}} & \frac{\varphi_0+\sigma}{\sqrt{2\epsilon}} - \frac{\mu'}{2\sqrt{2}\mu^{\frac{3}{2}}} & 0 \\ \frac{\psi_0}{\sqrt{2\mu}} + \frac{\epsilon'}{2\sqrt{2}\epsilon^{\frac{3}{2}}} & -\frac{\psi_0}{\sqrt{2\mu}} + \frac{\epsilon'}{2\sqrt{2}\epsilon^{\frac{3}{2}}} & 0 \\ 0 & 0 & \frac{\psi_0}{\sqrt{\mu}} \end{pmatrix}, \quad \widetilde{K}(x_3,t) = -\begin{pmatrix} \frac{\varphi'}{\sqrt{2\epsilon}} & \frac{\varphi'}{\sqrt{2\epsilon}} & 0 \\ \frac{\psi'}{\sqrt{2\mu}} & -\frac{\psi'}{\sqrt{2\mu}} & 0 \\ 0 & 0 & \frac{\psi'}{\sqrt{\mu}} \end{pmatrix}, \ \widehat{J} = (0, J_2, 0) \tag{6}$$

where $*$ is the transposition symbol.

Multiplying equation (5) from the left by the inverse matrix A_0^{-1}, we obtain

$$\left(I_3 \frac{\partial}{\partial t} + B_1 \frac{\partial}{\partial x_1} + B_2 \frac{\partial}{\partial x_3} + B_3 \right) U = \int_0^t K(x_3,\tau) U(x, t-\tau) d\tau + \overline{J}(x_1, x_3, t). \tag{7}$$

Here and in what follows, I_3 stands for the identity matrix of order 3 and $B_j = A_0^{-1} A_j, \quad j = \overline{1,3}$.

In accordance with (5), we have

$$B_1 = \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2\epsilon\mu}} \\ 0 & 0 & \frac{1}{\sqrt{2\epsilon\mu}} \\ \frac{1}{\sqrt{2\epsilon\mu}} & \frac{1}{\sqrt{2\epsilon\mu}} & 0 \end{pmatrix}, \ B_2 = \frac{1}{\sqrt{\epsilon\mu}} \Lambda_0, \ \ \Lambda_0 = diag(-1, 1, 0),$$

$$B_3 = \begin{pmatrix} \frac{\varphi_0+\sigma}{2\epsilon} + \frac{\psi_0}{2\mu} + \frac{\mu'}{4\mu\sqrt{\epsilon\mu}} + \frac{\epsilon'}{4\epsilon\sqrt{\epsilon\mu}} & \frac{\varphi_0+\sigma}{2\epsilon} - \frac{\psi_0}{2\mu} - \frac{\mu'}{4\mu\sqrt{\epsilon\mu}} + \frac{\epsilon'}{4\epsilon\sqrt{\epsilon\mu}} & 0 \\ \frac{\varphi_0+\sigma}{2\epsilon} - \frac{\psi_0}{2\mu} + \frac{\mu'}{4\mu\sqrt{\epsilon\mu}} - \frac{\epsilon'}{4\epsilon\sqrt{\epsilon\mu}} & \frac{\varphi_0+\sigma}{2\epsilon} + \frac{\psi_0}{2\mu} - \frac{\mu'}{4\mu\sqrt{\epsilon\mu}} - \frac{\epsilon'}{4\epsilon\sqrt{\epsilon\mu}} & 0 \\ 0 & 0 & \frac{\psi_0}{\mu} \end{pmatrix}, \quad \overline{J} = A_0^{-1} \widehat{J},$$

$$\overline{K}(x_3,t) = A_0^{-1} \widetilde{K}(x_3,t) = -\begin{pmatrix} \frac{\varphi'}{2\epsilon} + \frac{\psi'}{2\mu} & \frac{\varphi'}{2\epsilon} - \frac{\psi'}{2\mu} & 0 \\ \frac{\varphi'}{2\epsilon} - \frac{\psi'}{2\mu} & \frac{\varphi'}{2\epsilon} + \frac{\psi'}{2\mu} & 0 \\ 0 & 0 & \frac{\psi'}{\mu} \end{pmatrix}.$$

Introduce the new variable z using the formula

$$z = \theta(x_3) = \int_0^{x_3} \sqrt{\epsilon\mu} d\xi. \tag{8}$$

Denote by $\theta^{-1}(z)$ the function inverse to $\theta(x_3)$ and let

$$\bar{U}(x_1, z, t) := U(x_1, \theta^{-1}(z), t), \ J(x_1, z, t) := \overline{J}(x_1, \theta^{-1}(z), t),$$

$$C_1(z) := B_1(\theta^{-1}(z)), \ C_2(z) := B_3(\theta^{-1}(z)), \ K(z,t) := \overline{K}(\theta^{-1}(z), t) = (k_{ij})_{i,j=1}^3.$$

Then (7) takes the form

$$\left(I_3 \frac{\partial}{\partial t} + \Lambda_0 \frac{\partial}{\partial z} + C_1 \frac{\partial}{\partial x_1} + C_2 \right) V = \int_0^t K(z,\tau) V(x_1, z, t-\tau) d\tau + J(x_1, z, t). \tag{9}$$

Statement of the problem

In the direct problem given matrices $K, \ C_1, \ C_2$, and vector-function J, it is required, in the domain

$$D = \{(x_1, z, t) : 0 < z < L, t > 0, \ x_1 \in \mathbb{R}\},$$

find a vector-function $V(z,t)$ satisfying equation (9) for the following initial and boundary conditions:

$$V_i(x_1, z, t)\big|_{t=0} = \phi_i(x_1, z), \ i = \overline{1,3}, \tag{10}$$

$$V_1(x_1, z, t)|_{z=L} = g_1(x_1, t), \ V_2(x_1, z, t)|_{z=0} = g_2(x_1, t), \tag{11}$$

where $\phi(x_1, z) = (\phi_1, \phi_2, \phi_3)(x_1, z), \ g(x_1, t) = (g_1, g_2)(x_1, t)$ are some given functions.

Inverse problem: Find the functions $\varphi(t), \ \psi(t), \ t > 0$, that are involved in the matrix K, if the additional conditions

$$V_1(x_1, z, t)|_{z=0} = h_1(x_1, t), \ V_2(x_1, z, t)|_{z=L} = h_2(x_1, t). \tag{12}$$

are given for a solution to problem (9)-(11). Moreover, we assume that $\varphi(0)$ and $\psi(0)$ are given as well.

The problems of finding the memories from one second-order integro-differential equation have been widely studied (see [5]-[26]). The numerical solution of direct and inverse problems for such equations were under study in (see [26-36]). As a rule, the second-order equations are derived from systems of first-order partial differential equations under some additional assumptions.

The inverse problem of finding the kernels of the integral terms from a system of first-order integrodifferential equations of general form with two independent variables was studied in (see [38]-[39]). Some theorem of local existence and global uniqueness was obtained.

It seems quite natural to carry out the study of inverse problems of finding the kernels of the integral terms of a system of integro-differential equations directly in terms of the system itself. The present article is a natural continuation of this circle of problems and, to a certain extent, generalizes the results of (see [38]-[39]) to the case of the system of Maxwell's equations with memory (1), (2).

Suppose that functions $J(x_1, z, t)$, $\phi_i(x_1, z)$, and $g_i(x_1, t)$, occurring on the right-hand side of (9) and the data (10), (11) have some compact support with respect to x_1 for every fixed z and t. The existence for (9) of a finite dependence domain and the property of having compact support with respect to x_1 of the right-hand side of (9) and the data (10), (11) imply that solutions to problem (9)-(11) have the compact support with respect to x_1.

We investigate the properties of solutions to this problem. More exactly, we will confine ourselves to the study of the Fourier transform of a solution with respect to x_1. Denote

$$\widetilde{V}(\eta, z, t) = \int\limits_{\mathbb{R}} V(x_1, z, t)e^{i\eta x_1}dx_1,\ \ \widetilde{J}(\eta, z, t) = \int\limits_{\mathbb{R}} J(x_1, z, t)e^{i\eta x_1}dx_1, \tag{13}$$

where η is the parameter of the transform.

In terms of the function $\widetilde{V}$, we write problem (9)-(11) as follows:

$$\left(I_3\frac{\partial}{\partial t} + \Lambda_0\frac{\partial}{\partial z} + C(\eta, z)\right)\widetilde{V} = \int\limits_0^t K(z,\tau)\widetilde{V}(\eta, z, t-\tau)d\tau + \widetilde{J}(\eta, z, t), \tag{14}$$

here $C(\eta, z) = C_2 - i\eta C_1 = (c_{ij})_{i,j=1}^3$.

Fix η and for convenience introduce the notation $\widetilde{V}(\eta, z, t) = \widetilde{V}(z, t)$. We will adopt these notations for the Fourier transforms of the functions occurring in the initial, boundary conditions (10), (11):

$$\widetilde{V}_i\big|_{t=0} \equiv \widetilde{\phi}_i(z),\ \ i = 1, 2, 3, \tag{15}$$

$$\widetilde{V}_1|_{z=L} = \widetilde{g}_1(t),\ \ \widetilde{V}_2|_{z=0} = \widetilde{g}_2(t), \tag{16}$$

and additional conditions (12)

$$\widetilde{V}_1|_{z=0} = \widetilde{h}_1(t),\ \ \widetilde{V}_2|_{z=L} = \widetilde{h}_2(t). \tag{17}$$

The direct problem

Let $\Omega = \{(z, t) : 0 < z < L,\ t > 0\}$ be the projection of the domain D to the plane of the variables $z,\ t$. Consider an arbitrary point $(z, t) \in \Omega$ on the plane of the variables $\xi,\ \tau$ and draw a characteristic of the i equation of system (14) through (z, t) till the intersection with the boundary of Ω in the domain $\tau < t$. The equation is taking as

$$\xi = z + \gamma_i(\tau - t), \tag{18}$$

here

$$\gamma_i = \begin{cases} -1,\ i = 1, \\ 1,\ i = 2, \\ 0,\ i = 3. \end{cases}$$

For $\gamma_2 = 1$ this point lies either on the interval $[0, L]$ of the axis $t = 0$, or on the straight line $z = 0$, and for $\gamma_1 = -1$, either on the interval $[0, L]$ or on the straight line $z = L$.

Integrating equations (14) over characteristic (18) from (z_0^i, t_0^i) to (z, t), we find

$$\widetilde{V}_i(z, t) = \widetilde{V}_i(z_0^i, t_0^i) + \int\limits_{t_0^i}^t \left[\widetilde{J}_i(\xi, \tau) - \sum_{j=1}^3 c_{ij}(\xi)\widetilde{V}_j(\xi, \tau)\right]\bigg|_{\xi = z + \gamma_i(\tau - t)} d\tau +$$

$$+\int\limits_{t_0^i}^{t}\int\limits_{0}^{\tau}\sum_{j=1}^{3}k_{ij}(\xi,\tau)\widetilde{V}_j(\xi,\tau-\alpha)d\alpha\Big|_{\xi=z+\gamma_i(\tau-t)}d\tau,\ i=1,2,3. \tag{19}$$

Let we find t_0^i in (19) and consequence it depends on the coordinates of (z,t). It is easy to observe that $t_0^i(z,t)$ has the form

$$t_0^i(z,t)=\begin{cases}\begin{cases}t+z-L,\ t\geq z-L,\\ 0,\ 0<t<z-L,\quad i=1,\end{cases}\\ \begin{cases}t-z,\ t\geq z,\\ 0,\ 0<t<z,\quad i=2;\end{cases}\\ 0,\quad i=3\end{cases}$$

Then the condition that the pair (z_0^i,t_0^i) satisfy to the equation (18) implies

$$z_0^i(z,t)=\begin{cases}\begin{cases}L,\ t\geq z-L,\\ t+z,\ 0<t<z-L,\quad i=1,\end{cases}\\ \begin{cases}0,\ t\geq z,\\ z-t,\ 0<t<z,\quad i=2;\end{cases}\\ z,\quad i=3.\end{cases}$$

The free terms of the integral equations (18) are defined through the initial and boundary conditions (15) and (16) as follows:

$$\widetilde{V}_i(z_0^i,t_0^i)=\begin{cases}\begin{cases}\widetilde{g}_1(t+z-L),\ t\geq z-L,\\ \widetilde{\phi}_1(t+z),\ 0<t<z-L,\quad i=1,\end{cases}\\ \begin{cases}\widetilde{g}_2(t-z),\ t\geq z,\\ \widetilde{\phi}_2(z-t),\ 0<t<z,\quad i=2;\end{cases}\\ \widetilde{\phi}_3(z),\quad i=3.\end{cases}$$

It is required that $\widetilde{V}_i(z_0^i,t_0^i)$ be continuous in Ω. Note that, for these conditions to be fulfilled, the given functions $\widetilde{\phi}_i$ and $\widetilde{g}_i$ must satisfy the metting conditions at the angular points of Ω:

$$\widetilde{\phi}_1(L)=\widetilde{g}_1(0),\quad \widetilde{\phi}_2(0)=\widetilde{g}_2(0). \tag{20}$$

Here and below, the values of $\widetilde{g}_i$ at $t=0$ and $\widetilde{\phi}_i$ at $z=0$ and $z=L$ are understood as the limit values at these points as the argument tends from the side of the point where these functions are defined.

Suppose that all given functions in (19) are continuous functions of their arguments in Ω. Then we have a closed system of Volterra-type integral equations with continuous kernels and free terms. As usual, such a system has a unique solution in the bounded subdomain

$$\Omega_T=\big\{(z,t):0\leq z\leq L,\ 0\leq t\leq T\big\}$$

of Ω, where $T>0$ is fixed number.

Introduce the vector-function

$$w(z,t)=\frac{\partial}{\partial t}\widetilde{V}(z,t).$$

For obtaining a problem for $w(z,t)$ similar to (14)- (16), we differentiate (14) and the boundary conditions (16) with respect to t and find the condition for $t=0$ by means of (14) and the initial conditions (15). We get

$$\left(I_3\frac{\partial}{\partial t}+\Lambda_0\frac{\partial}{\partial z}+C(z)\right)w(z,t)=K(z,t)\widetilde{\phi}(z)+\int\limits_0^t K(z,\tau)w(z,t-\tau)d\tau+\frac{\partial}{\partial t}\widetilde{J}(z,t) \tag{21}$$

$$w(z,t)\Big|_{t=0}=\Phi(z), \tag{22}$$

$$w_1(z,t)\Big|_{z=L} = \frac{d}{dt}\widetilde{g}_1(t),\ w_2(z,t)\Big|_{z=0} = \frac{d}{dt}\widetilde{g}_2(t), \tag{23}$$

where $\Phi(z) = (\Phi_1, \Phi_2, \Phi_3)(z)$ and

$$\Phi_i(z) = \widetilde{J}_i(z,0) - \gamma_i \frac{\partial}{\partial z}\widetilde{\phi}_i(z) - \sum_{j=1}^{3} c_{ij}(z)\widetilde{\phi}_j(z), \quad i = 1,2,3. \tag{24}$$

Once again, integration along the corresponding characteristics reduces (21)-(23) to the integral equations

$$w_i(z,t) = w_i(z_0^i, t_0^i) + \int_{t_0^i}^{t} \left[\frac{\partial}{\partial t}\widetilde{J}_i(\xi,\tau) - \sum_{j=1}^{3} c_{ij}(\xi) w_j(\xi,\tau) + \sum_{j=1}^{3} k_{ij}(\xi,\tau)\widetilde{\phi}_j(\xi)\right]\Bigg|_{\xi=z+\gamma_i(\tau-t)} d\tau +$$

$$+\int_{t_0^i}^{t}\int_0^{\tau}\sum_{j=1}^{3} k_{ij}(\xi,\alpha) w_j(\xi,\tau-\alpha)d\alpha\Big|_{\xi=z+\gamma_i(\tau-t)} d\tau, \quad i = 1,2,3. \tag{25}$$

For the functions w_i, the additional conditions (17) takes as

$$w_1(z,t)\Big|_{z=0} = \frac{d}{dt}\widetilde{h}_1(t), \quad w_2(z,t)\Big|_{z=L} = \frac{d}{dt}\widetilde{h}_2(t). \tag{26}$$

In equations (25), the functions $w(z_0^i, t_0^i)$ are defined as follows:

$$w(z_0^i, t_0^i) = \begin{cases} \begin{cases} \frac{d}{dt}\widetilde{g}_1(t+z-L),\ t \geq z-L, \\ \Phi_1(t+z),\ 0 < t < z-L, \quad i = 1, \end{cases} \\ \begin{cases} \frac{d}{dt}\widetilde{g}_2(t-z),\ t \geq z, \\ \Phi_2(z-t),\ 0 < t < z, \quad i = 2; \end{cases} \\ \Phi_3(z), \quad i = 3. \end{cases}$$

Suppose the fulfillment of the conditions

$$\Phi_1(L) = \left[\frac{d}{dt}\widetilde{g}_1(t)\right]_{t=0}, \quad \Phi_2(0) = \left[\frac{d}{dt}\widetilde{g}_2(t)\right]_{t=0}. \tag{27}$$

It is not hard to see that the fitting conditions for the initial data (22) and the boundary data (23) coincide with (27) at the angular points of Ω. It is clear that if the same equalities (27) are fulfilled then (25) have unique continuous solutions $w_i(z,t)$ and the same $(\partial/\partial t)\widetilde{V}_i(z,t)$.

Thus, we proved the following:

Theorem 1. Suppose that

$$\hat{\varepsilon}(x_3) \in C^1[0,\infty),\ \hat{\mu}(x_3) \in C^1[0,\infty),\ \widetilde{\phi}(x_3) \in C^1[0,\infty],$$

$$\widetilde{g}(t) \in C^1[0,\infty),\ K(z,t) \in C^1[0,\infty),\ \widetilde{J}(x_3,t) \in C^1(\Omega)$$

and conditions (20) and (27) are fulfilled. Then there is a unique solution to problem (14)-(16) in Ω.

Analyzing of the inverse problem.

Consider an arbitrary point $(z,0) \in \Omega$ and draw the characteristic (18) through $(z,0)$ till the intersection with the lateral boundaries of Ω. Iterating the i component of equation (21), using the data (26), we obtain

$$w_i(z,0) = \frac{d}{dt}\widetilde{h}_i(t_i(z)) + \int_0^{(t_i(z))} \left[\frac{\partial}{\partial t}\widetilde{J}_i(\xi,\tau) - \sum_{j=1}^{3} c_{ij}(\xi) w_j(\xi,\tau) + \sum_{j=1}^{3} k_{ij}(\xi,\tau)\widetilde{\phi}_j(\xi)\right]\Bigg|_{\xi=z+\gamma_i\tau} d\tau +$$

$$+\int_0^{(t_i(z))}\int_0^{\tau}\sum_{j=1}^{3} k_{ij}(\xi,\alpha) w_j(\xi,\tau-\alpha)d\alpha\Big|_{\xi=z+\gamma_i\tau} d\tau, \quad i = 1,2, \tag{28}$$

where

$$t_i(z)=\frac{1}{\gamma_i}\begin{cases} L-z, & i=1,\\ -z, & i=2.\end{cases}$$

Integrating the 3 component of equation (21) leads to the integral equations

$$w_3(z,t)=\Phi_3(z)+\int\limits_0^t\Big[-\sum_{j=1}^3 c_{3j}(z)w_j(z,\tau)+\sum_{j=1}^3 k_{3j}(z,\tau)\widetilde{\phi}_j(z)\Big]d\tau+$$

$$+\int\limits_0^t\int\limits_0^\tau\sum_{j=1}^3 k_{3j}(z,\alpha)w_j(z,\tau-\alpha)d\alpha d\tau. \tag{29}$$

Taking into the initial data (22) and additional conditions (26), rewrite (28) as

$$\int\limits_0^{(t_i(z))}\sum_{j=1}^3 k_{ij}(z+\gamma_i\tau,\tau)\widetilde{\phi}_j(z+\gamma_i\tau)d\tau+\int\limits_0^{(t_i(z))}\int\limits_0^\tau\sum_{j=1}^3 k_{ij}(z+\gamma_i\tau,\alpha)w_j(z+\gamma_i\tau,\tau-\alpha)d\alpha d\tau=$$

$$=\Phi_i(z)-\frac{d}{dt}\widetilde{h}_i(t_i(z))-\int\limits_0^{(t_i(z))}\Big[\frac{\partial}{\partial t}\widetilde{J}_i(z+\gamma_i\tau,\tau)-\sum_{j=1}^3 c_{ij}(z+\gamma_i\tau)w_j(z+\gamma_i\tau,\tau)\Big]d\tau,\quad i=1,2,$$

$$\int\limits_0^t\sum_{j=1}^3 k_{3j}(0,\tau)\widetilde{\phi}_j(0)d\tau+\int\limits_0^t\int\limits_0^\tau\sum_{j=1}^3 k_{3j}(0,\alpha)w_j(0,\tau-\alpha)d\alpha d\tau=\frac{d}{dt}\widetilde{h}_3(t)-$$

$$-\Phi_3(z)+\int\limits_0^t\sum_{j=1}^3 c_{3j}(0)w_j(0,\tau)d\tau.$$

Differentiate the first equations with respect to z, and the second, with respect to t. Then

$$\sum_{j=1}^3 k_{ij}(z+\gamma_i t_i(z),t_i(z))\widetilde{\phi}_j(z+\gamma_i t_i(z))-\gamma_i\int\limits_0^{t_i(z)}\sum_{j=1}^3\frac{\partial}{\partial z}\big(k_{ij}(z+\gamma_i\tau,\tau)\widetilde{\phi}_j(z+\gamma_i\tau)\big)d\tau+$$

$$+\int\limits_0^{t_i(z)}\sum_{j=1}^3 k_{ij}(z+\gamma_i t_i(z),\tau)w_j(z+\gamma_i t_i(z),t_i(z)-\tau)d\tau-$$

$$-\gamma_i\int\limits_0^{t_i(z)}\int\limits_0^\tau\sum_{j=1}^3\frac{\partial}{\partial z}\left(k_{ij}(z+\gamma_i\tau,\alpha)w_j(z+\gamma_i\tau,\tau-\alpha)\right)d\alpha d\tau=-\gamma_i\frac{d}{dz}\Phi_i(z)-\frac{d^2}{dt^2}\widetilde{h}_i(t_i(z))-$$

$$-\Big[\frac{\partial}{\partial t}\widetilde{J}_i(z+\gamma_i t_i(z),t_i(z))-\sum_{j=1}^3 c_{ij}(z+\gamma_i t_i(z))w_j(z+\gamma_i t_i(z),t_i(z))\Big]+\gamma_i\int\limits_0^{t_i(z)}\Big[\frac{\partial^2}{\partial t\partial z}\widetilde{J}_i(z+\gamma_i\tau,\tau)-$$

$$-\sum_{j=1}^3\frac{\partial}{\partial z}\left(c_{ij}(z+\gamma_i\tau)w_j(z+\gamma_i\tau,\tau)\right)\Big]d\tau,\ \ i=1,2. \tag{30}$$

Now, replace $t_i(z)$ by t in (30). We get

$$\sum_{j=1}^3 k_{1j}(0,t)\widetilde{\phi}_j(0)=P_1(t)+\int\limits_0^t\sum_{j=1}^3\frac{\partial}{\partial z}\left(c_{1j}(t-\tau)w_j(t-\tau,\tau)\right)d\tau$$

$$-\int\limits_0^t\sum_{j=1}^3 k_{1j}(0,\tau)w_j(0,t-\tau)d\tau-\int\limits_0^t\sum_{j=1}^3\frac{\partial}{\partial z}\big(k_{1j}(t-\tau,\tau)\widetilde{\phi}_j(t-\tau)\big)d\tau-$$

$$-\int\limits_0^t\int\limits_0^\tau\sum_{j=1}^3\frac{\partial}{\partial z}\left(k_{1j}(t-\tau,\alpha)w_j(t-\tau,\tau-\alpha)\right)d\alpha d\tau, \tag{31}$$

$$\sum_{j=1}^3 k_{2j}(L,t)\widetilde{\phi}_j(L)=P_2(L-t)-\int\limits_0^t\sum_{j=1}^3\frac{\partial}{\partial z}\left(c_{2j}(L-(t-\tau))w_j(L-(t-\tau),\tau)\right)d\tau$$

$$-\int_0^t \sum_{j=1}^3 k_{2j}(L,\tau)w_j(L,t-\tau)d\tau + \int_0^t \sum_{j=1}^3 \frac{\partial}{\partial z}\big(k_{2j}(L-(t-\tau),\tau)\widetilde{\phi}_j(L-(t-\tau))\big)d\tau +$$

$$+\int_0^t \int_0^\tau \sum_{j=1}^3 \frac{\partial}{\partial z}\left(k_{2j}(L-(t-\tau),\alpha)w_j(L-(t-\tau),\tau-\alpha)\right)d\alpha d\tau, \tag{32}$$

where $P_i(z)$ are defined by the formulas

$$P_i(z) = \frac{d}{dz}\Phi_i(z) - \frac{d^2}{dt^2}\widetilde{h}_i(t_i(z)) - \frac{\partial}{\partial t}\widetilde{J}_i(z+\gamma_i t_i(z), t_i(z)) +$$

$$+\sum_{j=1}^3 c_{ij}(z+\gamma_i t_i(z))w_j(z+\gamma_i t_i(z), t_i(z)) + \gamma_i \int_0^{t_i(z)} \frac{\partial^2}{\partial t \partial z}\widetilde{J}_i(z+\gamma_i\tau,\tau)d\tau, \ i=1,2.$$

Let's rewrite the equations (31), (32) in the following form:

$$\frac{1}{2\epsilon}\left(\phi_1(0)+\phi_2(0)\right)\varphi^{'}(t) + \frac{1}{2\mu}\left(\phi_1(0)-\phi_2(0)\right)\psi^{'}(t) = -P_1(t) - \int_0^t \sum_{j=1}^3 \frac{\partial}{\partial z}\left(c_{1j}(t-\tau)w_j(t-\tau,\tau)\right)d\tau +$$

$$+\int_0^t \sum_{j=1}^3 k_{1j}(0,\tau)w_j(0,t-\tau)d\tau + \int_0^t \sum_{j=1}^3 \frac{\partial}{\partial z}\big(k_{1j}(t-\tau,\tau)\widetilde{\phi}_j(t-\tau)\big)d\tau +$$

$$+\int_0^t \int_0^\tau \sum_{j=1}^3 \frac{\partial}{\partial z}\left(k_{1j}(t-\tau,\alpha)w_j(t-\tau,\tau-\alpha)\right)d\alpha d\tau, \tag{33}$$

$$\frac{1}{2\epsilon}\left(\phi_1(L)+\phi_2(L)\right)\varphi^{'}(t) + \frac{1}{2\mu}\left(-\phi_1(L)+\phi_2(L)\right)\psi^{'}(t) = -P_2(L-t) +$$

$$+\int_0^t \sum_{j=1}^3 \frac{\partial}{\partial z}\left(c_{2j}(L-(t-\tau))w_j(L-(t-\tau),\tau)\right)d\tau +$$

$$+\int_0^t \sum_{j=1}^3 k_{2j}(L,\tau)w_j(L,t-\tau)d\tau - \int_0^t \sum_{j=1}^3 \frac{\partial}{\partial z}\big(k_{2j}(L-(t-\tau),\tau)\widetilde{\phi}_j(L-(t-\tau))\big)d\tau -$$

$$-\int_0^t \int_0^\tau \sum_{j=1}^3 \frac{\partial}{\partial z}\left(k_{2j}(L-(t-\tau),\alpha)w_j(L-(t-\tau),\tau-\alpha)\right)d\alpha d\tau. \tag{34}$$

Let us introduce the following notation:

$$F(\nu_i,\widetilde{\phi}(\nu_i)) := \begin{pmatrix} \frac{1}{2\epsilon}\left(\widetilde{\phi}_1(0)+\widetilde{\phi}_2(0)\right) & \frac{1}{2\mu}\left(\widetilde{\phi}_1(0)-\widetilde{\phi}_2(0)\right) \\ \frac{1}{2\epsilon}\left(\widetilde{\phi}_1(L)+\widetilde{\phi}_2(L)\right) & \frac{1}{2\mu}\left(-\widetilde{\phi}_1(L)+\widetilde{\phi}_2(L)\right) \end{pmatrix}, \tag{35}$$

Reckoning with (35), rewrite (25) as follows:

$$w_i(z,t) = w_i(z_0^i,t_0^i) + \int_{t_0^i}^t \left[\frac{\partial}{\partial t}\widetilde{J}_i(\xi,\tau) - \sum_{j=1}^3 c_{ij}(\xi)w_j(\xi,\tau) + \sum_{j=1}^2 F_{ij}(\xi;\widetilde{\phi})\Psi_j(\tau)\right]\Big|_{\xi=z+\gamma_i(\tau-t)} d\tau +$$

$$+\int_{t_0^i}^t \int_0^\tau \sum_{j=1}^2 F_{ij}(\xi;w(\xi,\tau-\alpha))\Psi_j(\alpha)d\alpha\Big|_{\xi=z+\gamma_i(\tau-t)} d\tau, \ i=1,2, \tag{36}$$

$$w_3(z,t) = \Phi_3(z) + \int_0^t \Big[-\sum_{j=1}^3 c_{3j}(z)w_j(z,\tau) + \frac{\widetilde{\phi}_3(z)}{\mu}\Psi_2(\tau)\Big]d\tau + \int_0^t \int_0^\tau \frac{w_3(z,\tau-\alpha)}{\mu}\Psi_2(\alpha)d\alpha d\tau, \tag{37}$$

here

$$F(z;w(z,t)) = \Big(F_{ij}(z;w(z,t))\Big)_{ij=1}^2.$$

Using (35), we can also rewrite (33) and (34) so that

$$\sum_{j=1}^{2} F_{ij}(\nu_i;\widetilde{\phi}(\nu_i))\Psi_j(t) = -P_i(\bar{t}_i(t)) + \gamma_i \int\limits_0^t \sum_{j=1}^{3} \big[\frac{\partial}{\partial z} c_{ij}(-\gamma_i(t-\tau)) w_j(-\gamma_i(t-\tau),\tau)+$$

$$+c_{ij}(-\gamma_i(t-\tau))\frac{\partial}{\partial z} w_j(-\gamma_i(t-\tau),\tau)\big]d\tau + \int\limits_0^t \sum_{j=1}^{2} F_{ij}(\nu_i;\frac{d}{dt}\widetilde{h}_j(-\gamma_i(t-\tau)))\Psi_j(\tau)d\tau+$$

$$+\gamma_i \int\limits_0^t \sum_{j=1}^{2} \frac{\partial}{\partial z} F_{ij}(\nu_i-\gamma_i(t-\tau);\widetilde{\phi}_j(\nu_i-\gamma_i(t-\tau)))\Psi_j(\tau)d\tau+$$

$$+\gamma_i \int\limits_0^t \int\limits_0^\tau \sum_{j=1}^{2} \frac{\partial}{\partial z} F_{ij}(\nu_i-\gamma_i(t-\tau);w_j(\nu_i-\gamma_i(t-\tau),\tau-\alpha))\Psi_j(\alpha)d\alpha d\tau, \tag{38}$$

here

$$\nu_i = \begin{cases} 0, & i=1, \\ L, & i=2, \end{cases} \qquad \bar{t}_i(t) = \begin{cases} t, & i=1 \\ L-t, & i=2. \end{cases}$$

Let $\Psi(t) = (\varphi'(t),\psi'(t))^*$ be the vector-function composed of the derivatives of the unknown functions of the inverse problem, where $\Psi_i(t)$ are the entries of this vector-function.

In what follows, we assume the fulfillment of the condition

$$\det F(\nu_i;\widetilde{\phi}) \neq 0, \tag{39}$$

which is equivalent to the inequalities

$$\widetilde{\phi}_1(0)\widetilde{\phi}_1(L) \neq \widetilde{\phi}_2(0)\widetilde{\phi}_2(L).$$

Now, solving (36) with respect to $\Psi_i(t)$, we obtain

$$\Psi_i(t) = \frac{1}{\det F(\nu_i;\widetilde{\phi})} \sum_{j=1}^{2} \Big[-P_j(\bar{t}_j(t)) + \gamma_i \int\limits_0^t \sum_{l=1}^{3} \big[\frac{\partial}{\partial z} c_{jl}(-\gamma_j(t-\tau)) w_l(-\gamma_j(t-\tau),\tau)+$$

$$+c_{jl}(-\gamma_j(t-\tau))\frac{\partial}{\partial z} w_l(-\gamma_j(t-\tau),\tau)\big]d\tau + \int\limits_0^t \sum_{l=1}^{2} F_{jl}(\nu_j;\frac{d}{dt}\widetilde{h}_l(-\gamma_j(t-\tau)))\Psi_l(\tau)d\tau\Big]\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})+$$

$$+\frac{1}{\det F(\nu_i;\widetilde{\phi})} \sum_{j=1}^{2} \Big[\gamma_j \int\limits_0^t \sum_{l=1}^{2} \frac{\partial}{\partial z} F_{jl}(\nu_j-\gamma_j(t-\tau);\widetilde{\phi}_l(\nu_j-\gamma_j(t-\tau)))\Psi_l(\tau)d\tau\Big]\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})+$$

$$+\frac{1}{\det F(\nu_i;\widetilde{\phi})} \sum_{j=1}^{2} \Big[\gamma_j \int\limits_0^t \int\limits_0^\tau \sum_{l=1}^{2} \frac{\partial}{\partial z} F_{jl}(\nu_j-\gamma_j(t-\tau);w_l(\nu_j-\gamma_{ij}(t-\tau),\tau-\alpha))\Psi_l(\alpha)d\alpha d\tau\Big]\mathcal{F}_{ji}(\nu_i;\widetilde{\phi}), \tag{40}$$

where $\mathcal{F}_{ji}$ are the algebraic complements to the entries F_{ji} of F, $i=1,2$.

Equations (40) contain the unknown functions $\frac{\partial}{\partial z} w_j$, $j=\overline{1,3}$. For them we obtain integral equations from (36) and (37) by differentiating in z. Moreover,

$$\frac{\partial}{\partial z} w_i(z,t) = \frac{\partial}{\partial z} w_i(z_0^i,t_0^i) - \frac{\partial}{\partial z} t_0^i \Big[\frac{\partial}{\partial t}\widetilde{J}_i(z_0^i,t_0^i) - \sum_{j=1}^{3} c_{ij}(z_0^i) w_j(z_0^i,t_0^i) + \sum_{j=1}^{2} F_{ij}(z_0^i;\widetilde{\phi})\Psi_j(t_0^i)\Big]+$$

$$\int\limits_{t_0^i}^t \Big[\frac{\partial}{\partial t \partial z}\widetilde{J}_i(\xi,\tau) - \sum_{j=1}^{3} \frac{d}{dz} c_{ij}(\xi) w_j(\xi,\tau) - \sum_{j=1}^{3} c_{ij}(\xi)\frac{\partial}{\partial z} w_j(\xi,\tau)+$$

$$+\sum_{j=1}^{2} \frac{\partial}{\partial z} F_{ij}(\xi;\widetilde{\phi})\Psi_j(\tau)\Big]\Big|_{\xi=z+\gamma_i(\tau-t)} d\tau - \frac{\partial}{\partial z} t_0^i \int\limits_0^{t_0^i} \sum_{j=1}^{2} F_{ij}(z_0^i;H_j(z_0^i,t_0^i-\tau))\Psi_j(\tau)d\tau+$$

$$+\int\limits_{t_0^i}^t \int\limits_0^\tau \sum_{j=1}^{2} \frac{\partial}{\partial z} F_{ij}(\xi;w(\xi,\tau-\alpha))\Psi_j(\alpha)d\alpha\Big|_{\xi=z+\gamma_i(\tau-t)} d\tau,\ i=1,2, \tag{41}$$

$$\frac{\partial}{\partial z}w_3(z,t)=\frac{d}{dz}\Phi_3(z)+\int\limits_0^t\Big[-\sum_{j=1}^{3}\Big(\frac{d}{dz}c_{3j}(z)w_j(z,\tau)+c_{3j}(z)\frac{\partial}{\partial z}w_j(z,\tau)\Big)+$$
$$+\frac{d}{dz}\frac{\widetilde{\phi}_3(z)}{\mu}\Psi_2(t)\Big]d\tau+\int\limits_0^t\int\limits_0^\tau\frac{\partial}{\partial z}\frac{w_3(z,\tau-\alpha)}{\mu}\Psi_2(\alpha)d\alpha d\tau, \tag{42}$$

where

$$H_j\left(z_0^i,t_0^i-\tau\right)=\begin{cases}\frac{d}{dt}h_j\left(\frac{L-z}{\gamma_j}-\tau\right), & j=2,\\ \frac{d}{dt}g_j\left(\frac{L-z}{\gamma_j}-\tau\right), & j=1.\end{cases}$$

The main result and the proof

The main result of the present article is as follows:

Theorem 2. Suppose the fulfillment of the conditions of Theorem 1 and also the conditions

$$\widetilde{\phi}(z)\in C^2\left[0,\ L\right],\ \widetilde{g}(t)\in C^2\left[0,\ \infty\right),\ \widetilde{h}(t)\in C^2(0,\infty),\ \widetilde{J}(z,t)\in C^2\left(\Pi\right)$$

condition (39), and the fitting conditions (20) and (27). Then, for every $L>0$, on the interval $\left[0,L\right]$, there exists a unique solution to the inverse problem (21)-(23), (26) of the class $\Psi(t)\in C^1\left[0,L\right]$.

Proof. Equations (36),(37) and (40)-(42) supplemented with the initial and boundary value conditions from (21) constitute the closed system of equations on the unknown $w_i(z,t)$, $\Psi_j(t)$, and $\frac{\partial}{\partial z}w_i(z,t)$, $i=1,2,3$, $j=1,2$. Now, consider the square

$$\Omega_0:=\left\{(z,t):0\le z\le L,\,0\le t\le L\right\}.$$

Equation (36),(37) and (40)-(42) show that the values of $w_i(z,t)$, $\Psi_j(t)$, and $\frac{\partial}{\partial z}w_i(z,t)$ for $(z,t)\in\Omega_0$ are expressed in terms of the integrals of some combinations of these functions over segments lying in Ω_0.

Write (36),(37) and (40)-(42) as a closed system of Volterra-type integral equations. For this introduce the vector-functions $\upsilon(z,t)=\left(\upsilon_i^1,\upsilon_j^2,\upsilon_i^3\right)$, $i=1,2,3$, $j=1,2$, by defining their components by the equalities

$$\upsilon_i^1(z,t)=w_i(z,t),\ \upsilon_i^2(z,t)=\Psi_i(t),$$
$$\upsilon_i^3(z,t)=\frac{\partial}{\partial z}w_i(z,t)+\beta_i\sum_{j=1}^{2}F_{ij}(z_0^i;\widetilde{\phi})\Psi_j(t_0^i)\frac{\partial}{\partial z}t_0^i,$$

here

$$\beta_i=\begin{cases}1, & i=1,2,\\ 0, & i=3.\end{cases}$$

Then the system (36),(37) and (40)-(42) takes the operator form

$$\upsilon=\mathcal{A}\upsilon, \tag{43}$$

where A is the operator $\mathcal{A}=\left(\mathcal{A}_i^1,\ \mathcal{A}_j^2,\ \mathcal{A}_i^3\right)$, $i=1,2,3$, $j=1,2$ that is defined in accordance with the right-hand sides of (36),(37) and (40)-(42) by the equalities

$$\mathcal{A}_i^1\upsilon=\upsilon_i^{01}(z,t)+\int\limits_{t_0^i}^t\Big[-\sum_{j=1}^{3}c_{ij}(z+\gamma_i(\tau-t))\upsilon_j^1(z+\gamma_i(\tau-t),\tau)+\sum_{j=1}^{2}F_{ij}(z+\gamma_i(\tau-t);\widetilde{\phi})\upsilon_j^2(\tau)\Big]d\tau+$$
$$+\int\limits_{t_0^i}^t\int\limits_0^\tau\sum_{j=1}^{2}F_{ij}(z+\gamma_i(\tau-t);\upsilon_j^1(z+\gamma_i(\tau-t),\tau-\alpha))\upsilon_j^2(\alpha)d\alpha d\tau,\ i=1,2, \tag{44}$$

$$\mathcal{A}_3^1\upsilon=\upsilon_3^{01}(z,t)+\int\limits_0^t\Big[-\sum_{j=1}^{3}c_{3j}(z)\upsilon_j^1(z,\tau)+\frac{\widetilde{\phi}_3(z)}{\mu}\upsilon_2^2(\tau)\Big]d\tau+\int\limits_0^t\int\limits_0^\tau\frac{\upsilon_3^1(z,\tau-\alpha)}{\mu}\upsilon_2^2(\alpha)d\alpha d\tau, \tag{45}$$

$$\mathcal{A}_i^2\upsilon=\upsilon_i^{02}(z,t)+\frac{1}{\det F(\nu_i;\widetilde{\phi})}\int\limits_0^t\sum_{j=1}^{2}\sum_{l=1}^{3}\gamma_j\frac{\partial}{\partial z}c_{jl}(-\gamma_j(t-\tau))\upsilon_l^1(-\gamma_j(t-\tau),\tau)d\tau\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})+$$
$$+\frac{1}{\det F(\nu_i;\widetilde{\phi})}\int\limits_0^t\sum_{j=1}^{2}\sum_{l=1}^{3}\gamma_jc_{jl}(-\gamma_j(t-\tau))\Big[\upsilon_l^3(-\gamma_j(t-\tau),\tau)-\beta_l\sum_{p=1}^{2}F_{lp}(z_0^l;\widetilde{\phi})\upsilon_p^2(t_0^l)\frac{\partial}{\partial z}t_0^j\Big]d\tau\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})+$$

$$+\frac{1}{\det F(\nu_i;\widetilde{\phi})}\int\limits_0^t\sum_{j=1}^2\sum_{l=1}^2\gamma_j\frac{\partial}{\partial z}F_{jl}(\nu_j-\gamma_j(t-\tau);\widetilde{\phi}_l(\nu_j-\gamma_j(t-\tau)))\upsilon_l^2(\tau)d\tau\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})+$$

$$+\frac{1}{\det F(\nu_i;\widetilde{\phi})}\int\limits_0^t\sum_{j=1}^2\sum_{l=1}^2F_{jl}(\nu_j;\frac{d}{dt}\widetilde{h}_l(-\gamma_j(t-\tau)))\upsilon_l^2(\tau)d\tau\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})+$$

$$+\frac{1}{\det F(\nu_i;\widetilde{\phi})}\int\limits_0^t\int\limits_0^\tau\sum_{j=1}^2\sum_{l=1}^2\gamma_j\frac{\partial}{\partial z}F_{jl}(\nu_j-\gamma_j(t-\tau);\upsilon_l^1(\nu_j-\gamma_{ij}(t-\tau),\tau-\alpha))\upsilon_l^2(\alpha)d\alpha d\tau\mathcal{F}_{ji}(\nu_i;\widetilde{\phi}), \tag{46}$$

$$\mathcal{A}_i^3\upsilon=\upsilon_i^{03}(z,t)-\int\limits_{t_0^i}^t\Bigg[\sum_{j=1}^3\frac{d}{dz}c_{ij}(\xi)\upsilon_j^1(\xi,\tau)+\sum_{j=1}^3c_{ij}(\xi)\Big(\upsilon_j^3(\xi,\tau)-\beta_j\sum_{l=1}^2F_{jk}(z_0^j;\widetilde{\phi})\upsilon_l^2(t_0^j)\frac{\partial}{\partial z}t_0^j\Big)$$

$$-\sum_{j=1}^2\frac{\partial}{\partial z}F_{ij}(\xi;\widetilde{\phi})\upsilon_j^2(\tau)\Bigg]\Bigg|_{\xi=z+\gamma_i(\tau-t)}d\tau-\frac{\partial}{\partial z}t_0^i\int\limits_0^{t_0^i}\sum_{j=1}^2F_{ij}(z_0^i;H_j(z_0^i,t_0^i-\tau))\upsilon_j^2(\tau)d\tau+$$

$$+\int\limits_{t_0^i}^t\int\limits_0^\tau\sum_{j=1}^2\frac{\partial}{\partial z}F_{ij}(\xi;\upsilon_j^1(\xi,\tau-\alpha))\upsilon_j^2(\alpha)d\alpha\Bigg|_{\xi=z+\gamma_i(\tau-t)}d\tau,\ i=1,2, \tag{47}$$

$$\mathcal{A}_3^3\upsilon=\upsilon_3^{03}(z,t)-\int\limits_{t_0^i}^t\Bigg[\sum_{j=1}^3\frac{d}{dz}c_{3j}(\xi)\upsilon_j^1(\xi,\tau)+\sum_{j=1}^3c_{3j}(\xi)\Big(\upsilon_j^3(\xi,\tau)-\beta_j\sum_{l=1}^2F_{jk}(z_0^j;\widetilde{\phi})\upsilon_l^2(t_0^j)\frac{\partial}{\partial z}t_0^j\Big)+$$

$$+\frac{d}{dz}\frac{\widetilde{\phi}_3(z)}{\mu}\upsilon_2^2(\tau)\Bigg]d\tau+\int\limits_0^t\int\limits_0^\tau\Bigg[\frac{1}{\mu}\upsilon_3^3(z,\tau-\alpha)\upsilon_2^2(\alpha)-\frac{1}{\mu^2}\upsilon_3^1(z,\tau-\alpha)\upsilon_2^2(\alpha)\Bigg]d\alpha d\tau. \tag{48}$$

In these formulas, we used the notations

$$\upsilon_i^{01}(z,t)=w_i(z_0^i,t_0^i)+\int\limits_{t_0^i}^t\frac{\partial}{\partial t}\widetilde{J}_i(z+\gamma_i(\tau-t),\tau)d\tau,\ i=1,2;\quad \upsilon_3^{01}(z,t)=\Phi_3(z)$$

$$\upsilon_i^{02}(z,t)=-\frac{1}{\det F(\nu_i;\widetilde{\phi})}\sum_{j=1}^2P_j(\bar{t}_j(t))\mathcal{F}_{ji}(\nu_i;\widetilde{\phi}),\quad \upsilon_i^{03}(z,t)=\frac{\partial}{\partial z}w_i(z_0^i,t_0^i)-\frac{\partial}{\partial z}t_0^i\frac{\partial}{\partial t}\widetilde{J}_i(z_0^i,t_0^i)+$$

$$+\frac{\partial}{\partial z}t_0^i\sum_{j=1}^6c_{ij}(z_0^i)w_j(z_0^i,t_0^i)+\int\limits_{t_0^i}^t\frac{\partial}{\partial t\partial z}\widetilde{J}_i(z+\gamma_i(\tau-t),\tau)d\tau,\ i=1,2;\quad \upsilon_3^{03}(z,t)=\frac{d}{dz}\Phi_3(z).$$

Endow the set of continuous functions $C_s(\Omega_0)$ with the norm

$$\|\upsilon\|_s=\max_{1\le i\le 8,\ 1\le l\le 3}\sup_{(z,t)\in\Omega_0}\left|\upsilon_i^l(z,t)e^{-st}\right|$$

where$s\ge 0$ is a number to be chosen below. Obviously, for $s=0$ this space coincides with the set of continuous functions with the norm $\|\upsilon\|_s$. By the inequality,

$$e^{-sL}\|\upsilon\|_s\le\|\upsilon\|_s\le\|\upsilon\|,$$

the norms $\|\upsilon\|_s$ and $\|\upsilon\|$ are equivalent for ant fixed $L\in(0,\infty)$.

Further, consider the set of functions $S\left(\upsilon^0,r\right)\subset C_s(\Omega_0)$, satisfying the inequality

$$\|\upsilon-\upsilon^0\|_s\le r, \tag{49}$$

where the vector-function

$$\upsilon^0(z,t)=\left(\upsilon_i^{01}(z,t),\ \upsilon_i^{02}(t),\ \upsilon_i^{03}(z,t),\ i=1,2,3,\ j=1,2\right)$$

is defined by the free terms of the operator equation (43). It is not hard to observe that the following estimate holds for $\upsilon\in S\left(\upsilon^0,r\right)$:

$$\|\upsilon\|_s\le\|\upsilon^0\|_s+r\le\|\upsilon^0\|+r:=r_0.$$

Thus, r_0 is known.

Introduce the notations

$$\widetilde{\phi}_0 := \max_{1\le i\le 3}\left\|\widetilde{\phi}_i\right\|_{C^2[0,L]},\ g_0 := \max_{1\le i\le 3}\|g_i\|_{C^2[0,L]},\ J_0 := \max_{1\le i\le 3}\left\|\widetilde{J}_i\right\|_{C^2[\Pi_0]},$$

$$h_0 := \max_{1\le i\le 3}\|h_i\|_{C^2[0,L]},\ c_0 := \max_{1\le i\le 3,\ 1\le j\le 3}\{c_{ij}(z)\},\ \Gamma_0 := \max\{g_0,h_0\},\ P_0 := \min\{|\mathcal{F}(0)|,|\mathcal{F}(L)|\},$$

$$\Upsilon_0\widetilde{\phi}_0 = \max_{1\le i\le n}\left\|F_{ij}(z+\gamma_i(\tau-t);\widetilde{\phi})\right\|_{C^1[0,L]},\ Q_0 := \max\left\{\max_{1\le i\le n}|F_i(0)|,\ \max_{1\le i\le n}|F_i(L)|\right\}.$$

The operator $\mathcal{A}$ takes $C_s(\Omega_0)$ into itself. Show that for a suitable choice of s (note that $L>0$ is an arbitrary fixed number) it is a contraction operator on $S\left(\upsilon^0,r\right)$. Let us first verify that $\mathcal{A}$ takes the set $S\left(\upsilon^0,r\right)$ into itself; i.e., the condition $\upsilon(z,t)\in S\left(\upsilon^0,r\right)$ implies that $\mathcal{A}\upsilon\in S\left(\upsilon^0,r\right)$, if s satisfies some constraints. Indeed, given $(z,t)\in\Omega_0$ and $\upsilon\in S\left(\upsilon^0,r\right)$ we have

$$\left|\left(\mathcal{A}_i^1\upsilon-\upsilon_i^{01}\right)e^{-st}\right| = \left|\int\limits_{t_0^i}^{t}\left[\sum_{j=1}^{2}F_{ij}(z+\gamma_i(\tau-t);\widetilde{\phi})e^{-s(t-\tau)}\upsilon_j^2(\tau)e^{-s\tau}-\right.\right.$$

$$\left.-\sum_{j=1}^{3}c_{ij}(z+\gamma_i(\tau-t))e^{-s(t-\tau)}\upsilon_j^1(z+\gamma_i(\tau-t),\tau)e^{-s\tau}\right]d\tau+$$

$$\left.+\int\limits_{t_0^i}^{t}\int\limits_{0}^{\tau}\sum_{j=1}^{2}F_{ij}(z+\gamma_i(\tau-t);\upsilon_j^1(z+\gamma_i(\tau-t),\tau-\alpha))e^{-s(\tau-\alpha)}\upsilon_j^2(\alpha)e^{-s\alpha}d\alpha d\tau\right|\le$$

$$\le\left[(2\Upsilon_0\widetilde{\phi}_0+3c_0)\|\upsilon\|_s+2\Upsilon_0\|\upsilon\|_s^2\tau\right]\int_0^t e^{-s(t-\tau)}d\tau\le\frac{1}{s}\left((2\Upsilon_0\widetilde{\phi}_0+3c_0)+\Upsilon_0Lr_0\right)r_0 := \frac{1}{s}\alpha_{11},$$

$$\left|\left(\mathcal{A}_3^1\upsilon-\upsilon_3^{01}\right)e^{-st}\right| = \left|\int\limits_{0}^{t}\left[-\sum_{j=1}^{3}c_{3j}(z)e^{-s(t-\tau)}\upsilon_j^1(z,\tau)e^{-s\tau}+\frac{\widetilde{\phi}_3(z)}{\mu}\upsilon_2^2(\tau)e^{-s\tau}\right]d\tau+\right.$$

$$\left.+\int\limits_{0}^{t}\int\limits_{0}^{\tau}\frac{\upsilon_3^1(z,\tau-\alpha)}{\mu}e^{-s(\tau-\alpha)}\upsilon_2^2(\alpha)e^{-s\alpha}d\alpha d\tau\right|\le$$

$$\le\frac{1}{s}\left[3c_0+\mu_0^{-1}\widetilde{\phi}_0+2\Upsilon_0\|\upsilon\|_s\tau\right]\|\upsilon\|_s\le\frac{1}{s}\left[3c_0+\mu_0^{-1}\widetilde{\phi}_0+2\Upsilon_0r_0\tau\right]\|\upsilon\|_sr_0 := \frac{1}{s}\alpha_{12},$$

$$\left|\left(\mathcal{A}_i^2\upsilon-\upsilon_i^{02}\right)e^{-st}\right| =$$

$$=\left|\frac{1}{\det F(\nu_i;\widetilde{\phi})}\int\limits_0^t\sum_{j=1}^{2}\sum_{l=1}^{3}\gamma_j\frac{\partial}{\partial z}c_{jl}(-\gamma_j(t-\tau))e^{-s(t-\tau)}\upsilon_l^1(-\gamma_j(t-\tau),\tau)e^{-s\tau)}d\tau\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})+\right.$$

$$+\frac{1}{\det F(\nu_i;\widetilde{\phi})}\int\limits_0^t\sum_{j=1}^{2}\sum_{l=1}^{3}\gamma_jc_{jl}(-\gamma_j(t-\tau))e^{-s(t-\tau)}\left[\upsilon_l^3(-\gamma_j(t-\tau),\tau)-\right.$$

$$\left.-\beta_l\sum_{p=1}^{2}F_{lp}(z_0^l;\widetilde{\phi})\upsilon_p^2(t_0^l)\frac{\partial}{\partial z}t_0^j\right]e^{-s\tau)}d\tau\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})+$$

$$+\frac{1}{\det F(\nu_i;\widetilde{\phi})}\int\limits_0^t\sum_{j=1}^{2}\sum_{l=1}^{2}\gamma_j\frac{\partial}{\partial z}F_{jl}(\nu_j-\gamma_j(t-\tau);\widetilde{\phi}_l(\nu_j-\gamma_j(t-\tau)))e^{-s(t-\tau)}\upsilon_l^2(\tau)e^{-s\tau)}d\tau\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})+$$

$$+\frac{1}{\det F(\nu_i;\widetilde{\phi})}\int\limits_0^t\sum_{j=1}^{2}\sum_{l=1}^{2}F_{jl}(\nu_j;\frac{d}{dt}\widetilde{h}_l(-\gamma_j(t-\tau)))e^{-s(t-\tau)}\upsilon_l^2(\tau)e^{-s\tau)}d\tau\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})+$$

$$+\frac{1}{\det F(\nu_i;\widetilde{\phi})}\int\limits_0^t\int\limits_0^\tau\sum_{j=1}^{2}\sum_{l=1}^{2}\gamma_j\frac{\partial}{\partial z}F_{jl}(\nu_j-\gamma_j(t-\tau);\upsilon_l^1(\nu_j-\gamma_{ij}(t-\tau),\tau-\alpha))e^{-s(\tau-\alpha)}\times$$

$$\left.\times\upsilon_l^2(\alpha)e^{-s\alpha}d\alpha d\tau\mathcal{F}_{ji}(\nu_i;\widetilde{\phi})\right|\le\frac{6P_0}{sQ_0}\left(2c_0\left(1+\Upsilon_0\widetilde{\phi}_0\right)+\Upsilon_0\widetilde{\phi}_0+\Upsilon_0\Gamma_0+\Upsilon_0Lr_0\right)r_0 := \frac{1}{s}\alpha_2,$$

$$\left|\left(\mathcal{A}_i^3\upsilon-\upsilon_i^{03}\right)e^{-st}\right|=\Bigg|-\int\limits_{t_0^i}^{t}\Bigg[\sum_{j=1}^{3}\frac{d}{dz}c_{ij}(\xi)e^{-s(t-\tau)}\upsilon_j^1(\xi,\tau)e^{-s\tau)}+$$

$$\sum_{j=1}^{3}c_{ij}(\xi)e^{-s(t-\tau)}\left(\upsilon_j^3(\xi,\tau)-\beta_j\sum_{l=1}^{2}F_{jk}(z_0^j;\widetilde{\phi})\upsilon_l^2(t_0^j)\frac{\partial}{\partial z}t_0^j\right)e^{-s\tau)}-$$

$$-\sum_{j=1}^{2}\frac{\partial}{\partial z}F_{ij}(\xi;\widetilde{\phi})e^{-s(t-\tau)}\upsilon_j^2(\tau)e^{-s\tau)}\Bigg]\Bigg|_{\xi=z+\gamma_i(\tau-t)}d\tau-$$

$$-\frac{\partial}{\partial z}t_0^i\int\limits_0^{t_0^i}\sum_{j=1}^{2}F_{ij}(z_0^i;H_j(z_0^i,t_0^i-\tau))e^{-s(t-\tau)}\upsilon_j^2(\tau)e^{-s\tau)}d\tau+$$

$$+\int\limits_{t_0^i}^{t}\int\limits_0^{\tau}\sum_{j=1}^{2}\frac{\partial}{\partial z}F_{ij}(\xi;\upsilon_j^1(\xi,\tau-\alpha))e^{-s(\tau-\alpha)}\upsilon_j^2(\alpha)e^{-s\alpha}d\alpha\Bigg|_{\xi=z+\gamma_i(\tau-t)}d\tau\Bigg|\le$$

$$\le\frac{3}{s}\left[2c_0\left(1+\Upsilon_0\widetilde{\phi}_0\right)+\Upsilon_0\widetilde{\phi}_0+\Upsilon_0\Gamma_0+\Upsilon_0Lr_0\right]r_0:=\frac{1}{s}\alpha_{31},$$

$$\left|\left(\mathcal{A}_3^3\upsilon-\upsilon_3^{03}\right)e^{-st}\right|=\Bigg|-\int\limits_{t_0^i}^{t}\Bigg[\sum_{j=1}^{3}\frac{d}{dz}c_{3j}(\xi)e^{-s(t-\tau)}\upsilon_j^1(\xi,\tau)e^{-s\tau)}+$$

$$+\sum_{j=1}^{3}c_{3j}(\xi)e^{-s(t-\tau)}\left(\upsilon_j^3(\xi,\tau)-\beta_j\sum_{l=1}^{2}F_{jk}(z_0^j;\widetilde{\phi})\upsilon_l^2(t_0^j)\frac{\partial}{\partial z}t_0^j\right)e^{-s\tau)}+\frac{d}{dz}\frac{\widetilde{\phi}_3(z)}{\mu}e^{-s(t-\tau)}\upsilon_2^2(\tau)e^{-s\tau)}\Bigg]d\tau+$$

$$+\int\limits_0^{t}\int\limits_0^{\tau}\left[\frac{1}{\mu}\upsilon_3^3(z,\tau-\alpha)e^{-s(\tau-\alpha)}\upsilon_2^2(\alpha)e^{-s\alpha}-\frac{1}{\mu^2}\upsilon_3^1(z,\tau-\alpha)e^{-s(\tau-\alpha)}\upsilon_2^2(\alpha)e^{-s\alpha}\right]d\alpha d\tau\Bigg|\le$$

$$\le\frac{3}{s}\left[2c_0\left(1+\Upsilon_0\widetilde{\phi}_0\right)+\mu_0^{-1}\widetilde{\phi}_0+2\mu_0^{-2}Lr_0\right]r_0:=\frac{1}{s}\alpha_{32}.$$

These together with (43) ? (44)-(48) imply the estimates

$$\|\mathcal{A}\upsilon-\upsilon^0\|_s=\max\Bigg\{\max_{1\le i\le 3}\sup_{(z,t)\in\Omega_0}\left|\left(\mathcal{A}_i^1\upsilon-\upsilon_i^{01}\right)e^{-st}\right|,\ \max_{1\le i\le 3}\sup_{t\in[0,L]}\left|\left(\mathcal{A}_i^2\upsilon-\upsilon_i^{02}\right)e^{-st}\right|,$$

$$\max_{1\le i\le 3}\sup_{t\in[0,L]}\left|\left(\mathcal{A}_i^3\upsilon-\upsilon_i^{03}\right)e^{-st}\right|\Bigg\}\le\frac{1}{s}\alpha_0,$$

where $\alpha_0:=\max\left(\alpha_{11},\ \alpha_{12},\ \alpha_2,\ \alpha_{31},\ \alpha_{32}\right)$ Choosing $s>(1/r)\alpha_0$, we obtain that $\mathcal{A}$ takes $S\left(\upsilon^0,\rho\right)$ into itself.

Now, take $\upsilon,\widetilde{\upsilon}\in S\left(\upsilon^0,r\right)$ and estimate the norm of the difference $\mathcal{A}\upsilon-\mathcal{A}\widetilde{\upsilon}$. Using the obvious inequality

$$\left|\upsilon_i^k\upsilon_i^l-\widetilde{\upsilon}_i^k\widetilde{\upsilon}_i^l\right|e^{-st}\le\left|\upsilon_i^k-\widetilde{\upsilon}_i^k\right|\left|\upsilon_i^l\right|e^{-st}+\left|\widetilde{\upsilon}_i^k\right|\left|\upsilon_i^l-\widetilde{\upsilon}_i^l\right|e^{-st}\le 2r_0\|\upsilon-\widetilde{\upsilon}\|_s$$

and estimates for the integrals analogous to those above, we arrive at

$$\left|\left(\mathcal{A}_i^1\upsilon-\mathcal{A}_i^1\widetilde{\upsilon}\right)e^{-st}\right|=\Bigg|\int\limits_{t_0^i}^{t}\Bigg[\sum_{j=1}^{2}F_{ij}(z+\gamma_i(\tau-t);\widetilde{\phi})e^{-s(t-\tau)}\left(\upsilon_j^2(\tau)-\widetilde{\upsilon}_j^2(\tau)\right)e^{-s\tau}-$$

$$-\sum_{j=1}^{3}c_{ij}(z+\gamma_i(\tau-t))e^{-s(t-\tau)}\left(\upsilon_j^1(z+\gamma_i(\tau-t),\tau)-\widetilde{\upsilon}_j^1(z+\gamma_i(\tau-t),\tau)\right)e^{-s\tau}\Bigg]d\tau+$$

$$+\int\limits_{t_0^i}^{t}\int\limits_0^{\tau}\sum_{j=1}^{2}\Big[F_{ij}(z+\gamma_i(\tau-t);\upsilon_j^1(z+\gamma_i(\tau-t),\tau-\alpha))e^{-s(\tau-\alpha)}\upsilon_j^2(\alpha)e^{-s\alpha}-$$

$$-F_{ij}(z+\gamma_i(\tau-t);\widetilde{\upsilon}_j^1(z+\gamma_i(\tau-t),\tau-\alpha))e^{-s(\tau-\alpha)}\widetilde{\upsilon}_j^2(\alpha)e^{-s\alpha}\Big]d\alpha d\tau\Bigg|\le$$

$$\leq \frac{1}{s}\left((2\Upsilon_0\widetilde{\phi}_0 + 3c_0) + 2\Upsilon_0 L r_0\right)\|\upsilon - \widetilde{\upsilon}\|_s := \frac{1}{s}\beta_{11}\|\upsilon - \widetilde{\upsilon}\|_s$$

$$\left|\left(\mathcal{A}_3^1\upsilon - \mathcal{A}_3^1\widetilde{\upsilon}\right)e^{-st}\right| = \left|\int_0^t \left[-\sum_{j=1}^{3} c_{3j}(z)e^{-s(t-\tau)}\left(\upsilon_j^1(z,\tau) - \widetilde{\upsilon}_j^1(z,\tau)\right)e^{-s\tau} + \right.\right.$$

$$\left. + \frac{\widetilde{\phi}_3(z)}{\mu}\left(\upsilon_2^2(\tau) - \widetilde{\upsilon}_2^2(\tau)\right)(\tau)e^{-s\tau}\right]d\tau +$$

$$\left. + \int_0^t\int_0^\tau \frac{\upsilon_3^1(z,\tau-\alpha) - \widetilde{\upsilon}_3^1(z,\tau-\alpha)}{\mu}e^{-s(\tau-\alpha)}\left(\upsilon_2^2(\alpha) - \widetilde{\upsilon}_2^2(\alpha)\right)e^{-s\alpha}d\alpha d\tau\right| \leq$$

$$\leq \frac{1}{s}\left[3c_0 + \mu_0^{-1}\widetilde{\phi}_0 + 4\Upsilon_0 r_0 L\right]\|\upsilon - \widetilde{\upsilon}\|_s := \frac{1}{s}\beta_{12}\|\upsilon - \widetilde{\upsilon}\|_s,$$

Likewise, we obtain the estimates

$$\left|\left(\mathcal{A}_i^2\upsilon - \mathcal{A}_i^2\widetilde{\upsilon}\right)e^{-st}\right| \leq \frac{6P_0}{sQ_0}\left(2c_0\left(1 + \Upsilon_0\widetilde{\phi}_0\right) + \Upsilon_0\widetilde{\phi}_0 + \Upsilon_0\Gamma_0 + 2\Upsilon_0 L r_0\right)\|\upsilon - \widetilde{\upsilon}\|_s := \frac{1}{s}\beta_2\|\upsilon - \widetilde{\upsilon}\|_s,$$

$$\left|\left(\mathcal{A}_i^3\upsilon - \mathcal{A}_i^3\widetilde{\upsilon}\right)e^{-st}\right| \leq \frac{3}{s}\left[2c_0\left(1 + \Upsilon_0\widetilde{\phi}_0\right) + \Upsilon_0\widetilde{\phi}_0 + \Upsilon_0\Gamma_0 + 2\Upsilon_0 L r_0\right]\|\upsilon - \widetilde{\upsilon}\|_s := \frac{1}{s}\beta_{31}\|\upsilon - \widetilde{\upsilon}\|_s$$

$$\left|\left(\mathcal{A}_3^3\upsilon - \mathcal{A}_3^3\widetilde{\upsilon}\right)e^{-st}\right| \leq \frac{3}{s}\left[2c_0\left(1 + \Upsilon_0\widetilde{\phi}_0\right) + \mu_0^{-1}\widetilde{\phi}_0 + 4\mu_0^{-2}L r_0\right]\|\upsilon - \widetilde{\upsilon}\|_s := \frac{1}{s}\beta_{32}\|\upsilon - \widetilde{\upsilon}\|_s$$

Hence,

$$\|\mathcal{A}\upsilon - \mathcal{A}\widetilde{\upsilon}\|_s = \max\left\{\max_{1\leq i\leq 3}\sup_{(z,t)\in\Pi_0}\left|\left(\mathcal{A}_i^1\upsilon - \mathcal{A}_i^1\widetilde{\upsilon}\right)e^{-st}\right|,\ \max_{1\leq i\leq 2}\sup_{t\in[0,L]}\left|\left(\mathcal{A}_i^2\upsilon - \mathcal{A}_i^2\widetilde{\upsilon}\right)e^{-st}\right|,\right.$$

$$\left.\max_{1\leq i\leq 3}\sup_{t\in[0,L]}\left|\left(\mathcal{A}_i^3\upsilon - \mathcal{A}_i^3\widetilde{\upsilon}\right)e^{-st}\right|\right\} \leq \frac{1}{s}\beta_0\|\upsilon - \widetilde{\upsilon}\|_s,$$

where $\beta_0 := \max\left(\beta_{11},\ \beta_{12},\ \beta_2,\ \beta_{31},\ \beta_{32}\right)$.

Now, choosing $s > \beta_0$,, we conclude that $\mathcal{A}$ contracts the distance between υ and $\widetilde{\upsilon}$ by $S\left(\upsilon^0, \rho\right)$.

As follows from the estimates above, if s is chosen so that

$$s > s^* := max\{\alpha_0, \beta_0\},$$

then $\mathcal{A}$ is a contraction on $S\left(\upsilon^0, \rho\right)$. In this event, by the Banach Principle (see [39], p. 87-97), equation (43) has a unique solution in $S\left(\upsilon^0, \rho\right)$ for every fixed $L > 0$. Theorem 2 is proved. □

Knowing $\varphi^{'}(t)$ and $\psi^{'}(t)$, we can find the functions $\varphi(t)$ and $\psi(t)$:

$$\varphi(t) = \varphi(0) + \int_0^t \varphi^{'}(\tau)d\tau,\ \ \psi(t) = \psi(0) + \int_0^t \psi^{'}(\tau)d\tau.$$

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