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MUNDARIJA *** СОДЕРЖАНИЕ *** CONTENTS

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LAPLAS INTEGRAL ALMASHTIRISHI VA UNING DIFFERENSIAL TENGLAMALARGA QO‘YILGAN MASALALARNI YECHISHDA QO‘LLANILISHI

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Annotatsiya. *Ushbu maqola ilm-fan sohasida keng tatbiqqa ega bo‘lgan Laplas almashtirishi va bu tushunchadan differensial tenglamalarga qo‘yilgan masalalarni yechish usullariga bag‘ishlanadi. Dastlab almashtirish va uning ba’zi xossalar, so‘ng uning tatbiqlari haqida ma’lumotlar keltirilgan. Maqolada keltirilgan ma’lumotlardan matematika yo‘nalishida tahsil olayotgan bakalavrlar, magistrlar foydalanishlari mumkin.*

Kalit so‘zlar: *integral almashtirishlar, Laplas almashtirishi, differensial tenglamalar, Koshi masalasi.*

ПРЕОБРАЗОВАНИЕ ЛАПЛАСА И ЕГО ПРИМЕНЕНИЕ ПРИ РЕШЕНИИ ЗАДАЧ, ПОСТАВЛЕННЫХ ДИФФЕРЕНЦИАЛЬНЫМ УРАВНЕНИЯМ

Аннотация. *Данная статья посвящена преобразованию Лапласа, которое широко используется в области науки, и методам решения задач, поставленных дифференциальным уравнениями при помощи этого понятия. Сначала дано определения преобразования Лапласа и его некоторые свойства, а затем показано его применение. Информация, представленная в статье, может быть использована бакалаврами и магистрами, изучающими математику.*

Ключевые слова: *интегральные преобразования, преобразования Лапласа, дифференциальные уравнения, задача Коши.*

LAPLACE TRANSFORMATION AND ITS APPLICATION IN SOLVING PROBLEMS POSED BY DIFFERENTIAL EQUATIONS

Abstract. *This article is devoted to the Laplace transform, which is widely used in the field of science, and methods for solving problems posed by differential equations using this concept. First, the definition of the Laplace transform and some of its properties are given, and then its applications are shown. The information presented in the article can be used by bachelors and masters students studying mathematics.*

Keywords: *integral transformations, Laplace transforms, differential equations, Cauchy problem.*

Kirish. Integral almashtirishlar matematik amal bo'lib, u matematik ifodalarni boshqa ekvivalent oddiy shaklga aylantiradi. Oddiy va ayniqsa xususiy hosilali differensial tenglamalarni yechishning eng kuchli vositalaridan biri bu integral almashtirishlar usulidir. Furye, Laplas, Xankel va boshqa almashtirishlar elastiklik nazariyasi, issiqlik o'tkazuvchanlik, elektrodinamika va matematik fizikaning boshqa bo'limlari masalalarini hal qilish uchun qo'llaniladi. Integral almashtirishlardan foydalanish differensial, integral yoki integro-differensial tenglamani algebraik tenglamaga, xususiy hosilali differensial tenglamalar holida differensial tenglamani oddiy differensial tenglamaga keltiradi. Hozirgi vaqtda kasr tartibli hosila qatnashgan tenglamalarga qo‘yilgan masalalarni yechishda almashtirishlardan keng foydalaniloqda ([2]-[7] ga qarang).

Ko'plab turdagi almashtirishlar mavjud, ammo Laplas va Furye almashtirishlari eng ko‘p qo‘llaniladigan va mashhur almashtirishlardandir. Biz Laplas almashtirishiga to’xtalamiz. Laplas almashtirishi algebraik usullarni qo'llash orqali differensial tenglamani yechish imkonini beruvchi almashtirishlardandir. Ushbu almashtirishni birinchi marta 1790 yilda fransuz matematigi Laplas ehtimollar nazariyasi ustidagi ishlarida kiritgan.

Laplas almashtirishi va uning xossalari

Ta`rif [1]: Faraz qilaylik, $y = f(t)$ funksiya quyidagi shartlarni qanoatlantirsin:

1°. $t \geq 0$ da $f(t)$ bo'lakli uzluksiz ya'ni cheklita 1-tur uzilishga ega.

2°. $t < 0$ da $f(t) = 0$.

3°. Bu funksiya biror ko'rsatkichli funksiyadan oshmaydi, ya`ni shunday $M > 0$ son olish va $Re\, s \geq 0$ ixtiyoriy $t > 0$ uchun

$$|f(t)| \leq Me^{st} \tag{1}$$

shartni qanoatlantiruvchi inf $Re\, s = s_0 \geq 0$ ga $f(t)$ funksiyaning o'sish ko'rsatkichi deyiladi.

Ta`rif [1]: Yuqoridagi 3ta shartlarni qanoatlantiruvchi haqiqiy argumentli kompleks qiymatli $f(t)$ funksiya ***original*** deyiladi.

Faraz qilaylik, $f: R \to \mathbb{C}$ akslantirish va $s = \sigma + i\gamma$ berilgan bo'lsin.

Ta`rif* [1]*: Quyidagi xosmas integralga

$$F(s) = \int_0^{+\infty} e^{-st} f(t)dt \tag{2}$$

$F(s)$ funksiyaning ***Laplas almashtirishi*** deyiladi va quyidagicha yoziladi:

$$f(t) \risingdotseq F(s).$$

Misol: $f(t) = 1$ funksiyaning Laplas almashtirishini aniqlang.

$$F(s) = \int_0^{+\infty} e^{-st}\, 1dt = \frac{1}{s} e^{-st} \Big|_0^{+\infty} = \frac{1}{s}(1 - \lim_{t\to\infty} e^{-st}) = \frac{1}{s},$$

$$1 \risingdotseq \frac{1}{s}.$$

Ta`rif* [1]*: $F(s)$ ga $f(t)$ originalning ***tasviri*** deyiladi va quyidagicha belgilanadi:

$$\mathcal{L}\{f(t)\}(s) = F(s).$$

Misol: $f(t) = e^{at}$ funksiyaning tasvirini aniqlang.

$$F(s) = \int_0^{+\infty} e^{-st}\, e^{at} dt = \int_0^{+\infty} e^{(a-s)t}\, dt = \frac{1}{a-s} e^{(a-s)t} \Big|_0^{+\infty} = \frac{1}{s-a},$$

$$e^{at} \risingdotseq \frac{1}{s-a}.$$

Laplas almashtirishining xossalari

1°. $f(t)$ ning originali F va $g(t)$ ning originali G bo'lsin, u holda $\forall \alpha.\beta \in \mathbb{C}(R)$

$$\alpha f + \beta g \risingdotseq \alpha F + \beta G$$

ga teng.

Misol: $f(t) =$sinωt. $F(s) =?$

$sin\omega t = \frac{1}{2i}(e^{i\omega t} - e^{-i\omega t})$ va $e^{at} \risingdotseq \frac{1}{s-a}$ ga ko'ra

$$\mathcal{L}\{sin\omega t\}(s) = \frac{1}{2i}\left(\frac{1}{s - i\omega} - \frac{1}{s + i\omega}\right) = \frac{1}{2i} \cdot \frac{2i\omega}{s^2 + \omega^2} = \frac{\omega}{s^2 + \omega^2}$$

2°.O'xshashlik teoremasi: $\forall \alpha > 0$ ($\alpha = const$) uchun

$$f(\alpha t) \risingdotseq \frac{1}{\alpha} F\left(\frac{s}{\alpha}\right)$$

o'rinli.

Misol: $\mathcal{L}\{sint\}(s) = \frac{1}{s^2+1}$.

$$\mathcal{L}\{sin\alpha t\}(s) = \frac{1}{\alpha} \cdot \frac{1}{\left(\frac{s}{\alpha}\right)^2 + 1}$$

3°.Originalni differensiallash. Agar $f \in \mathbb{C}$ (t>0) da uzluksiz va $f'(t)$ yoki $f^{(n)}(t)$ −ham original bo'lsa, u holda quyidagi munosabatlar o'rinli:

1) $\mathcal{L}\{f'(t)\}(s) = sF(s) - f'(0)$,

2) $\mathcal{L}\{f^{(n)}(t)\}(s) = s^n F(s) - \sum_{k=0}^{n-1} s^k f^{(n-k-1)}(0)$.

Isbot. $\mathcal{L}\{f^{(n)}(t)\}(s) = \int_0^{+\infty} e^{-st} f^{(n)}(t)dt =$

$$= e^{-st} f^{(n-1)}(t)\Big|_{0}^{+\infty} + s \cdot \int_{0}^{+\infty} e^{-st} f^{(n-1)}(t)\, dt =$$

$$= -f^{(n-1)}(0) + s \cdot \left[e^{-st} f^{(n-2)}(t)\Big|_{0}^{+\infty} + s \cdot \int_{0}^{+\infty} e^{-st} f^{(n-2)}(t)\, dt \right] =$$

$$= -f^{(n-1)}(0) - s \cdot f^{(n-2)}(0) + s^2 \int_{0}^{+\infty} e^{-st} f^{(n-2)}(t)\, dt = \ldots$$

$$= s^n \int_{0}^{+\infty} e^{-st} f(t)\, dt - \left[f^{(n-1)}(0) + s \cdot f^{(n-2)}(0) + \cdots s^{n-1} f(0) \right].$$

4°.Originalni integrallash

$$\int_0^t f(\tau) d\tau \risingdotseq \frac{F(s)}{s}.$$

5°.Tasvirni differensiallash

$$F^{(n)}(s) \risingdotseq (-1)^n t^n f(t).$$

6°.Tasvirni integrallash $\left(\int_p^{+\infty} F(s) ds < +\infty \right)$

$$\int_{p}^{+\infty} F(s) ds \risingdotseq \frac{f(t)}{t}.$$

7°.O‘ramaning Laplas almashtirishi.

$$\mathcal{L}\{f * g\} = \mathcal{L}\{f\} \cdot \mathcal{L}\{g\}$$

f va g original funksiyalarning Laplas o‘ramasi deb quyidagi integralga

$$f * g = \int_{0}^{t} f(\rho) g(t-\rho) d\rho$$

aytiladi.

Bizga $\int_0^{+\infty} e^{-st} f(t) dt < \infty$ va $\int_0^{+\infty} e^{-st} g(t) dt < \infty$ yaqinlashuvchi funksiyalar berilgan bo‘lsin.

$\mathcal{L}\{f * g\} = \int_0^{+\infty} e^{-st} \{f * g\}(t) dt = \int_0^{+\infty} e^{-st} \int_0^t f(y)\, g(t-y) dy dt =$

$$= \begin{vmatrix} 0 \le t < +\infty \\ 0 \le y \le t \end{vmatrix} = \int_{0}^{+\infty} f(y) \int_{0}^{+\infty} e^{-st} g(t-y) dt dy$$

$$= \int_{0}^{+\infty} f(y) e^{-sy} \int_{y}^{+\infty} e^{-s(t-y)}\, g(t-y) dt dy = \begin{vmatrix} t-y=z \\ dt = dz \end{vmatrix} =$$

$$= \int_{0}^{+\infty} f(y) e^{-sy} \int_{0}^{+\infty} e^{-sz}\, g(z) dz dy =$$

$= \int_0^{+\infty} f(y) e^{-sy} dy \cdot \int_0^{+\infty} e^{-sz}\, g(z) dz = L\{f\} \cdot L\{g\}$

Misol. Riman-Liuvill kasr integralining Laplas almashtirishini topaylik.

$$I_{0+}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{0}^{t} (t-\tau)^{\alpha-1} f(\tau) d\tau = \frac{t^{\alpha-1}}{\Gamma(\alpha)} * f(t).$$

O‘ramaning Laplasiga ko‘ra, $\mathcal{L}\{I_{0+}^{\alpha} f(t)\} = \mathcal{L}\left\{\frac{t^{\alpha-1}}{\Gamma(\alpha)}\right\} \cdot \mathcal{L}\{f\}$

$$\mathcal{L}\{t^{\alpha}\} = \int_{0}^{+\infty} e^{-st}\, t^{\alpha} dt = \begin{vmatrix} ts = z \\ dt = \frac{dz}{s} \end{vmatrix} = \int_{0}^{+\infty} e^{-z} \frac{z^{\alpha}}{s^{\alpha}} \frac{dz}{s} =$$

$$= \frac{1}{s^{\alpha+1}} \int_0^{+\infty} e^{-z} z^{\alpha} dt = \frac{1}{s^{\alpha+1}} \int_0^{+\infty} e^{-z} z^{\alpha+1-1} dt = \frac{1}{s^{\alpha+1}} \cdot \Gamma(\alpha+1).$$

$\frac{1}{s^{\alpha+1}} \cdot \Gamma(\alpha+1) \risingdotseq t^{\alpha}$ va $\frac{t^{\alpha-1}}{\Gamma(\alpha)} \risingdotseq \frac{1}{s^{\alpha}}$.

$\mathcal{L}\{I_{0+}^{\alpha} f(t)\} = \mathcal{L}\left\{\frac{t^{\alpha-1}}{\Gamma(\alpha)}\right\} \cdot \mathcal{L}\{f\} = F(s) \cdot \frac{1}{s^{\alpha}}$ ga teng.

Misollar. Quyidagi funksiyalarning Laplas almashtirishini toping.

1. $f(t) = t^2 - 2$. $F(s) = ?$

$\frac{1}{s^{\alpha+1}} \cdot \Gamma(\alpha+1) \risingdotseq t^{\alpha}$ va $1 \risingdotseq \frac{1}{s}$ ga ko‘ra bu funksiyaning Laplasi. U holda

$t^2 - 2 \risingdotseq \frac{\Gamma(2+1)}{s^{(2+1)}} - \frac{2}{s}$ va $\Gamma(n+1) = n!$ ekanligidan $t^2 - 2 \risingdotseq \frac{4}{s^3} - \frac{2}{s}$

2. $f(t) = 4t^3 + t^2$. $F(s) = ?$

$$4t^3 + t^2 \risingdotseq 4 \cdot \frac{\Gamma(4)}{s^4} + \frac{\Gamma(3)}{s^3} = \frac{24}{s^4} + \frac{4}{s^3}$$

3. $f(t) = sin\omega t.$ $F(s) = ?$

$sin\omega t = \frac{1}{2i}(e^{i\omega t} - e^{-i\omega t})$ va $e^{at} \risingdotseq \frac{1}{s-a}$ ga ko‘ra

$$\mathcal{L}\{sin\omega t\}(s) = \frac{1}{2i}\left(\frac{1}{s-i\omega} - \frac{1}{s+i\omega}\right) = \frac{1}{2i} \cdot \frac{2i\omega}{s^2+\omega^2} = \frac{\omega}{s^2+\omega^2}.$$

4. $f(t) = cos\omega t.$ $F(s) = ?$

$cos\omega t = \frac{1}{2}(e^{i\omega t} + e^{-i\omega t})$ va $e^{at} \risingdotseq \frac{1}{s-a}$ ga ko‘ra

$$\mathcal{L}\{cos\omega t\}(s) = \frac{1}{2}\left(\frac{1}{s-i\omega} + \frac{1}{s+i\omega}\right) = \frac{1}{2} \cdot \frac{2s}{s^2+\omega^2} = \frac{s}{s^2+\omega^2}.$$

3 va 4 misollarga o‘xshab $\mathcal{L}\{sh\omega t\}(s) = \frac{\omega}{s^2-\omega^2}$ va $\mathcal{L}\{ch\omega t\}(s) = \frac{s}{s^2-\omega^2}$ lar topiladi.

5. $f(t) = cos^2 t.$ *F(s)=?*

$cos^2 t = \frac{1+cos2t}{2} = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}\left(e^{2it} + e^{-2it}\right) = \frac{1}{2} + \frac{1}{4}\left(e^{2it} + e^{-2it}\right)$

$e^{at} \risingdotseq \frac{1}{s-a}$ va $1 \risingdotseq \frac{1}{s}$ ga ko‘ra bu funksiyaning Laplasi quyidagicha topiladi:

$\mathcal{L}\{cos^2 t\}(s) = \frac{1}{2s} + \frac{1}{4} \cdot \left(\frac{1}{s-2i} + \frac{1}{s+2i}\right) = \frac{1}{2s} + \frac{1}{4} \cdot \frac{2s}{s^2+4} =$

$$= \frac{1}{2s} + \frac{1}{2} \cdot \frac{s}{s^2+4} = \frac{1}{2}\left(\frac{1}{s} + \frac{s}{s^2+4}\right)$$

6. $f(t) = sin^2 2t$ *F(s)=?*

$sin^2 2t = \frac{1-cos4t}{2} = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2}\left(e^{4it} + e^{-4it}\right) = \frac{1}{2} - \frac{1}{4}\left(e^{4it} + e^{-4it}\right)$

$e^{at} \risingdotseq \frac{1}{s-a}$ va $1 \risingdotseq \frac{1}{s}$ ga ko‘ra bu funksiyaning Laplasi quyidagicha topiladi:

$\mathcal{L}\{sin^2 2t\}(s) = \frac{1}{2s} - \frac{1}{4} \cdot \left(\frac{1}{s-4i} + \frac{1}{s+4i}\right) = \frac{1}{2s} + \frac{1}{4} \cdot \frac{2s}{s^2+16} =$

$$= \frac{1}{2s} + \frac{1}{2} \cdot \frac{s}{s^2+16} = \frac{1}{2}\left(\frac{1}{s} + \frac{s}{s^2+16}\right).$$

Quyida ba’zi differensial tenglamalarga qo‘yilgan Koshi masalasini Laplas almashtirishi yordamida yechilishini ko‘rsatamiz:

1. Boshlang’ich shart bilan berilgan quyidagi differensial tenglamani Laplas usulida yechaylik.

$$y'' - 3y' - 4y = 4x - 5 \quad y(0) = -1, \ y'(0) = 2$$

Bunda quyidagi xossalardan foydalanamiz:

$$\mathcal{L}\{f'(t)\}(s) = sF(s) - f'(0)$$

$$\mathcal{L}\left\{f^{(n)}(t)\right\}(s) = s^n F(s) - \sum_{k=0}^{n-1} s^k f^{(n-k-1)}(0).$$

$$y'' \risingdotseq s^2 F(s) - y'(0) - sy(0)$$

$$y' \risingdotseq sF(s) - y(0)$$

$$y \risingdotseq F(s)$$

$y'' - 3y' - 4y = 4x - 5$ tenglama quyidagicha ko’rinishga keladi:

$$s^2F(s) - y'(0) - sy(0) - 3\big(sF(s) - y(0)\big) - 4F(s) = \frac{4}{s^2} - \frac{5}{s}$$

bu yerdan F(s) ni topamiz .

$$s^2F(s) - 2 - (-1)s - 3(sF(s) - (-1)) - 4F(s) = \frac{4}{s^2} - \frac{5}{s}$$

$$s^2F(s) - 3sF(s) - 4F(s) = -s + 5 + \frac{4}{s^2} - \frac{5}{s}$$

$$(s^2 - 3s - 4)F(s) = \frac{-s^3 + 5s^2 - 5s + 4}{s^2}$$

$$F(s) = \frac{-s^3 + 5s^2 - 5s + 4}{s^2(s+1)(s-4)}$$

Bu ifodani soddalashtirib teskari Laplas qo’llab bo’ladigan darajaga keltiramiz.

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s-4} = \frac{-s^3 + 5s^2 - 5s + 4}{s^2(s+1)(s-4)}$$

$$As(s+1)(s-4) + B(s+1)(s-4) + Cs^2(s-4) + Ds^2(s+1) = -s^3 + 5s^2 - 5s + 4$$

$$A(s^3 = 3s^2 - 4s) + B(s^2 - 3s - 4) + C(s^3 - 4s^2) + D(s^3 + s^2) = -s^3 + 5s^2 - 5s + 4$$

$$A = 2, B = -1, C = -3, D = 0$$

Bizga quyidagi ifoda hosil bo’ldi:

$$F(s) = \frac{2}{s} - \frac{1}{s^2} - \frac{3}{s+1}$$

Laplas operatori xossalariga ko’ra

$$\frac{1}{s} \risingdotseq 1,\ \ \frac{1}{s^2} \risingdotseq x,\ \frac{1}{s+1} \risingdotseq e^{-x},\ F(s) \risingdotseq y$$

Teskari Laplasga keltirsak, bizga quyidagi yechimga kelinadi:

$$y(x) = 2 \cdot 1 - x - 3 \cdot e^{-x}$$

$$y(x) = 2 - x - 3e^{-x}$$

2. Yuqoridagi tenglamaga o’xshab quyidagi tenglamaga qo‘yilgan masala ham yechiladi:
$y'' + 4y = (8x + 4)e^{2x}$, $y(0) = 4, y'(0) = -1$

Laplas operatorining ba’zi xossalaridan foydalanib F(s) ni topamiz.

$$F(s) \risingdotseq y, \quad y'' \risingdotseq s^2F(s) - y'(0) - sy(0)$$

$$s^2F(s) - y'(0) - sy(0) + 4F(s) = \frac{8}{(s-2)^2} + \frac{4}{s-2}$$

$$s^2F(s) - (-1) - 4s + 4F(s) = \frac{8}{(s-2)^2} + \frac{4}{s-2}$$

$$s^2F(s) + 4F(s) = 4s - 1 + \frac{8}{(s-2)^2} + \frac{4}{s-2}$$

$$(s^2 + 4)F(s) = \frac{(4s-1)(s^2 - 4s + 4) + 8 + 4(s-2)}{(s-2)^2}$$

$$F(s) = \frac{4s^3 - 17s^2 + 24s - 4}{(s^2+4)(s-2)^2}$$

$$\frac{A}{s-2} + \frac{B}{(s-2)^2} + \frac{Cs + D}{s^2 + 4} = \frac{4s^3 - 17s^2 + 24s - 4}{(s^2+4)(s-2)^2}$$

$$A(s^2+4)(s-2) + B(s^2+4) + (Cs + D)(s^2 - 4s + 4) = 4s^3 - 17s^2 + 24s - 4$$

$$A(s^3 - 2s^2 + 4s - 8) + B(s^2 + 4) + C(s^3 - 4s^2 + 4s) + D(s^2 - 4s + 4) = 4s^3 - 17s^2 + 24s - 4$$

$$A = 0, B = 1, C = 4, D = -2$$

$$F(s) = \frac{1}{(s-2)^2} + \frac{4s - 2}{s^2 + 4} = \frac{1}{(s-2)^2} + \frac{4s}{s^2 + 4} - \frac{2}{s^2 + 4}$$

Bundan teskari Laplasga o’tsak,

$$F(s) \risingdotseq y, \qquad \frac{1}{(s-2)^2} \risingdotseq xe^{2x}$$

$$\frac{s}{s^2 + 2^2} \risingdotseq cos2x, \qquad \frac{1}{s^2 + 2^2} \risingdotseq \frac{1}{2} sin2x$$

quyidagi tenglama yechimiga kelamiz:

$$y(x) = xe^{2x} + 4cos2x - sin2x$$

Xulosa. Ushbu maqolada keltrilgan ma’lumotlar o‘z ilmiy ishlarida Laplas almashtirishi tushunchasidan foydalanadigan tadqiqotchilarga dastlabki ma’lumotlarni olish imkonini beradi.

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