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BIR ZARRACHALI DISKRET SHREDINGER OPERATORI XOS QIYMATI HAQIDA

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Ma'lumki, μ va λ haqiqiy parametrlarga bog'liq bo'lgan ikki zarrachali ... Shredinger operatori xos qiymatlari sonini aniqlash va bu xos qiymatlarning muhim ... tashqaridagi joylashuv o'rnini aniqlash kabi masalalar [1], [2] ishlarda o'rganilgan.

[3], [4] ishlarda $h(\lambda) = -\Delta + \lambda V$ Shredinger operatori yetarlicha kichik $\lambda > 0$ ... potensialga qo'yilgan ba'zi shartlarda bog'langan holat (xos qiymat)ga ega ekanligi ... Ushbu xos qiymatlarning $\lambda \to 0$ dagi asimptotikalari o'rganilgan.

$L^{2,j}(\mathbb{T}) - \mathbb{T} = (-\pi, \pi]$ bir o'lchamli torda aniqlangan kvadrati bilan integrallanuvchi ... funksiyalar gilbert fazosi va $H_{\mu\lambda}$ tashqi maydon bilan bir va ikki qadamda ta'sirlashuvchi ... zarrachali diskret Shredinger operatori bo'lsin. Ushbu operator $L^{2,j}(\mathbb{T})$ fazoda ... aniqlanadi:

$$(H_{\mu\lambda} f)(q) = \varepsilon(q) f(q) - \mu \int_{\mathbb{T}} cosqcostf(t)\,dt - \lambda \int_{\mathbb{T}} cos2qcos2tf(t)\,dt$$

bunda $\varepsilon(q) = (1 - cosq)$ va $f \in L^{2,j}(\mathbb{T})$.

Veyl teoremasiga asosan $H_{\mu\lambda}$ operatorning muhim spektri $\sigma_{ess}(H_{\mu\lambda})$ μ va λ parametrlarga bog'liq bo'lmasdan H_0 operatorning $\sigma_{ess}(H_0)$ muhim spektri bilan ustma-ust tushadi va ... tenglik o'rinli:

$$\sigma_{ess}(H_{\mu\lambda}) = \sigma_{ess}(H_0) = [0,2].$$

Teorema. $\mu, \lambda \in \mathbb{R}$ bo'lsin. Agar $\lambda + \mu + 3\mu\lambda > 0$ bo'lsa, $H_{\mu\lambda}$ operatorning muhim spektrdan chapda yagona xos qiymati mavjud.

Foydalanilgan adabiyotlar

[1] S.N. Lakaev, G.A. Muminova: O chisle i mestonoxojdenii sobstvennix znacheniy dvux... gamiltoniana na odnomernoy reshetke, Uzb.Mat.Jur.4(2010)

[2] S.N. Lakaev, Ender Ozdemir: The existence and location of eigenvalues of the one particle ... Schroedinger operators, arXiv:submit/1138572[math-ph] 15 Dec. 2014.

[3] Barry Simon: The Bound State of Weakly Coupled Schroedinger operators in one Dimension ... Annals of Physics 108, 288-300 (1977)

[4] M.Klaus:On the Bound State Schroedinger operators in one and Two Dimensions. Annals of ... Physics 97,279-288 (1976)

ON ESTIMATES FOR CONVOLUTION OPERATORS

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In this paper, we assume $1 \le p \le 2$ and $\frac{1}{p} + \frac{1}{p'} = 1$, e.g. p, p' are conjugate exponents ... We shall consider the problem on $L^p \to L^{p'}$ (where and further $L^q := L^q(R^3)$) boundedness ... Fourier multiplier operators of the following type:

$$M_k := F^{-1} e^{i\varphi(\xi)} a_k(\xi) F,$$

where F is the Fourier transform operator, $\varphi \in C^{\omega}(R^n \setminus \{0\})$ is a non-vanishing real analytic, homogenous function of order 1, $a_k \in C^{\infty}(R^n)$ is a classical symbol of a pseudo-differential operator of order $-k$. For simplicity we assume that a_k is a smooth homogeneous function of order $-k$ for large $|\xi|$. Following, M. Sugimoto [6] we assume $\varphi(\xi) > 0$ for any $\xi \neq 0$ and set:

$$S := \{\xi \in R^n \setminus \{0\} : \varphi(\xi) = 1\}. \qquad (1)$$

Clearly, the case, when $\varphi(\xi) < 0$ for any $\xi \neq 0$, can be treated by using quite similar arguments. By the classical Euler homogeneity relation S is a compact, closed analytic hyper-surface without singularities. The main problem is the following: for which $k > k(p)$ the operator M_k is extended as a bounded operator from L^p to $L^{p'}$?

In the case when $S = S^{n-1}$ is the unite sphere centered at the origin, which is related to the classical wave equation, the $L^p \to L^{p'}$ boundedness of M_k is obtained by Stricharts [4]. Further, this result has been extended to the case when S has everywhere non-vanishing Gaussian curvature by Brenner [2]. Under these conditions S is a strictly convex hyper-surface. Some estimates are obtained for such kind of convolution operators for the case when S is convex (not necessarily strictly convex) and also for some classes of non-convex hyper-surfaces in R^3 by M. Sugimoto [5]-[6]. However, as remarked in [6] the problem still is remained open for some classes of hyper- surfaces. In this paper we have some estimates which cover the open cases indicated by M.Sugimoto in [6].

We consider estimates for the convolution operators in the case when $n = 3$. Since S is a compact set, we can localize the operator. Thus, we can write the convolution operator as a sum of finite number of operators for which the associated symbol is concentrated in a sufficiently small conic neighborhood of points of the unite sphere. So, following [6], we shall assume that $a(\xi)$ is supported in a sufficiently small conic neighborhood Γ of a particular point $v \in S^2$ (where S^2 is the unite sphere centered at the origin of R^3), and $\varphi(\xi) \in C^{\omega}(\Gamma)$. Without loss of generality we may assume $v = (0,0,1) \in S^2$ and $\varphi(0,0,1) = 1, \nabla\varphi(0,0,1) = (0,0,1)$. Then, in the neighborhood Γ the surface S can be expressed as

$$S \cap \Gamma = \{\xi \in \Gamma : \varphi(\xi) = 1\} = \{(y, 1 + h(y)), y \in U\},$$

where $h \in C^{\omega}(U)$ (here $U \in R^2$ is an open neighborhood of the origin) is a real analytic function. After possible linear change of variables we may and shall assume that h is a real analytic function satisfying the conditions: $h(0,0) = 0, \nabla h(0,0) = (0,0)$ Let's reduce the classification of function h given by M. Sugimoto[6]. For the sake of being definite we assume that rank$\left\{\frac{\partial^2 h(0,0)}{\partial x_k \partial x_j}\right\}_{k,j=1}^{2} = 0$. Otherwise, the result can be derived from the main Theorem by Brenner [2]. Then after possible rotation of coordinate axes we may suppose that the function h satisfies the following conditions:

$$\frac{\partial^2 h(0,0)}{\partial x_1^2} \neq 0, \frac{\partial^2 h(0,0)}{\partial x_1 \partial x_2} = 0, \frac{\partial^2 h(0,0)}{\partial x_2^2} = 0. \qquad (2)$$

Due to the classical implicit function Theorem it is easy to see that the equation $\frac{\partial h(x_1, x_2)}{\partial x_1} = 0$ has a unique solution:

$$x_1 = b_1(x_2) \qquad (3)$$

with a real analytic function b_1 satisfying the conditions: $b_1(0) = b_1'(0) = 0$. Let's define the new function of one variable given by the function h:

$$b_0(x_2) := h(b_1(x_2), x_2). \qquad (4)$$

The following definition was given in the paper [6]. It gives a classification of singularities of the function h.

Definition1. Let h be a real analytic function at the origin satisfying (2) and b_0, b_1 be the real analytic functions defined by (3), (4). Then we define δ_j to be the smallest integer $m \geq 2$ such that $b_j^{(m)}(0) \neq 0$, and we say that h is of type I if $\delta_0 < \infty$, type II if $\delta_0 = \infty$, and $\delta_1 < \infty$, type III if $\delta_0 = \delta_1 = \infty$.

First, we give a simple statement, which follows from the classical division Theorem.

Proposition 1. Assume that h is a smooth function satisfying the assumptions (2). Then the function h can be written in the following form on a sufficiently small neighborhood of the origin:

$$h(x_1, x_2) = u(x_1, x_2)(x_1 - b_1(x_2))^2 + b_0(x_2),$$

where u, b_0, b_1 are smooth functions satisfying the conditions: $u(0,0) \neq 0$, and where $b_1(x_2) = cx_2^{\delta_0} + O(x_2^{\delta_0+1})$ $(c \neq 0)$ unless it is a flat function, $b_0(x_2) = cx_2^{\delta_1} + O(x_2^{\delta_1+1})$ $(c \neq 0)$ unless it is a flat function.

The following statement characterize the classification given by M. Sugimoto[6].

Proposition2. Let h be a real analytic function at the origin satisfying (1) and b_0, b_1 be the real analytic functions defined by (3), (4). Then h is of type I if and only if h has A_{k-1} type singularities at the origin; h is of type II or III if and only if h has A_∞ type singularities at the origin. Moreover, if h has A_∞ type singularities at the origin, then it is the type II if and only if the projection of the zero set of the Gaussian curvature to the plane R^2 is a smooth curve which has no tangent line with infinite order of contact.

See [1] for the definition of A type singularities. In this paper we mainly dealt with A type singularities.

If b_0 is a flat function (we have $b_0(x_2) \equiv 0$ whenever h is a real analytic function) and it has A_∞ type singularities. It is easy to see that in the latter case, the curve in the plane given by the equation $x_1 = b_1(x_2)$, coincides with the projection of the zero set of the Gaussian curvature. Hence, the number $\delta_1 - 1$ coincides with order of contact with the curve and tangent line at the origin.

Note that singularities of the so-called phase function defined by

$$\Phi(x_1, x_2, s_1, s_2) := h(x_1, x_2) + s_1 x_1 + s_2 x_2,$$

depend on classification of the function $h(x)$. In the monograph [1], there were given characterizations of singularities of that function and also for the phase function in the case both in a generic position up to some number of variables and up to some Milnor number.

It should be noted that behavior of the convolution operator depends on the Fourier transform of measures supported on the surface S. We introduce a more general signed measure supported on family of hyper-surfaces. Let $(S_a), S_a \subset R^{n+1}$ be a family of smooth hyper-surfaces smoothly depending on a parameter $a \in R^m$, and let $\psi \in C_0^\infty(R^{n+1} \times R^m)$ be a smooth function with compact support. The Fourier transform of the signed measure $d\mu_a = \psi\, dS_a$ is determined by the integral

$$d\hat{\mu}_a(\xi) = \int_{S_a} e^{ix\xi} d\mu_a(x)$$

The following result proved in the paper[6].

Theorem 1. Suppose that $h(x)$ is of type II with $\delta_1 \geq 3$ and satisfies

$$\frac{\partial^{\mu}}{\partial x_2^{\mu}}\left\{\frac{\partial^{\nu} h(b_1(x_2),x_2)}{\partial x_1^{\nu}}\right\}_{x_2=0} = 0 \qquad (6)$$

for $\mu = 2,3,...,\delta_1 - 1$ and $\nu = 2,3,...$ then M_k is $L^p \to L^{p'}$ bounded if

$$k > \max\left\{6\left(\frac{1}{p}-\frac{1}{2}\right)-\frac{1}{2}, \left(5-\frac{1}{\delta_1}\right)\left(\frac{1}{p}-\frac{1}{2}\right)\right\}.$$

Actually, the conditions (6) can be formulated in terms of the function u.

Lemma. Suppose that $h(x)$ is of type II with $\delta_1 \geq 2$. Then the conditions (6) are equivalent to the following conditions posed to the function u:

$$\frac{\partial^{\mu+\nu} u(0,0)}{\partial x_2^{\mu}\partial x_1^{\nu}} = 0, \text{ for } \mu = 1,2,3,..,\delta_1 - 1 \text{ and } \nu = 0,12,3,...$$

In the other words, if we consider power series expansion of the function u, then all exponents with non-zero coefficients lie above the line given by the equation $x_2 = \mu$ but trivial term.

The lemma can be proved by using straightforward calculations.

The following Theorem gives a solution of the problem posed by M. Sugimoto in the paper [6].

Theorem 3. Suppose that $h(x)$ is of type II with $\delta_1 \geq 3$ then M_k is $L^p \to L^{p'}$ bounded if

$$k > \max\left\{6\left(\frac{1}{p}-\frac{1}{2}\right)-\frac{1}{2}, \left(5-\frac{1}{\delta_1}\right)\left(\frac{1}{p}-\frac{1}{2}\right)\right\}.$$

Theorem 3 is a generalization of the Theorem 2, because it gives estimate for the convolution operators without the conditions (6). A proof of the Theorem 3 is based on estimates for the measure (5). Our estimates are based on the theory of singularities and quite different than the methods used in the paper [6]. The measure (5) can be written as the following oscillatory integral:

$$J(\lambda, z) := \int_{R^2} e^{i\lambda\Phi(x_1,x_2,s_1,s_2)} g(x_1,x_2)dx_1dx_2,$$

where $\Phi(x_1,x_2,s_1,s_2) := h(x_1,x_2) + s_1x_1 + s_2x_2$, and g is a smooth function concentrated in a sufficiently small neighborhood of the origin.

If the point (s_1,s_2) belongs to the outside of the some ball centered at the origin then the phase function has no critical points provided the support of the function g is concentrated in a sufficiently small neighborhood of the origin. Hence, we shall consider the oscillatory integral when $(s_1,s_2) \in V$, where V is a sufficiently small neighborhood of the origin. We prove the following inequality which was proved in the paper [6] under the conditions (6):

$$\|J(\lambda,\cdot)\|_{L^{\delta_0+1}(V)} \leq C_{g,\varepsilon}\lambda^{-\left(\frac{1}{2}+\frac{2}{\delta_1+1}\right)+\varepsilon},,$$

where $C_{g,\varepsilon}$ is a constant depending on a positive number ε and maximum of derivatives of the function g up to the second order. A required estimate for the norm $\|J(\lambda,\cdot)\|_{L^{\delta_0+1}(R^2\setminus V)}$ follows from the integration by parts formula, because the phase function has no critical points provided amplitude function g is concentrated in a sufficiently small neighborhood of the origin. Thus, combining the obtained two estimates we get the following result:

$$\|J(\lambda,\cdot)\|_{L^{\delta_0+1}(R^2)} \leq C_{g,\varepsilon}\lambda^{-\left(\frac{1}{2}+\frac{2}{\delta_1+1}\right)+\varepsilon},.$$

From the last estimate follows a proof of the Theorem 3 by using standard interpolation arguments.

But, as indicated in [6] there was an open problem whether we can remove the assumption $\delta_1 \geq 3$ and (5) or not. The following Theorem gives a solution of that problem.

Theorem 4. Let $S = \{(x, h(x)+1)\}$ be the surface and h be a real analytic function with $h(0,0)=0, \nabla h(0,0)=0$. If $K(0,0)=0$ and $\nabla K(0,0) \neq 0$ (where K is the Gaussian curvature of the surface) then M_k is $L^p \to L^{p'}$ bounded if

$$k > \max\left\{6\left(\frac{1}{p}-\frac{1}{2}\right)-\frac{1}{2}, \frac{14}{3}\left(\frac{1}{p}-\frac{1}{2}\right)\right\}.$$

provided the cone neighborhood Γ is chosen sufficiently small.

Remark. Actually a solution of the problem posed by M. Sugimoto [6] follows from the more general Theorem 4. If $\delta_1 = 2$ then the surface satisfies the conditions of Theorem 4. A proof of Theorem 4 follows from the following results of the paper [3].

Theorem 5. [3] Let $(S_a), S_a \subset R^{n+1}$, be a family of analytic hyper-surfaces depending on a parameter $a \in \Sigma$ (where $\Sigma \subset R^m$ is a compact set). If for some fixed $a = a_0$ and for any $x \in S_{a_0} \bigcap Supp(d\mu_a)$ the relation $|K(a_0,x)| + |\nabla K(a_0,x)| \neq 0$, holds, then the maximal function

$$M_{a_0}(\omega) := \sup_{r>0} r^{\frac{n}{2}} \left| d\hat{\mu}_{a_0}(r\omega) \right|$$

belongs to $L^{4-0}(S^n) := \bigcap_{p<4} L^p(S^n)$. Moreover, there exists a neighborhood V of a_0 such that for any $a \in V \cap \Sigma$ and for any $p < 4$ the integral

$$\int_{S^n} M_a^p d\omega$$

is bounded on $V \cap \Sigma$. The result is sharp in the sense that if the hyper-surface S satisfies the conditions: $K(x_0)=0$ and $\nabla K(x_0) \neq 0$ at some point x_0, ψ is a smooth function supported in a sufficiently small neighborhood of the point x_0 and $\psi(x_0) \neq 0$, then the maximal function corresponding to $d\hat{\mu}(r\omega)$ does not belong to $L^4(S^n)$.

Now, we show that if the function h is of type II and $\delta_1 = 2$ then the corresponding surface satisfies the condition of the Theorem 4. Actually, the following result holds true.

Proposition 6. Let $S \subset R^3$ be a smooth surface containing the origin of R^3 given as the graph of the function h. If the surface S satisfies the conditions $K(0,0)=0$ and $\nabla K(0,0) \neq 0$, then either the phase function $\Phi(x_1, x_2, s_1, s_2)$ is the R_+-universal deformation of A_2 type singularities, either the function h is of type II with $\delta_1 = 2$. Conversely, if $\Phi(x_1, x_2, s_1, s_2)$ is the R_+-universal deformation of A_2 type singularities or the function h is of type II with $\delta_1 = 2$ then for the graph of the function h is the surface satisfying the conditions $K(0,0)=0$ and $\nabla K(0,0) \neq 0$.

References

1. Arnol'd V.I., Gusein-Zade S.M., Varchenko V.N., Singularities of differentiable maps, classification of critical points, Caustics and Wave Fronts, Birkhäuser, Boston-Basel-Stuttgart.

2. Brenner P., On $L^p \to L^{p'}$ estimates for the wave-equation, Math. Z.145, pp.251-254.

3. Ikromov I.A. , Integrality of the Fourier transforms of measures concentrated on hypersurfaces, Functional analysis and its applications, Vol. 49, No 3, pp. 74-79, 2015.

4. Strichartz R. Convolutions with kernels having singularities on a sphere, Trans. Amer. Math. Soc. 148, pp.461-471, 1970.

5. Sugimoto M., A priori estimates for higher order hyperbolic equations, Math. Z. 215, pp.

6. Sugimoto M., Estimates for hyperbolic equations of space dimension 3, Journal of functional analysis 160, pp.382-407, 1998.