МАТЕМАТИЧЕСКИЙ ЦЕНТР В АКАДЕМГОРОДКЕ

«Современные проблемы дифференциальных уравнений и их приложения»

Международная научная конференция

Ташкент, 23-25 ноября 2023 года

ТЕЗИСЫ ДОКЛАДОВ

ЧАСТЬ II

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НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ УЗБЕКИСТАНА ИМЕНИ МИРЗО УЛУГБЕКА
ИНСТИТУТ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО АКАДЕМИИ НАУК РЕСПУБЛИКИ УЗБЕКИСТАН
ИНСТИТУТ МАТЕМАТИКИ И МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ (КАЗАХСТАН)
МАТЕМАТИЧЕСКИЙ ЦЕНТР МИРОВОГО УРОВНЯ "МАТЕМАТИЧЕСКИЙ ЦЕНТР В АКАДЕМГОРОДКЕ"(РОССИЯ)
ФЕРГАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
ТЕРМЕЗСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

ТЕЗИСЫ ДОКЛАДОВ

международной научной конференции на тему

СОВРЕМЕННЫЕ ПРОБЛЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ И ИХ ПРИЛОЖЕНИЯ

Ташкент, 23-25 ноября, 2023 год

II

ВЫЧИСЛИТЕЛЬНАЯ МАТЕМАТИКА И МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ

for functions φ with given values (1) in the Sobolev space $L_2^{(3)}(0,1)$.

The error of the approximation formula (1) defines a functional $(\ell,\varphi)=\varphi(z)-P_\varphi(z)$ (called the error functional) at a fixed point $x=z$. Then the error of the approximation formula (2) is estimated as follows

$$|(\ell,\varphi)| \le \|\ell\|_{L_2^{(3)*}} \|\varphi\|_{L_2^{(3)}}.$$

The problem is to find coefficients C_i, $C_{i,1}$, $C_{i,2}$, $i=0,1,...,N$ which give the minimum to the norm of the error functions ℓ. These coefficient are called optimal and the interpolation formula of the form (2) with these coefficients is called optimal interpolation formula with derivatives.

In the present paper we get explicit expressions of the coefficients for the optimal interpolation formula of the form (2).

References

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The norm for the error functional of the quadrature formula with derivative in the Sobolev space

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We consider a quadrature formula of the form

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{\beta=0}^{N} C_\beta^0\varphi(h\beta) + \sum_{\beta=0}^{N} C_\beta^1\varphi'(h\beta) \qquad (1)$$

where C_β^0 are the coefficients given in [1] and $\omega \in \mathbb{R}\backslash\{0\}$, $h=\frac{1}{N}$ and where the coefficients C_β^1 are the unknowns to be determined.

We suppose that a function φ belongs to $L_2^{(2)}(0,1)$ and

$$L_2^{(2)} = \{\varphi : [0,1] \to \mathbb{C} \mid \varphi' \text{ is absolute continuous and } \varphi'' \in L_2(0,1)\}.$$

The inner product for the functions φ and ψ in this space is defined as

$$\langle \varphi,\psi\rangle_{L_2^{(2)}} = \int_0^1 \varphi''(x)\cdot\bar{\psi}''(x)dx,$$

where $\bar{\psi}$ is the complex conjugate function to the function ψ and the corresponding of the function φ is defined by the formula

$$\|\varphi\|_{L_2^{(2)}} = \langle\varphi,\varphi\rangle^{1/2}.$$

The difference between the quadrature sum and integral

$$(\ell,\varphi)=\int_0^1 e^{2\pi i\omega x}\varphi(x)dx-\sum_{\beta=0}^{N}\left(C_\beta^0\varphi(h\beta)+C_\beta^1\varphi'(h\beta)\right)=\int_{-\infty}^{\infty}\ell(x)\varphi(x)dx, \qquad (2)$$

is called *the error* and the corresponding error functional has the form

$$\ell(x)=e^{2\pi i\omega x}\varepsilon_{[0,1]}(x)-\sum_{\beta=0}^{N}\left(C_\beta^0\delta(x-h\beta)-C_\beta^1\delta'(x-h\beta)\right). \qquad (3)$$

Using the Cauchy-Schwarz inequality, we get the following

$$|(\ell,\varphi)|\le\|\ell\|_{L_2^{(2)*}(0,1)}\cdot\|\varphi\|_{L_2^{(2)}(0,1)}.$$

Therefore, the absolute value of the error (2) of the quadrature formula (1) is estimated by the norm of the error functional (3)

$$\|\ell\|_{L_2^{(2)*}(0,1)}=\sup_{\varphi,\,\|\varphi\|_{L_2^{(2)}(0,1)}\neq 0}\frac{|(\ell,\varphi)|}{\|\varphi\|_{L_2^{(2)}(0,1)}}.$$

The main aim of this work is to find the minimum of the norm for the error functional ℓ by coefficients C_β^1 in the space $L_2^{(2)}$. That is the problem is to find the coefficients C_β^1

$$\inf_{C_\beta^1}\|\ell\|_{L_2^{(2)*}}.$$

If the coefficients C_β^1 give the minimum $\|\ell\|_{L_2^{(2)*}}$, then C_β^1 are called *optimal coefficients* and together with these coefficients, formula (1) is called the optimal quadrature formula Thus, to obtain the optimal quadrature formula of the form (1) in the space $L_2^{(2)}(0,1)$, we need to solve the following problem.

Problem 1. Find the norm of the error functional (3) of the quadrature formula (1) in the space $L_2^{(2)*}$

Theorem 1. The norm for the error functional (3) of the quadrature formula (1) has the following form in the space $L_2^{(2)}$

$$\|\ell\|^2_{L_2^{(2)*}}=\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_\beta^0\overline{C_\gamma^0}G_2(h\beta-h\gamma)+\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}\left(\overline{C_\beta^1}C_\gamma^0+C_\beta^1\overline{C_\gamma^0}\right)G_2'(h\beta-h\gamma)$$
$$-\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_\beta^1\overline{C_\gamma^1}G_2''(h\beta-h\gamma)-\sum_{\beta=0}^{N}\int_0^1\left(\overline{C_\beta^0}e^{2\pi i\omega x}+C_\beta^0e^{-2\pi i\omega x}\right)G_2(x-h\beta)dx$$
$$+\sum_{\beta=0}^{N}\int_0^1\left(\overline{C_\beta^1}e^{2\pi i\omega x}+C_\beta^1e^{-2\pi i\omega x}\right)G_2'(x-h\beta)dx+\int_0^1\int_0^1 e^{2\pi i\omega(x-y)}G_2(x-y)dxdy,$$

where

$$G_2(x)=\frac{|x|^3}{12}.$$

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Aplication to cryptograpy of pseudo invertible matrices in Excel

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Abstract
In this article we consider the example of usage of Moore-Penrose pseudoinverse to encrypt message in software package MS Excel.

Definition 1.[1] Let be a matrix of size m n (m - rows and n - columns) of rank r ($rgA = r$) over the field of real numbers R. $A^+ \in R_r^{n\times m}$ - transposed matrix The pseudo-inverse (generalized Moore-Penrose inverse) to matrix A is determined by the relations:

1) $AA^+A = A$; $A^+AA^+ = A^+$ (A^+ is a weak inversion in a multiplicative semigroup);
2) $(AA^+)^* = AA^+$; $(A^+A)^* = A^+A$ (this means that AA^+ and A^+A are Hermitian matrices);

Here A^* is the Hermitian conjugate matrix A (for matrices over the field of real numbers $A^* = A^T$)

Theorem 1.[1] For any rectangular matrix A of arbitrary rank there is a unique pseudoinverse matrix A^+

Remark 1. To simplify the problem, we narrow the class of rectangular matrices to the following form: If the columns of matrix A are linearly independent, then matrix A^*A is invertible. In this case, the pseudoinverse matrix is given by the formula

$$A^+ = (A^*A)^{-\ 1}A^*.$$

Using pseudo-inverse matrices in Excel, we can give a basic example of the ciphering system

Example We demonstrate an algorithm of encryption using Excel formulas and a modest message "cann't encrypt"to do this, we will create a new document in Excel and on the first sheet we will create an ABC array, in which only those characters that are included in the alphabet will be used ("SPACE"and "' "will be placed in 27 and 28 position). We can take a random matrix A as a key and using operation $= MDETERM(A^*A)$ it can be checked if it satisfies the condition of ivertibility to be used as a key.

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 10 \\ 5 & 7 & 16 \\ 1 & 8 & 6 \end{bmatrix}$$