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ABSTRACTS

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The extremal function of the optimal quadrature formula with derivative

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Abstract. This work is devoted to the process of constructing an optimal quadrature formula with derivative in the sense of Sard in the Hilbert space $W_2^{(2,1)}(0,1]$ of periodic and complex-valued functions for numerical calculation of Fourier integrals. Here a quadrature sum consists of a linear combination of the given function value on a uniform mesh. The error of a quadrature formula is estimated from above by the functional norm of the error based on the Cauchy-Schwarz inequality. To calculate the norm, the concept of an extremal function is used. The extremal function corresponding to the error functional is found using the Riesz representation theorem.

We consider a quadrature formula of the form

$$\int_0^1 e^{2\pi i w x}\varphi(x)dx \cong \sum_{k=1}^{N} C_k \varphi(hk) + \sum_{k=1}^{N} A_k \varphi'(hk), \tag{1}$$

where ω is non-zero integer, $\varphi \in W_2^{(2,1)}[0,1)$, C_k are coefficients given in the work [3]

$$C_k = \frac{2}{4\pi^2\omega^2+1}\cdot\frac{e^{2h}-2e^h\cos(2\pi\omega h)+1}{e^{2h}-1}\cdot e^{2\pi i\omega hk} \quad \text{for each } k=1,2,\dots,N,$$

and A_k are unknown coeffisients to be determined and $h=\frac{1}{N}$.

The difference between the integral and the quadrature sum is called *the error* of the quadrature formula (1)

$$(\ell,\varphi) = \int_0^1 e^{2\pi i w x}\varphi(x)dx - \sum_{k=1}^{N} C_k \varphi(hk) - \sum_{k=1}^{N} A_k \varphi'(hk), \tag{2}$$

where δ is the Dirac delta-function. That is, the value of the functional ℓ on a function φ gives the error of quadrature formula (1). This difference defines a linear functional

$$\ell(x) = \left(e^{2\pi i w x}\varepsilon_{(0,1]}(x) - \sum_{k=1}^{N} C_k \delta(x-hk) + \sum_{k=1}^{N} A_k \delta'(x-hk) \right) * \phi_0(x), \tag{3}$$

is called *the error functional* of the quadrature formula (1), and belongs to the space $W_2^{(2,1)*}(0,1]$ and $\varepsilon_{(0,1]}(x)$ is the characteristic function of the interval $(0,1]$, $\phi_0(x) = \sum_{\beta=-\infty}^{\infty}\delta(x-\beta)$.

Since the error functional ℓ is defined on the space $W_2^{(2,1)}(0,1]$, the following equality is valid as in the work [1]

$$(\ell,1)=0.$$

Using the Cauchy-Schwarz inequality, we obtain the following upper bound for the absolute value of the error (2)

$$|(\ell,\varphi)| \le \|\ell\|_{W_2^{(2,1)*}} \cdot \|\varphi\|_{W_2^{(2,1)}},$$

where

Therefore, in order to estimate the error of the quadrature formula (1) on functions of the space $W_2^{(2,1)}(0,1]$ it is necessary to find the norm of the error functional ℓ defined on the space $W_2^{(2,1)}(0,1]$.

Thus, for constructing the optimal quadrature formula of the form (1), we solve the following problem.

Problem 1. Find the extremal function corresponding to the error functional (3) of the quadrature formula (1).

The following theorem holds for the solution of Problem 1.

Theorem 1. *The generalized solution to the following differential equation is the extremal function ψ_ℓ corresponding to the error functional ℓ*

$$\overline{\psi_\ell}^{IV}(x)-\overline{\psi_\ell}^{II}(x)=\ell(x),$$

expressed as

$$\psi_\ell(x)=d_0+e^{-2\pi i\omega x}\cdot\kappa(\omega)-\sum_{k=1}^{N}\overline{C_k}\sum_{\beta\neq 0}\kappa(\beta)e^{2\pi i\beta(x-hk)}-\sum_{k=1}^{N}\overline{A_k}\sum_{\beta\neq 0}\kappa(\beta)\cdot(2\pi i\beta)e^{2\pi i\beta(x-hk)},$$

where d_0 is a complex number and

$$\kappa(\omega)=\frac{1}{(2\pi\omega)^4+(2\pi\omega)^2}.$$

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Вычисление нормы функционала погрешности оптимальных интерполяционных формул.

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Предположим, что нам дана таблица значений $\varphi\left(x_\beta\right)$, $\beta=0,1,...,N$ функции φ в точках $x_\beta\in\left[0,1\right]$. Требуется аппроксимировать функцию φ другой, более простой функцией P_φ ,т.е.

$$\varphi\left(x\right)\cong P_\varphi\left(x\right)=\sum_{\beta=0}^{N}C_\beta\left(x\right)\varphi\left(x_\beta\right), \tag{1}$$

где $P_\varphi\left(x\right)=\sum_{\beta=0}^{N}C_\beta\left(x\right)\varphi\left(x_\beta\right)$ - интерполяционная формула и