



**NATIONAL UNIVERSITY OF UZBEKISTAN
SAMARKAND STATE UNIVERSITY
V.I. ROMANOVSKIY INSTITUTE OF MATHEMATICS
NATURAL SCIENCE PUBLISHING**

ABSTRACTS

OF VIII INTERNATIONAL SCIENTIFIC CONFERENCE

ACTUAL PROBLEMS OF APPLIED MATHEMATICS AND INFORMATION TECHNOLOGIES-AL-KHWARIZMI 2023

Dedicated to the 105th anniversary of the National University of Uzbekistan and the 1240th anniversary of Musa Al-Khwarizmi

**SamSU, SAMARKAND - UZBEKISTAN,
SEPTEMBER 25–26, 2023**

<https://apmath.nuu.uz>

**The National University of Uzbekistan
named after Mirzo Ulugbek**

V.I. Romanovskii institute of mathematics

**Samarkand state university
named after Sharof Rashidov**

Natural Science publishing

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**OF THE 8TH INTERNATIONAL CONFERENCE
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A sharp upper bound on the error of exponentially weighted optimal quadrature formulas in the Hilbert space of periodic functions

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We consider a quadrature formula of the following form

$$\int_0^1 e^{2\pi i \omega x} \varphi(x) dx \cong \sum_{k=1}^N C_k \varphi(hk), \quad (1)$$

where $\varphi(\cdot) \in \widetilde{W}_2^{(m,m-1)}(0,1]$, $\omega \in \mathbb{Z} \setminus \{0\}$ and $\omega h \in \mathbb{Z}$, C_k are coefficients of the quadrature formula, N is number of nodes and $h = 1/N$.

We denote by $\widetilde{W}_2^{(m,m-1)}(0,1]$ the subspace of $W_2^{(m,m-1)}[0,1]$ of complex-valued, 1-periodic functions (see [1] for details).

This space is equipped by the norm $\|\varphi\|_{\widetilde{W}_2^{(m,m-1)}} = \left(\int_0^1 (\varphi^{(m)}(x) + \varphi^{(m-1)}(x))^2 dx \right)^{\frac{1}{2}}$.

The error of quadrature formula (1) is the following difference $(\ell, \varphi) = \int_0^1 e^{2\pi i \omega x} \varphi(x) dx - \sum_{k=1}^N C_k \varphi(hk)$, and the corresponding error functional is

$$\ell(x) = e^{2\pi i \omega x} - \sum_{k=1}^N C_k \sum_{\beta=-\infty}^{\infty} \delta(x - hk - \beta). \quad (2)$$

The problem of constructing optimal quadrature formulas in the space $\widetilde{W}_2^{(m,m-1)}$ is calculation of the following quantity:

$$\left\| \overset{\circ}{\ell} \right\|_{\widetilde{W}_2^{(m,m-1)*}}^2 := \inf_{C_k} \sup_{\varphi, \|\varphi\| \neq 0} \frac{|(\ell, \varphi)|}{\|\varphi\|_{\widetilde{W}_2^{(m,m-1)}}}.$$

The main result of this work is:

Theorem 1. *On the space $\widetilde{W}_2^{(m,m-1)*}(0,1]$ for $m \geq 2$, the norm of the error functional ℓ for the optimal quadrature formulas (1) with $\omega h \in \mathbb{Z} \setminus \{0\}$ has the following form*

$$\left\| \overset{\circ}{\ell} \right\|_{\widetilde{W}_2^{(m,m-1)*}}^2 = \frac{1}{(2\pi\omega)^{2m} + (2\pi\omega)^{2m-2}}.$$

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