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ABSTRACTS

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III. SECTION. COMPUTATIONAL AND DISCRETE MATHEMATICS

A sharp upper bound on the error of exponentially weighted optimal quadrature formulas in the Hilbert space of periodic functions

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We consider a quadrature formula of the following form

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx \cong \sum_{k=1}^{N} C_k\varphi(hk), \tag{1}$$

where $\varphi(\cdot)\in \widetilde{W}_2^{(m,m-1)}(0,1]$, $\omega\in\mathbb{Z}\backslash\{0\}$ and $\omega h\in\mathbb{Z}$, C_k are coefficients of the quadrature formula, N is number of nodes and $h=1/N$.

We denote by $\widetilde{W}_2^{(m,m-1)}(0,1]$ the subspace of $W_2^{(m,m-1)}[0,1]$ of complex-valued, 1-periodic functions (see [1] for details).

This space is equipped by the norm $\|\varphi\|_{\widetilde{W}_2^{(m,m-1)}} = \left(\int\limits_0^1 \left(\varphi^{(m)}(x)+\varphi^{(m-1)}(x)\right)^2 dx\right)^{\frac{1}{2}}$.

The error of quadrature formula (1) is the following difference $(\ell,\varphi)=\int_0^1 e^{2\pi i\omega x}\varphi(x)dx - \sum_{k=1}^{N} C_k\varphi(hk)$, and the corresponding error functional is

$$\ell(x)=e^{2\pi i\omega x}-\sum_{k=1}^{N} C_k\sum_{\beta=-\infty}^{\infty}\delta(x-hk-\beta). \tag{2}$$

The problem of constructing optimal quadrature formulas in the space $\widetilde{W_2}^{(m,m-1)}$ is calculation of the following quantity:

$$\left\|\overset{\circ}{\ell}\right\|^2_{\widetilde{W}_2^{(m,m-1)*}} := \inf_{C_k}\sup_{\varphi,\|\varphi\|\neq 0}\frac{|(\ell,\varphi)|}{\|\varphi\|_{\widetilde{W_2}^{(m,m-1)}}}.$$

The main result of this work is:

Theorem 1. *On the space $\widetilde{W}_2^{(m,m-1)*}(0,1]$ for $m\geq 2$, the norm of the error functional ℓ for the optimal quadrature formulas (1) with $\omega h\in\mathbb{Z}\backslash\{0\}$ has the following form*

$$\left\|\overset{\circ}{\ell}\right\|^2_{\widetilde{W}_2^{(m,m-1)*}} = \frac{1}{(2\pi\omega)^{2m}+(2\pi\omega)^{2m-2}}.$$

References

1. Hayotov A.R. and Khayriev U.N. Optimal quadrature formulas in the space $\widetilde{W_2}^{(m,m-1)}$ of periodic functions // Vestnik KRAUNC. 2022, vol. 40, no. 3. pp. 200–215.