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**YASHIL TARAQQIYOT:
BARQAROR KELAJAK UCHUN
YECHIMLAR**

2025-YIL 16-APREL

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AN EFFECTIVE QUADRATURE FORMULA IN THE HILBERT SPACE OF PERIODIC FUNCTIONS

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**“YASHIL TARAQQIYOT: BARQAROR KELAJAK UCHUN YECHIMLAR” MAVZUSIDA
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*PhD, Applied mathematics and programming technologies,
Bukhara State University, 11, M.Ikbol street,
Bukhara, 200114, Uzbekistan,
V.I.Romanovskiy Institute of Mathematics,
Uzbekistan Academy of Sciences, 9, University street,
Tashkent 100174, Uzbekistan.*

khayrievu@gmail.com, u.n.xayriev@buxdu.uz

Hamroeva Zarifa Abdullaevna

*Bukhara State University, 11, M.Ikbol street,
Bukhara, 200114, Uzbekistan.*

Abstract. In this work, we deal with the construction of an effective quadrature formula for the numerical integration of the Fourier integrals in the Hilbert space $W_2^{(2,1)}(a,b)$ of complex-valued functions. This integrals are broadly used in science and technology, specifically, in the problems of Computed tomography (CT). It is widely known that when complete continuous X-ray data are accessible, CT images can be reconstructed exactly using the filtered back-projection formula. This formula consequentially uses the Radon transform, the Fourier transforms, and the back-projection formula. Fourier transforms play an important role in the filtered back-projection method of CT image reconstruction.

Keywords. Hilbert space, effective quadrature formula,

Below, we consider the problem of approximating the Fourier integrals

$$I(\varphi, \omega) = \int_a^b e^{2\pi i \omega x} \varphi(x) dx, \quad (1)$$

where $\omega \in \mathbb{R}$ such type of integrals for sufficiently large ω is called integrals with *highly* or *strongly oscillating integrals*.

we construct an effective quadrature formula for the numerical calculation of the integral (1) with real ω in the space $W_2^{(2,1)}(a,b)$ (for full information about this space, see [1]).

One of the expansions of the optimal quadrature formulas

$$\int_0^1 e^{2\pi i \omega x} \varphi(x) dx \cong \sum_{k=1}^N C_k \varphi(hk), \quad (2)$$

for the case $\omega \in \square$, is the approximation formula obtained by assuming the coefficients in Theorem 2.3 in work [1] as continuous functions with respect to $\omega \in \mathbb{R}$. To do this, we consider construction of the quadrature formula of the following form

$$\int_a^b e^{2\pi i \omega x} \varphi(x) dx \cong \sum_{k=0}^N C_{k,\omega}[a,b] \varphi(hk + a), \quad (3)$$

where $\omega \in \mathbb{R}$, $i^2 = -1$, $C_{k,\omega}[a,b]$ are coefficients and $h = (b-a)/N$ with $N \in \mathbb{N}$.

5-Sho'ba. Barqaror rivojlanish uchun zamonaviy texnologiyalar, sun'iy intellekt va innovatsion yechimlar

The following theorems are valid for the coefficients $C_{k,\omega}[a,b]$ of the effective quadrature formula (3).

Theorem 1. *The coefficients $C_{k,\omega}[a,b]$ of the effective quadrature formula (3) with $\omega \in \mathbb{R} \setminus \{0\}$ and $\omega h \notin \mathbb{Z}$ in the space $W_2^{(2,1)}(a,b)$ have the following forms*

$$C_{0,\omega}[a,b] = \frac{(b-a)K_{\omega,2}}{(2\pi\omega_1)^4 + (2\pi\omega_1)^2} \cdot e^{2\pi i\omega a},$$

$$C_{k,\omega}[a,b] = \frac{2(b-a)K_{\omega,2}}{(2\pi\omega_1)^4 + (2\pi\omega_1)^2} \cdot e^{2\pi i\omega(hk+a)} \quad \text{for } k = 1, 2, \dots, N-1,$$

$$C_{N,\omega}[a,b] = \frac{(b-a)K_{\omega,2}}{(2\pi\omega_1)^4 + (2\pi\omega_1)^2} \cdot e^{2\pi i\omega b},$$

and for $\omega = 0$ take the form

$$C_{0,\omega}[a,b] = \frac{h}{2},$$

$$C_{k,\omega}[a,b] = h \quad \text{for each } k = 1, 2, \dots, N-1,$$

$$C_{N,\omega}[a,b] = \frac{h}{2},$$

where

$$K_{\omega,2} = - \left[\frac{e^{2h} - 1}{e^{2h} + 1 - 2e^h \cos(2\pi\omega h)} + \frac{h}{\cos(2\pi\omega h) - 1} \right]^{-1}.$$

$$\text{and } \omega_1 = (b-a)\omega, \quad h = \frac{b-a}{N}.$$

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