O’ZBEKISTON RESPUBLIKASI OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

TASHKENT INTERNATIONAL UNIVERSITY

YASHIL TARAQQIYOT: BARQAROR KELAJAK UCHUN YECHIMLAR

2025-YIL 16-APREL

TASHKENT INTERNATIONAL UNIVERSITY

"YASHIL TARAQQIYOT: BARQAROR KELAJAK UCHUN YECHIMLAR"

mavzusidagi respublika ilmiy-amaliy konferensiyasi tezislar

TO’PLAMI

16-aprel 2025 yil

TASHKENT INTERNATIONAL UNIVERSITY

СБОРНИК

Тезисов Республиканской научно-практической конференции на тему

« ЗЕЛЁНОЕ РАЗВИТИЕ:РЕШЕНИЯ ДЛЯ УСТОЙЧИВОГО БУДУШЕГО»

16- апрель 2025 год

TASHKENT INTERNATIONAL UNIVERSITY

COLLECTION

Theses of the Republican Scientific and Practical Conference on the theme of

“GREEN DEVELOPMENT: SOLUTIONS FOR A SUSTAINABLE FUTURE”

April 16, 2025

TOSHKENT 2025

AN EFFECTIVE QUADRATURE FORMULA IN THE HILBERT SPACE OF PERIODIC FUNCTIONS

Khayriev Umedjon Narmon ugli,

PhD, Applied mathematics and programming technologies,
Bukhara State University, 11, M.Ikbol street,
Bukhara, 200114, Uzbekistan,
V.I.Romanovskiy Institute of Mathematics,
Uzbekistan Academy of Sciences, 9, University street,
Tashkent 100174, Uzbekistan.
khayrievu@gmail.com, u.n.xayriev@buxdu.uz
Hamroeva Zarifa Abdullaevna
Bukhara State University, 11, M.Ikbol street,
Bukhara, 200114, Uzbekistan.

Abstract. In this work, we deal with the construction of an effective quadrature formula for the numerical integration of the Fourier integrals in the Hilbert space $W_2^{(2,1)}(a,b)$ of complex-valued functions. This integrals are broadly used in science and technology, specifically, in the problems of Computed tomography (CT). It is widely known that when complete continuous X-ray data are accessible, CT images can be reconstructed exactly using the filtered back-projection formula. This formula consequentially uses the Radon transform, the Fourier transforms, and the back-projection formula. Fourier transforms play an important role in the filtered back-projection method of CT image reconstruction.

Keywords. Hilbert space, effective quadrature formula,

Below, we consider the problem of approximating the Fourier integrals

$$I(\varphi,\omega)=\int_a^b e^{2\pi i\omega x}\varphi(x)dx, \qquad (1)$$

where $\omega\in\mathrm{R}$ such type of integrals for sufficiently large ω is called integrals with *highly* or *strongly oscillating integrals.*

we construct an effective quadrature formula for the numerical calculation of the integral (1) with real ω in the space $W_2^{(2,1)}(a,b)$ (for full information about this space, see [1]).

One of the expansions of the optimal quadrature formulas

$$\int_0^1 e^{2\pi i\omega x}\varphi(x)dx\cong\sum_{k=1}^{N}C_k\varphi(hk), \qquad (2)$$

for the case $\omega\in\square$, is the approximation formula obtained by assuming the coefficients in Theorem 2.3 in work [1] as continuous functions with respect to $\omega\in\mathrm{R}$ .To do this, we consider construction of the quadrature formula of the following form

$$\int_a^b e^{2\pi i\omega x}\varphi(x)dx\cong\sum_{k=0}^{N}C_{k,\omega}[a,b]\varphi(hk+a), \qquad (3)$$

where $\omega\in\mathrm{R}$, $i^2=-1$, $C_{k,\omega}[a,b]$ are coefficients and $h=(b-a)/N$ with $N\in\mathrm{N}$.

The following theorems are valid for the coefficients $C_{k,\omega}[a,b]$ of the effective quadrature formula (3).

Theorem 1. *The coefficients $C_{k,\omega}[a,b]$ of the effective quadrature formula (3) with $\omega \in \mathbb{R} \setminus \{0\}$ and $\omega h \not\in Z$ in the space $W_2^{(2,1)}(a,b)$ have the following forms*

$$C_{0,\omega}[a,b] = \frac{(b-a)K_{\omega,2}}{(2\pi\omega_1)^4 + (2\pi\omega_1)^2} \cdot e^{2\pi i\omega a},$$

$$C_{k,\omega}[a,b] = \frac{2(b-a)K_{\omega,2}}{(2\pi\omega_1)^4 + (2\pi\omega_1)^2} \cdot e^{2\pi i\omega(hk+a)} \quad \text{for} \quad k = 1,2,\ldots,N-1,$$

$$C_{N,\omega}[a,b] = \frac{(b-a)K_{\omega,2}}{(2\pi\omega_1)^4 + (2\pi\omega_1)^2} \cdot e^{2\pi i\omega b},$$

and for $\omega = 0$ take the form

$$C_{0,\omega}[a,b] = \frac{h}{2},$$

$$C_{k,\omega}[a,b] = h \quad \text{for each } k = 1,2,\ldots,N-1,$$

$$C_{N,\omega}[a,b] = \frac{h}{2},$$

where

$$K_{\omega,2} = -\left[\frac{e^{2h}-1}{e^{2h}+1-2e^{h}\cos(2\pi\omega h)} + \frac{h}{\cos(2\pi\omega h)-1}\right]^{-1}.$$

and $\omega_1 = (b-a)\omega,\ \ h = \dfrac{b-a}{N}.$

REFERENCES

1. Kh. M. Shadimetov, A. R. Hayotov, U. N. Khayriev, Optimal quadrature formulas for approximating strongly oscillating integrals in the Hilbert space $W_2^{(m,m-1)}$ of periodic functions. Journal of Computational and Applied Mathatematics, 2025. 453:116133.http://dx.doi.org/10.1016/j.cam.2024.116133