EYLER INTEGRALLARINING TATBIQLARI

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ANNOTATSIYA

Ushbu maqolada Eyler integrallari hisoblangan beta va gamma funksiyalarning muhim xossalarini o'rganish va ularni turli xil integrallarni hisoblashga, Eyler integrallari yordamida aniq integrallarni hisoblashga hamda akademik litsey kursidagi integrallarni hisoblashga tadbiq qilishdan iborat.

Kalit so'zlar: gamma funksiya, beta funksiya, parametrga bog'liq integral, xosmas integral, Nyuton binomi.

APPLICATIONS OF EYLER INTEGRALS

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ABSTRACT

In this paper we examine the important properties of beta and gamma functions, which are Euler integrals, and their application to the calculation of various integrals, the calculation of definite integrals using Euler integrals, and the calculation of integrals in the academic high school course.

Keywords: gamma function, beta function, parameter-dependent integral, non-specific integral, Newton's binomial.

KIRISH

Ushbu maqolada Eyler integrallari hisoblangan beta va gamma funksiyalarning muhim xossalarini o'rganish va ularni turli xil integrallarni hisoblashga, akademik litsey kursidagi integrallarni hisoblashga hamda Eyler integrallari yordamida aniq integrallarni hisoblashga tadbiq qilishdan iborat. Maqolada Eyler integrallarining akademik litsey kursidagi integrallarni hisoblash tadbiqiga doir bir qator misollar yechib ko'rsatilgan.

ADABIYOTLAR TAHLILI VA METODOLOGIYA

Aytish joizki adabiyotlarda berilgan bir qator murakkab integrallarni hisoblash, maqolaning maqsad va vazifalarini tashkil etadi.

Maqolada Eyler integrallarining akademik litsey kursidagi integrallarni hisoblash tadbiqiga doir bir qator misollar yechib ko'rsatilgan va Eyler integrallarining muhim xossalari o'rganilib, bu xossalardan foydalanib, turli xildagi integrallarni hisoblash usullari ko'rsatilgan. [1-10] maqolalarda matematika fani bo'yicha o'quv mashg'ulotlarini ilg'or pedagogik texnologiyalar yordamida tashkil etish bo'yicha metodik tavsiyalar keltirilgan. [13-30] ishlarda aniq karrali integrallarning yaqinlashuvchi yoki uzoqlashuvchi ekanligi matematik analiz elementlari yordamida ko'rsatilgan.

1-Misol. Eyler integrallaridan foydalanib , quyidagi integralning qiymatini toping. $I=\int_0^1\sqrt{x-x^2}\,dx$

$$I=\int_0^1\sqrt{x-x^2}\,dx=\int_0^1\sqrt{x}\sqrt{(1-x)}\,dx=\int_0^1 x^{\frac{1}{2}}(1-x)^{\frac{1}{2}}\,dx=B(\frac{3}{2},\frac{3}{2})$$

Endi $\Gamma(a+1)=a\Gamma(a)$, $B(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$, $\Gamma(a)\Gamma(1-a)=\frac{\pi}{sina\pi}$ formulalardan foydalanib, $B\left(\frac{3}{2},\frac{3}{2}\right)$ ning qiymatini topamiz, ya'ni

$$B\left(\frac{3}{2},\frac{3}{2}\right)=\frac{\Gamma(\frac{3}{2})\Gamma(\frac{3}{2})}{\Gamma(\frac{3}{2}+\frac{3}{2})}=\frac{\{\Gamma\left(1+\frac{1}{2}\right)\}^2}{\Gamma(3)}=\frac{\pi}{8}$$

Binobarin, berilgan integralning qiymati $\frac{\pi}{8}$

2-Misol.

$$\int_0^{+\infty}\frac{\sqrt[4]{x}}{(1-x)^2}dx$$

integralni hisoblang.

Yechilishi. Berilgan integralni hisoblashda beta funksiyaning ikkinchi formulasi $B(a,b)=\int_0^{+\infty}\frac{t^{a-1}}{1+t^{a+b}}dt$ dan foydalanib, quyidagicha yozib olamiz.

$$\int_0^{+\infty}\frac{\sqrt[4]{x}}{(1+x)^2}dx=\int_0^{+\infty}\frac{x^{\frac{5}{4}-1}}{(1+x)^{\frac{5}{4}+\frac{3}{4}}}dx=B(\frac{5}{4},\frac{3}{4})$$

Endi $B(\frac{5}{4},\frac{3}{4})$ ning qiymatini $B(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$, $\Gamma(a)\Gamma(1-a)=\frac{\pi}{sina\pi}$ formulalar bilan topamiz.

$$B\left(\frac{5}{4},\frac{3}{4}\right)=\frac{\Gamma\left(\frac{5}{4}\right)\Gamma(\frac{3}{4})}{\Gamma(2)}=\frac{\Gamma\left(1+\frac{1}{4}\right)\Gamma(1-\frac{1}{4})}{1}=\frac{1}{4}\frac{\pi}{sin\frac{\pi}{4}}=\frac{\pi}{\frac{\sqrt{2}}{2}}*\frac{1}{4}=\frac{\pi}{2\sqrt{2}}$$

3-Misol.

$$\int_0^a x^2\sqrt{a^2-x^2}dx,\qquad (a>0)$$

integralning qiymatini toping.

Yechilishi: $x=a\sqrt{t}, t>0$ almashtirish bajarib, berilgan integralni hisoblaymiz.

$$\int_0^a x^2\sqrt{a^2-x^2}dx \left|x=a\sqrt{t}, \quad dx=\frac{a}{2\sqrt{t}}dt, x=0, t=0, x=a, t=1\right|$$

$$=\int_0^1 a^2t\sqrt{a^2-a^2t}\frac{a}{2\sqrt{t}}dt$$

$$=\frac{a^4}{2}\int_0^1 t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}dt=\frac{a^4}{2}B\left(\frac{3}{2},\frac{3}{2}\right)=\frac{a^4\Gamma^2(\frac{3}{2})}{2\Gamma(3)}=\frac{a^4\{\frac{1}{2}\Gamma(\frac{1}{2})\}^2}{2*2!}=\frac{a^4\frac{1}{4}\pi}{4}$$

$$=\frac{\pi a^4}{16}$$

4-misol.

$$\int_0^{+\infty}\frac{x^{m-1}}{1+x^n}dx$$

n>0 integralni hisoblang.

Yechilishi. Berilgan integralda $x=t^{\frac{1}{n}},\ t>0$ almashtirish bajarib berilgan integralni hisoblaymiz.

$$\int_0^{+\infty}\frac{x^{m-1}}{1+x^n}dx=\left|x=t^{\frac{1}{n}}, dx=\frac{1}{n}t^{\frac{1}{n}-1}dt, x=0, t=0, x=\infty, t=\infty\right|$$

$$=\frac{1}{n}\int_0^{+\infty}\frac{t^{\frac{m-1}{n}}}{1+t}t^{\frac{1}{n}-1}dt=\frac{1}{n}\int_0^{+\infty}\frac{t^{\frac{m}{n}-1}}{1+t}dt=\frac{1}{n}B\left(\frac{m}{n},\ 1-\frac{m}{n}\right)$$

$$=\frac{1}{n}\Gamma\left(1-\frac{m}{n}\right)\Gamma\left(\frac{m}{n}\right)=\frac{\pi}{nsin\frac{\pi m}{n}}$$

5-misol.

$$\int_0^1(ln\frac{1}{x})^p dx$$

integralni hisoblang.

Yechilishi. Bu integralni hisoblash uchun belgilash usulidan foydalanamiz.

$$\int_0^1(ln\frac{1}{x})^p dx=ln\frac{1}{x}=t, |dx=-e^{-t}dt, x=0, t=\infty; x=1, t=0|=-\int_{+\infty}^0 t^p\, e^{-t}dt$$

$$=\int_0^{+\infty}t^p\, e^{-t}dt=\Gamma(p+1)$$

6 – misol.

$$I=\int_0^{+\infty}\frac{ln^2x}{1+x^4}dx$$

integralni hisoblang.

Yechilishi. Berilgan integralda quyidagicha belgilash kiritamiz.

$$I=\int_0^{+\infty}\frac{ln^2x}{1+x^4}d=\left|x=t^4\ (t>0),dx=\frac{1}{4}t^{-\frac{3}{4}}dt,x=0,t=0,x=\infty,t=\infty\right|$$

$$=\int_0^{+\infty}\frac{\frac{1}{16}ln^2t}{1+t}\frac{1}{4}t^{-\frac{3}{4}}dt=\frac{1}{64}\int_0^{+\infty}\frac{t^{-\frac{3}{4}}ln^2t}{1+t}dt$$

Bu integralda esa $\frac{1}{64}\int_0^{+\infty}\frac{t^{a-1}}{1+t}$dt integraldan ikkinchi marta a bo'yicha hosila olib, a=$\frac{1}{4}$ qo'yilsa, hosil bo'ladi. demak,

$$I=\frac{1}{64}\frac{d^2}{da^2}\int_0^{+\infty}\frac{t^{a-1}}{(1+t)^{a+(1-a)}}]_{a=\frac{1}{4}}=\frac{1}{64}\frac{d^2}{da^2}(B(a,1-a))]_{a=\frac{1}{4}}=\frac{1}{64}\frac{d^2}{da^2}(\frac{\pi}{sina\pi})_{a=\frac{1}{4}}$$

$$=\frac{1}{64}\frac{d}{da}(-\frac{\pi^2cosa\pi}{sin^2a\pi})_{a=\frac{1}{4}}=\frac{1}{64}(\frac{\pi^3sin^3a\pi+\pi^3sin2a\pi*cosa\pi}{sin^4a\pi})_{a=\frac{1}{4}}$$

$$=\frac{1}{64}\frac{\pi^3sin^3\frac{\pi}{4}+\pi^3*sin\frac{\pi}{2}*cos\frac{\pi}{4}}{sin^4\frac{\pi}{4}}=\frac{3\sqrt{2}}{64}\pi^3$$

Shunday qilib, misolning javobi $\frac{3\sqrt{2}}{64}\pi^3$

Endi Eyler integrallari yordamida akademik litsey kursida uchrab turadigan ba'zi integrallarni hisoblash uchun quyidagi misollarni ko'rib chiqamiz.

1-Misol.

$$\int_0^1 x^2(1-x)^3dx$$

integralni hisoblaymiz.

Yechilishi. 1-usul. Berilgan integralni eng avval akademik litsey kursida hisoblaymiz. Buning uchun litsey o'quvchisi integral ostidagi ifodani qisqa ko'paytirish formulasidan foydalanib, ko'phad ko'rinishiga yozib, hisoblaydi, ya'ni

$$\int_0^1 x^2(1-x)^3dx=$$

$$=\int_0^1 x^2(1-3x+3x^2-x^3)dx$$

$$=\int_0^1(x^2-3x^3+3x^4-x^{5)}dx=(\frac{1}{3}x^3-\frac{3}{4}x^4+\frac{3}{5}x^5-\frac{1}{6}x^6)]_0^1$$

$$=\frac{1}{3}-\frac{3}{4}+\frac{3}{5}-\frac{1}{6}=\frac{1}{60}$$

2-usul.Endi yuqoridagi integralni Eyler integrallari yordamida hisoblashga o'tamiz. Bunda berilgan integralga e'tibor beradigan bo'lsak, u 1- tur Eyler integrali bo'lgan beta funksiyani ifoda qilayotganini payqash qiyin emas, ya'ni

$$\int_0^1 x^2(1-x)^3dx = \int_0^1 x^{3-1}(1-x)^{4-1}dx = B(3,4)$$

bo'ladi. Hosil bo'lgan B(3,4) funksiyaning qiymatini

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, \quad \Gamma(n+1) = n!$$

formulalardan foydalanib, osonlik bilan hisoblaymiz.

$$B(3,4) = \frac{\Gamma(3)\Gamma(4)}{\Gamma(3+4)} = \frac{2!*3!}{6!} = \frac{1}{60}$$

Demak, berilgan integralning qiymati ikkala usul bilan ham yechganda $\frac{1}{60}$ ga teng bo'ladi.

2-Misol

$$\int_0^1 x^3(1-x)^4dx$$

integralni hisoblang.

Yechilishi . 1-usul. Litsey o'quvchisi bu integralni hisoblashi uchun Nyuton binomi formulasini bilishiga to'g'ri keladi. Aks holda integral ostidagi ifodani ko'p had ko'rinishiga keltirish uchun bir qancha qiyinchiliklarga uchraydi. Quyidagi Nyuton binomi formulasiga asosan berilgan integralni hisoblaymiz.

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1*2}a^{n-2}b^2 + \cdots + \frac{n(n-1)(n-2)\dots(n-k+1)}{1*2*3*\dots*k}a^{n-k}b^k + \cdots + b^n$$

Demak,

$$\int_0^1 x^3(1-x)^4dx = \int_0^1 x^3(1-4x+6x^2-4x^3+x^4)dx = \int_0^1 (x^3-4x^4+6x^5-4x^6+x^7)\,dx = [\frac{1}{4}x^4 - \frac{4}{5}x^5 + \frac{6}{6}x^6 - \frac{4}{7}x^7 + \frac{1}{8}x^8]_0^1 = \frac{1}{4} - \frac{4}{5} + \frac{6}{6} - \frac{4}{7} + \frac{1}{8} = \frac{1}{280}$$

2-usul. Berilgan integralga Eyler integrallarni tadbiq etamiz.

$$\int_0^1 x^3(1-x)^4dx = B(4{,}5) = \frac{\Gamma(4)\Gamma(5)}{\Gamma(9)} = \frac{3!*4!}{8!} = \frac{1}{280}$$

Shunday qilib, integralning qiymati $\frac{1}{280}$ ga teng ekanligini topdik.

3- Misol.

$$\int_0^{\frac{\pi}{2}} sin^2x\, cos^2xdx$$

integralni hisoblaymiz.

Yechilishi. Bu integralning akademik litsey kursida hisoblash uchun litsey o'quvchisi ikkilangan burchak sinusi formulasidan foydalanib, integral ostida berilgan ifodani bitta funksiyaga keltiradi va darajani pasaytirish formulasini qo'llab, hisoblaydi.

$$\int_0^{\frac{\pi}{2}} sin^2x\, cos^2xdx == \frac{1}{4}\int_0^{\frac{\pi}{2}} sin^2 2xdx = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{1-cos4x}{2}dx = \frac{1}{8}[(x-\frac{1}{4}sin4x)]_0^{\frac{\pi}{2}} = \frac{\pi}{16}$$

2- usul. Endi berilgan integralni Eyler integrallari yordamida hisoblaymiz. Bunda quyidagi almashtirishlarni bajaramiz:

$$\int_0^{\frac{\pi}{2}} sin^2x\, cos^2xdx = \left|sinx = y, dy = cosxdx, x = 0, y = 0; x = \frac{\pi}{2}, y = 1\right|$$

$$= \int_0^1 y^2\sqrt{1-y^2}\,dy$$

integralni hosil qilamiz Bu integralda yana almashtirish usulini qo'llab, berilgan integralning qiymatini topamiz, ya'ni

$$\int_0^1 y^2\sqrt{1-y^2}dy = \left|y^2 = t, dy = \frac{1}{2\sqrt{t}}dt\,, y = 0, t = 0; y = 1, t = 1\right|$$

$$= \frac{1}{2}\int_0^1 t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}dt = \frac{1}{2}B\left(\frac{3}{2},\frac{3}{2}\right) = \frac{1}{2}\frac{\Gamma^2\left(\frac{3}{2}\right)}{\Gamma(3)} = \frac{1}{2}\frac{\left\{\frac{1}{2}\Gamma\left(\frac{1}{2}\right)\right\}^2}{2!} = \frac{\pi}{16}$$

Demak, integrakning qiymati $\frac{\pi}{16}$

4-Misol. $y = x^2$ va $y = x^3$ chiziqlar bilan chegaralangan shaklning yuzini topamiz.

Yechilishi. Berilgan egri chiziqlarni tenglashtirib, kesishish nuqtalarini topamiz va grafigini yasaymiz.

$$x^2 = x^3, \qquad x^2(x-1) = 0\,, x = 0, x = 1.$$

Ma'lumki, ta'lab qilingan shaklning yuzi aniq integral bilan topiladi.

$$S = \int_0^1 (x^2 - x^3)dx = [(\frac{1}{3}x^3 - \frac{1}{4}x^4)]_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

Endi bu integralni Eyler integrallari bilan hisoblaymiz.

$$\int_0^1 (x^2 - x^3)dx = \int_0^1 x^2(1-x)dx = B(3,2) = \frac{\Gamma(3)\Gamma(2)}{\Gamma(5)} = \frac{2!\ 1}{4!} = \frac{1}{12}$$

Demak, shaklning yuzi $\frac{1}{12}$ $kv.bir$

5-Misol.

$$\int_0^1 \sqrt{x}(x-1)^2 dx$$

Integralni hisoblang.

Yechilishi. 1-usul. Bu usulda integral akademik litsey programmasida quyidagicha topiladi.

$$\int_0^1 \sqrt{x}(x-1)^2 dx$$

$$= \int_0^1 \sqrt{x}(x^2 - 2x + 1)dx = [(\frac{2}{7}x^{\frac{7}{2}} - \frac{4}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}})]_0^1 = \frac{2}{7} - \frac{4}{5} + \frac{2}{3} = \frac{16}{105}$$

2-usul. Berilgan integral Eyler integrallari orqali quyidagicha hisoblanadi ya'ni

$$\int_0^1 \sqrt{x}(x-1)^2 dx = \int_0^1 x^{\frac{3}{2}-1}(1-x)^{3-1}dx =)B(\frac{3}{2},3)$$

Endi $\Gamma\left(n + \frac{1}{2}\right) = \frac{1*3*5*...*(2n-1)}{2^n}\sqrt{\pi}$, $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ formulalarni qo'llab, $B(\frac{3}{2},3)$ funksiyaning qiymatini topamiz.

$$B\left(\frac{3}{2},3\right) = \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(3)}{\Gamma(\frac{3}{2}+3)} = \frac{\Gamma\left(1+\frac{1}{2}\right)\Gamma(3)}{\Gamma(4+\frac{1}{2})} = \frac{\frac{1}{2}\sqrt{\pi} * 2!}{\frac{1 * 3 * 5 * 7}{2^4}\sqrt{\pi}} = \frac{16}{105}$$

Demak, berilgan misolning javobi; $\frac{16}{105}$

MUHOKAMA

Ushbu Eyler integrallarining afzalliklari: matematik bilimlarini chuqurlashtirish, fikrlash doirasini kengaytirish, tasavvur qobiliyatini o'stirish, Eyler integrallari hisoblangan beta va gamma funksiyalarning muhim xossalarini o'rganib ularni turli xil integrallarni hisoblashga tadbiq qilishga, fizika va mexanikaning ba'zi masalalarini yechishga hamda adabiyotlarda berilgan bir qator murakkab integrallarni hisoblashga o'rgatadi.

Kamchiliklari: Eyler integrallarining tatbiqlari qisqa yoritildi.

NATIJA

Ushbu Eyler integrallarining tatbiqining afzalliklari: talabalarning Eyler integrallarining muhim xossalarini tahlil qilish va isbotlashga, Eyler integrallarining akademik litsey kursidagi integrallarni hisoblash tadbiq qilishga imkon yaratadi, fikrlash doirasini kengaytiradi, tasavvurini o'stiradi hamda fanga nisbatan qiziqishini oshiradi.Natijada fan yuzasidan bilimlari yanada mustahkamlanadi.

Metodning kamchiliklari deyarli aniqlanmagan. Faqat o'qituvchi va o'quvchidan ozgina izlanish talab qilinadi.

XULOSA

Ma'lumki, hozirgi vaqtda mamlakatimiz Prezidenti tomonidan matematika fani va uni amaliyotga qo'llashni rivojlantirishga katta ahamiyat berilib, bir qator qarorlar imzolangan.Qarorlar ijrosini ta'minlashning negizida albatta fanni talabalarga qulay matematik usullardan foydalanib o'rgatish yotadi. Maqolada Eyler integrallarining akademik litsey kursidagi integrallarni hisoblashga qo'llash bo'yicha izlanishlar olib borilgan.

Eyler integrallari fizika va mexanikaning ba'zi masalalariga tadbiq qilinsa talabalarga qulaylik tug'diradi.

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