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Propagation of Natural Waves on a Multilayer Viscoelastic Cylindrical Body Containing the Surface of a Weakened Mechanical Contact

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Propagation of Natural Waves on a Multilayer Viscoelastic Cylindrical Body Containing the Surface of a Weakened Mechanical Contact

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Abstract. In this work, the study of the problems associated with the propagation of natural waves in multilayer viscoelastic cylindrical bodies with a weakened mechanical contact is discussed. A detailed analysis of well-known works devoted to this problem is given. A mathematical formulation, a technique, and an algorithm for studying the damping properties of natural waves in multilayered cylindrical mechanical systems with a weakened mechanical contact are developed. The solution of the considered problem was obtained by the method of separating the variables based on the theory of potential functions (special functions).The complex roots (phase velocities) of the dispersion transcendental equation for given wavenumbers are determined numerically by the Muller method. The phase and group velocities of a structurally heterogeneous mechanical system at various geometric and physical-mechanical parameters for the elements of the mechanical system are investigated. It was established that the real parts of the wave velocity will increase by only a few percent, and the imaginary parts for structurally heterogeneous mechanical systems radically change; the phase velocities (real parts of the complex velocity) of natural waves with an increasing wave numbers around the cylinder circumference of structurally heterogeneous mechanical systems first decrease and then begin to increase. A mechanical effect was discovered for structurally heterogeneous mechanical systems, which provides damping for the waves of the mechanical system as a whole.

1. Introduction

In geological engineering and mining mechanics, the concept of surfaces with a weakened mechanical contact (WMC) is established as surfaces along which a rigid connection is broken between adjacent sections of the environment, which in some cases leads to emergencies during mining and technical activities [1, 2]. WMC surfaces can be layer change boundaries with different lithologies, thin layers of soft rocks, tectonic fractures, surfaces of a change in sedimentary accumulation conditions, etc. The diagnosis and localization of WMC surfaces by acoustic and seismic means represent a task that can only be solved by understanding, in detail, the nature of the wave field propagating in long cylindrical layers, having an imperfect contact with the environment. In general, the degree of weakness is characterized by an effective coupling coefficient K_v, including in the boundary condition connecting the tangential voltage at the contact τ with displacement jump Δu, parallel to the border: $\tau = K_v \Delta u$ [3, 4]. A hard contact is obtained when $K_v \to \infty$, and a sliding contact is obtained when $K_v \to 0$. Thus, the sliding contact is the limiting case of a WMC, and it can be assumed that the main features of the field at $K_v = 0$ should have much in common with the fields, observed with a large weakening of the contact $K_v \leq 1$. A study on the dynamics of layered cylindrical bodies with rigid contacts has received much attention [5, 6]. Various aspects of the processes of excitation and propagation of waves in cylindrical bodies associated with various elastic and acoustic media are investigated in these works. In [7], the unsteady behaviour of a wave field propagating in a cylinder under the influence of pulsed pressures with allowance for WMCs was considered. In previous works [8, 9], the properties of surface waves in elastic cylinders in contact with an infinite elastic environment were studied.

In solving such problems, studying the roots of the dispersion equation is a necessary step in a number of practically important problems. In a number of problems in the works [6, 8–10], dispersion equations are constructed, and based on the Newton numerical method, numerical results for a rigid contact condition are obtained. In particular, the study of the asymptotic behaviour of the roots as the wave parameters tend to zero or infinity for the analysis of the propagation of unsteady waves under the influence of shock plays an important role in the works of [10, 11].

In previous works [12, 13], the acoustic spectroscopic properties of cylindrical waveguides were studied. Additionally, in many other works [14, 15], theoretical and applied problems of the dynamics of deformable bodies associated with WMCs were studied.

Along with this study, the dynamics of various heterogeneous elastic and viscoelastic systems, taking into account their features and working conditions, are the focus of works devoted to evaluating the natural vibrations and dynamic behaviour of a structure under various influences [24–34].

Here, a review of only some of the works that are devoted to assessing the dynamics of various heterogeneous systems with WMCs is provided. Currently, unlike elastic waveguides with WMCs, the features of dispersion of deformation waves in hereditary viscoelastic bodies, taking into account WMCs, have been insufficiently studied [16, 17].Therefore the study of wave propagation in elastic and viscoelastic cylindrical layers with WMCs is an urgent problem. Therefore, in this paper, the problem of wave propagation in a deformable layer in sliding contact with a viscoelastic half-space is considered. Such a model was previously considered in [18], which applied to the description of low-frequency waves without taking into account the rheological properties of the material at $K_v = 0$.

2. Methods

2.1 Statement of the problem and methods of the solution

The problem of the propagation of natural waves (i.e., oscillatory processes) in multilayered cylindrical bodies located in a viscoelastic (or acoustic) environment is considered. The equations of motion for a multilayered body and its environment, in the absence of mass forces, satisfy the integro-differential equations:

$$\tilde{\mu}_\kappa \nabla^2 \vec{u} + (\tilde{\lambda}_\kappa + \tilde{\mu}_\kappa) graddi\vartheta \vec{u} = \rho_\kappa \frac{\partial^2 \vec{u}}{\partial t^2} \text{ , (к =1, 2, 3..N, N+1)} \quad , \quad (1)$$

where $\vec{u}$ -is a vector of the displacements of the environment points; ρ_κ -is the material density of the κ-th layer; λ_κ, μ_κ -are integral operators describing the mechanical properties of the κ-th layer; and N -is the number of layers,

$$\tilde{\lambda}_\kappa\left[f(t)\right]=\lambda_{0\kappa}\left[f(t)-\int_0^t R_{\lambda\kappa}(t-\tau)f(\tau)d\tau\right];$$
$$\tilde{\mu}_\kappa\left[f(t)\right]=\mu_{0\kappa}\left[f(t)-\int_0^t R_{\mu\kappa}(t-\tau)f(\tau)d\tau\right], \tag{2}$$

here $f(t)$ -is an arbitrary function of time; $R_{\lambda\kappa}(t-\tau)$ and $R_{\mu\kappa}(t-\tau)$ -are relaxation cores; and $\lambda_{0\kappa}, \mu_{0\kappa}$ -are instantaneous Lame constants, associated with the mechanical properties of the material or environment (modulus of elasticity).

The integral terms in (2) are accepted as small. Furthermore, using the freezing procedure, the relations in (2) are replaced by the approximate relations of the form [4]

$$\bar{\lambda}_j\left[f(t)\right]=\lambda_{0j}\left[1-\Gamma^C_{\lambda j}(\omega_R)-i\Gamma^S_{\lambda j}(\omega_R)\right]f(t),$$

$$\bar{\mu}_\kappa\left[f(t)\right]=\mu_{0\kappa}\left[1-\Gamma^C_{\mu\kappa}(\omega_R)-i\Gamma^S_{\mu\kappa}(\omega_R)\right]f(t).$$

Here,

$$\Gamma_{\lambda\kappa}{}^C(\omega_R)=\int_0^\infty R_{\lambda\kappa}(\tau)\cos\omega_R\tau\, d\tau, \Gamma_{\mu\kappa}{}^C(\omega_R)=\int_0^\infty R_{\mu\kappa}(\tau)\cos\omega_R\tau\, d\tau,$$

$$\Gamma_{\lambda\kappa}{}^S(\omega_R)=\int_0^\infty R_{\lambda\kappa}(\tau)\sin\omega_R\tau\, d\tau, \Gamma_{\mu\kappa}{}^S(\omega_R)=\int_0^\infty R_{\mu\kappa}(\tau)\sin\omega_R\tau\, d\tau$$

the cosine and sine images of the Fourier, $R_{\lambda\kappa}(t-\tau)$ and $R_{\mu\kappa}(t-\tau)$- are relaxation cores, and ω_R -is a real value.

Hard contact conditions are set between the layers

$$\sigma_{rr}^{(1)}=\sigma_{rr}^{(2)};\ \ \sigma_{r\theta}^{(1)}=\sigma_{r\theta}^{(2)};\ \sigma_{rz}^{(1)}=\sigma_{rz}^{(2)};\ u_r^{(1)}=u_r^{(2)};\ \ u_\theta^{(1)}=u_\theta^{(2)};\ u_z^{(1)}=u_z^{(2)}. \tag{3}$$

or sliding (WMC) contact

$$\sigma_{rr}^{(1)}=\sigma_{rr}^{(2)};\ \ \sigma_{r\theta}^{(1)}=\sigma_{r\theta}^{(2)}=\sigma_{rz}^{(1)}=\sigma_{rz}^{(2)}=0;\ u_r^{(1)}=u_r^{(2)}. \tag{4}$$

If there is friction on the contact, then

$$\sigma_{rr}^{(1)}=\sigma_{rr}^{(2)};\ \ \sigma_{r\theta}^{(1)}=\sigma_{rz}^{(1)}=k\sigma_{rr}^{(1)},\ \sigma_{r\theta}^{(2)}=\sigma_{rz}^{(2)}=k\sigma_{rr}^{(2)};\ u_r^{(1)}=u_r^{(2)}, \tag{5}$$

where k -is the coefficient of friction.

The potentials of the displacements at infinity $r \to \infty$ satisfy the Somerfield radiation conditions:

$$\lim_{r\to\infty}\phi_{N+1}=0,\ \lim_{r\to\infty}(\sqrt{r})^{\kappa}\left(\frac{\partial\phi_{N+1}}{\partial r}+i\alpha_{N+1}\phi_{N+1}\right)=0,$$

$$\lim_{r\to\infty}\psi_{N+1}=0,\ \lim_{r\to\infty}(\sqrt{r})^{\kappa}\left(\frac{\partial\psi_{N+1}}{\partial r}+i\beta_{N+1}\psi_{N+1}\right)=0.$$

Equation (1) is solved for the potentials of the displacements. Then, for the displacement vector, the Green-Lemb decomposition is valid

$$\vec{u}_{\kappa}=grad\phi_{\kappa}+rot\vec{\psi}_{\kappa},div\vec{\psi}_{\kappa}=0, \quad (6)$$

here, $\boldsymbol{\phi}_{\kappa}$ -is the longitudinal wave potential, and $\vec{\psi}_{\kappa}(\psi_{x\kappa},\psi_{y\kappa},\psi_{z\kappa})$ -is the shear wave potential:

$$\begin{aligned}
&\nabla^2\phi_{\kappa}-\frac{1}{\bar{c}_{p\kappa}^2}\frac{\partial^2\phi_{\kappa}}{\partial t^2}=0;\\
&\nabla^2\psi_{Z\kappa}-\frac{1}{\bar{c}_{s\kappa}^2}\frac{\partial^2\psi_{Z\kappa}}{\partial t^2}=0;\\
&\nabla^2\psi_{\theta\kappa}-\frac{\psi_{\theta\kappa}}{r^2}+\frac{2}{r^2}\frac{\partial\psi_{r\kappa}}{\partial\theta}-\frac{1}{\bar{c}_{s\kappa}^2}\frac{\partial^2\psi_{\theta\kappa}}{\partial t^2}=0;\\
&\nabla^2\psi_{r\kappa}-\frac{\psi_{r\kappa}}{r^2}-\frac{2}{r^2}\frac{\partial\psi_{\theta\kappa}}{\partial\theta}-\frac{1}{\bar{c}_{s\kappa}^2}\frac{\partial^2\psi_{r\kappa}}{\partial t^2}=0.
\end{aligned} \quad (7)$$

Here, $\bar{c}_{s\kappa}^2=c_{s\kappa}^2\Gamma_{\kappa}^{\bullet},\bar{c}_{p\kappa}^2=c_{p\kappa}^2\Gamma_{\kappa}^{\bullet},\Gamma_{\kappa}^{\bullet}=1-\Gamma_{\kappa}^{C}(\omega_R)-i\Gamma_{\kappa}^{S}(\omega_R),c_{pk}^2=(\lambda_k+2\mu_k)/\rho_k;c_{sk}^2=\mu_k/\rho_k$.

The geometry of the object and the natural assumption about the nature of the wave motion along the *Oz* axis make it possible to substantially predict the shape of the desired scalar and vector functions. They should represent waves running along the *Oz* axis. Based on this consideration, the solution of the wave equation in equation (7) is sought in the form

$$\begin{aligned}
&\phi_k(r,\theta,z,t)=\sum_{n=0}^{\infty}\varphi_n(\alpha_k r)\begin{Bmatrix}\cos n\theta\\-\sin n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t};\\
&\psi_{rk}(r,\theta,z,t)=\sum_{n=0}^{\infty}\psi_{nr}(\beta_{\ k}r)\begin{Bmatrix}\sin n\theta\\-\cos n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t};\\
&\psi_{\theta k}(r,\theta,z,t)=\sum_{n=0}^{\infty}\psi_{n\theta}(\beta_{\ k}r)\begin{Bmatrix}\cos n\theta\\-\sin n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t};\\
&\psi_{zk}(r,\theta,z,t)=\textstyle\sum_{n=0}^{\infty}\psi_{nz}(\beta_{\ k}r)\begin{Bmatrix}\sin n\theta\\\cos n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t}.
\end{aligned} \quad (8)$$

Here, dimensionless coordinates $r=r_1/a_0,z=z_1/a_0$ -are introduced and used; n -is an integer; and γ_p -is the dimensionless wave propagation constant.

The unknown functions of the radial coordinate in work [8] satisfy the following ordinary differential equations:

Journal of Physics: Conference Series **1921** (2021) 012127 doi:10.1088/1742-6596/1921/1/012127

$$\frac{d^2\varphi_k}{dr^2}+\frac{1}{r}\frac{d\phi_k}{dr}+\left(\alpha_k^2-\frac{n^2}{r^2}\right)\phi_k=0;$$

$$\frac{d^2\psi_{zk}}{dr^2}+\frac{1}{r}\frac{d\psi_{zk}}{dr}+\left(\beta_k^2-\frac{n^2}{r^2}\right)\psi_{zk}=0;$$

$$\frac{d^2\psi_{\theta k}}{dr^2}+\frac{1}{r}\frac{d\psi_{\theta k}}{dr}+\frac{1}{r^2}\left(-n^2\psi_{\theta k}+2n\psi_{\theta k}-\psi_{\theta k}\right)\beta^2\psi_{\theta k}=0;$$

$$\frac{d^2\psi_{rk}}{dr^2}+\frac{1}{r}\frac{d\psi_{rk}}{dr}+\frac{1}{r^2}\left(-n^2\psi_{rk}+2n\psi_{\theta k}-\psi_{rk}\right)\beta^2\psi_{rk}=0. \tag{9}$$

Here $\alpha_k^2=\frac{\overline{\Omega}_k^2}{\gamma_k^2}-\gamma_p^2$; $\beta_k^2=\overline{\Omega}_k^2-\gamma_p^2$; $\overline{\Omega}_k=\frac{\omega\alpha_k}{\bar{c}_{sk}}$; $\gamma_k^2=\frac{2(1-\nu_k)}{1-2\nu_k}$.

The solution of the ordinary differential equations with complex coefficients in equation (9) is expressed in terms of the special functions of Bessel and Hankel [19].

2.2. The method of obtaining the dispersion equation

Consider a cylindrical coordinate system (r,z,θ) model of a mechanical system consisting of a multilayer pipe:

1.($\boldsymbol{r_1\le r\le r_2}$), 2.($\boldsymbol{r_2\le r\le r_3}$).....n-1.($\boldsymbol{r_{n-2}\le r\le r_{n-1}}$), n. ($\boldsymbol{r}\ \ \ge \boldsymbol{r_n}$).

Assume that the space inside the pipe ($\boldsymbol{r_0\le r\le r_1}$) is filled with liquid. Consider the problem of natural vibrations arising in such systems. The equations of motion of the environment for longitudinal and transverse potentials are presented in the form of (7) - (9). At the interfaces between the elastic environment and the liquid, the boundary conditions of continuity of the normal components of the displacements and stresses and tangential stresses equal to zero are satisfied.

The solution of equation (9) satisfying the condition of finiteness of the field on the axis $\boldsymbol{r=0}$ and the conditions of decreasing at infinity can be written in the form

$$\begin{aligned}
\phi_k(r,\theta,z,t)&=\sum_{n=0}^{\infty}\left[F_{kn}J_n(\alpha_k r)+E_{kn}N_n(\alpha_k r)\right]\begin{Bmatrix}\cos n\theta\\-\sin n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t};\\
\psi_{rk}(r,\theta,z,t)&=\sum_{n=0}^{\infty}\left[A_{1kn}J_n(\beta_k r)+A_{2kn}N_n(\beta_k r)\right]\begin{Bmatrix}\sin n\theta\\-\cos n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t};\\
\psi_{\theta k}(r,\theta,z,t)&=\sum_{n=0}^{\infty}\left[B_{1kn}J_{n-1}(\beta_k r)+B_{2kn}N_{n+1}(\beta_k r)\right]\begin{Bmatrix}\cos n\theta\\-\sin n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t};\\
\psi_{zk}(r,\theta,z,t)&=\sum_{n=0}^{\infty}\left[B_{1kn}J_{n-1}(\beta_k r)-B_{2kn}N_{n+1}(\beta_k r)\right]\begin{Bmatrix}\cos n\theta\\-\sin n\theta\end{Bmatrix}e^{\pm i\gamma_p z}e^{-i\omega t}.
\end{aligned} \tag{10}$$

Here, $\boldsymbol{J_n}$ and N_n - are Bessel and Neumann functions of the complex argument n-th order. Instead of the Bessel and Neumann functions, in the general case, the first-kind Hankel functions of the n-th order of the complex argument are used, namely, $\boldsymbol{H_n^{(1)}}$ and $\boldsymbol{H_n^{(2)}}$. Then, the generalized Hooke's law according to [20,21] takes the form
$(\mathbf{1} \to \boldsymbol{x}, \mathbf{2} \to \boldsymbol{y}, \mathbf{3} \to \boldsymbol{z})$

$$\begin{aligned}
\sigma_{11k} &= 2\bar{\mu}_k(\partial_1^2\varphi_k + \partial_1\partial_2\psi_{3k} - \partial_1\partial_3\psi_{2k}) + \bar{\lambda}_\kappa\Delta\varphi_\kappa; \\
\sigma_{22k} &= 2\bar{\mu}_k(\partial_2^2\varphi_k + \partial_2\partial_3\psi_{1k} - \partial_1\partial_2\psi_{3k}) + \bar{\lambda}_\kappa\Delta\varphi_\kappa; \\
\sigma_{33k} &= 2\bar{\mu}_k(\partial_3^2\varphi_k + \partial_1\partial_3\psi_{2k} - \partial_2\partial_3\psi_{1k}) + \bar{\lambda}_\kappa\Delta\varphi_\kappa; \\
\sigma_{12k} &= \bar{\mu}_k(2\partial_1\partial_2\varphi_k + \partial_1\partial_3\psi_{1k} - \partial_2\partial_3\psi_{2k} + \partial_2^2\ \psi_{3k} - \partial_1^2\psi_{3k}); \\
\sigma_{13k} &= \bar{\mu}_k(2\partial_1\partial_3\varphi_k + \partial_2\partial_3\psi_{3k} - \partial_1\partial_2\psi_{1k} + \partial_1^2\ \psi_{2k} - \partial_3^2\psi_{2k}); \\
\sigma_{23k} &= \bar{\mu}_k(2\partial_2\partial_3\varphi_k + \partial_1\partial_2\psi_{2k} - \partial_1\partial_3\psi_{3k} + \partial_3^2\ \psi_{1k} - \partial_2^2\psi_{1k}),
\end{aligned} \tag{11}$$

where

$$\Delta = \partial_1^2 + \partial_2^2 + \partial_3^2,\ \partial_1 = \frac{\partial}{\partial x}, \partial_2 = \frac{\partial}{\partial y}, \partial_3 = \frac{\partial}{\partial z}\ .$$

Substituting the expressions in (10) into the boundary conditions in (4) with allowance (11) gives a system of *6n* linear independent equations with complex coefficients with *6n* unknowns. The problem of the propagation of natural waves comes down to the problem of natural values with complex output parameters, i.e.,

$$(C(\omega_R, \omega_I, c_{p\kappa}, c_{s\kappa}, \gamma_p, \gamma_\kappa, D) - \omega^2 A)V = 0,$$

where matrix A, in the general case, has a block-diagonal structure. Matrix C consists of block structure matrices, the elements of which consist of a combination of the Bessel (or Hankel) function of the complex argument

$$C = \begin{pmatrix} c_{1j} & \dots\dots\dots\dots\dots\dots & c_{1n} \\ . & \dots\dots\dots\dots\dots & . \\ c_{nj} & \dots\dots\dots\dots\dots\dots & c_{nn} \end{pmatrix}$$

Here, the elements$\mathrm{c}_{\boldsymbol{1j}}, \dots\dots\dots, \mathrm{c}_{\boldsymbol{nn}}$ –are elements of complex matrices with dimensions (6k x 6k). The conditions for the existence of a nontrivial solution lead to the dispersion equation, which determines the phase velocity of normal waves as an implicit function of the complex frequency and phase velocity

$$C(\omega_R, \omega_I, c_{p\kappa}, c_{s\kappa}, \gamma_p, \gamma_\kappa, D) - \omega^2 A = 0, \tag{12}$$

where k = 1,2... n, and D -is geometric parameter.

It is known that the roots of dispersion equation (12) describe the field of normal waves arising in a viscoelastic mechanical system with WMC (one or several). Complex roots corresponds to damped natural vibrations. If an elastic mechanical system is considered, then $R_{\lambda k} = 0, R_{\mu k} = 0$, and in wave propagation processes, the waves change their amplitude only due to geometric divergence and dispersion, but the complex roots describe leakage waves that exhibit additional exponential attenuation with scattering due to the re-emission of energy from the layer (for a body, not connected to an infinite environment). For these reasons, studying the behaviour of roots on the complex plane of variables ω_I and ω_R, as functions of dimensionless parameters $\gamma_p r_1$, represents an important part of decision research.

The roots of equation (12) can be divided into two classes. The first class includes those that, for $\gamma_p r_1 = 0$, are at a finite distance from the origin. All the other roots belong to the second class. In formulas (7) - (8), the frequency is determined by the expression $\omega = \gamma_p r_1 Im\omega + \gamma_p r\, Re\, \omega$, and then the roots of the first class describe damped oscillations, the spectrum of which begins at a frequency equal to zero. The roots of the second class correspond to vibrations starting from the boundary frequencies [20]

$$\omega_n = \lim_{\gamma_p \to \infty}\left(\gamma_p \,\mathrm{Im}\,\omega \bar{C}_{s1}\right) \approx n\pi C_{s1}\Gamma_{k\mu} r_1^{-1}. \tag{13}$$

The root behaviour under conditions $\gamma_p r_1 << 0$, $\gamma_p r|\omega_I| < 1$ ($R_{\mu k} = 0$) was studied in detail in the work [20]. For a specific example, we consider the propagation and attenuation of natural waves in two-layered cylindrical bodies (when k = 1.2) with a liquid (k = 0) located in an infinite environment (k = 3). The above mechanical system represents a well model [21]. The complex phase velocity is denoted by c_f, and the group velocity is determined by the complex quantity ϑ_{gr}.The real parts of the natural waves express the group velocity, and the imaginary parts - the attenuation coefficients of the group waves. Wave attenuation is determined by the following formula for δ_z

$$\delta_z = \frac{2\pi\omega(\mathrm{Im}\, c_f)}{(\mathrm{Re}\, c_f)^2}.$$

3. Results and discussion

A C ++ program is compiled to calculate the complex roots corresponding to a decaying wave along the cavity lining. The complex roots of equation (12) are found by the Muller method. The frequencies of the corresponding elastic problem are used as an initial approximation. Depending on the ratio of the thickness of the first $\Delta r_1(r_2 - r_1)$ and second $\Delta r_2(r_3 - r_2)$ pipes, waves of a special nature are selected, caused by the imperfect contact ($r = r_3$) of a two-layer body with the environment (k = 3). The roots of equation (12) exist for arbitrary relations between the physical constants of the environment in which there are purely imaginary roots of equation (12) related to the segment $i \le Im\,\omega \le i\delta_0^{-1}$ [21]. Outside the interval, there are real roots of the dispersion equation (for elastic mechanical systems). Furthermore, due to the symmetry of the roots with respect to the real roots, we will consider only the upper half-plane $Im\omega = \omega_I < 0$. Figure 1 shows the dependences of the real and imaginary parts of the frequencies. It can be seen from the figure1 that, in addition to complex roots, in low-frequency regions, there are imaginary roots, which correspond to the aperiodic motions of the mechanical system. The roots moving with growth $\gamma_p r_1$ from point $i\delta_1^{-1}$ down the imaginary axis correspond to normal waves and describe the phase velocity dispersion $\vartheta = r_1 Im\omega$. As a function of the wavenumber γ_p or frequency ω, waves of this nature have been studied in detail in the problems of wave propagation in layered environments in [21]. We present the results for two variants of environmental models called the low-speed ($c_{p1} = 2500 m/sec$ and $c_{p1} = 1500 m/sec$) (figures 1 and 2) and high-speed cases ($c_{p1} =$

Journal of Physics: Conference Series **1921** (2021) 012127 doi:10.1088/1742-6596/1921/1/012127

$5000m/sec$ and $c_{s1} = 2500m/sec$) (figures 3 and 4). In both cases, the qualitative behaviour of the kinematic and dynamic characteristics of the wave turn out to be almost the same.

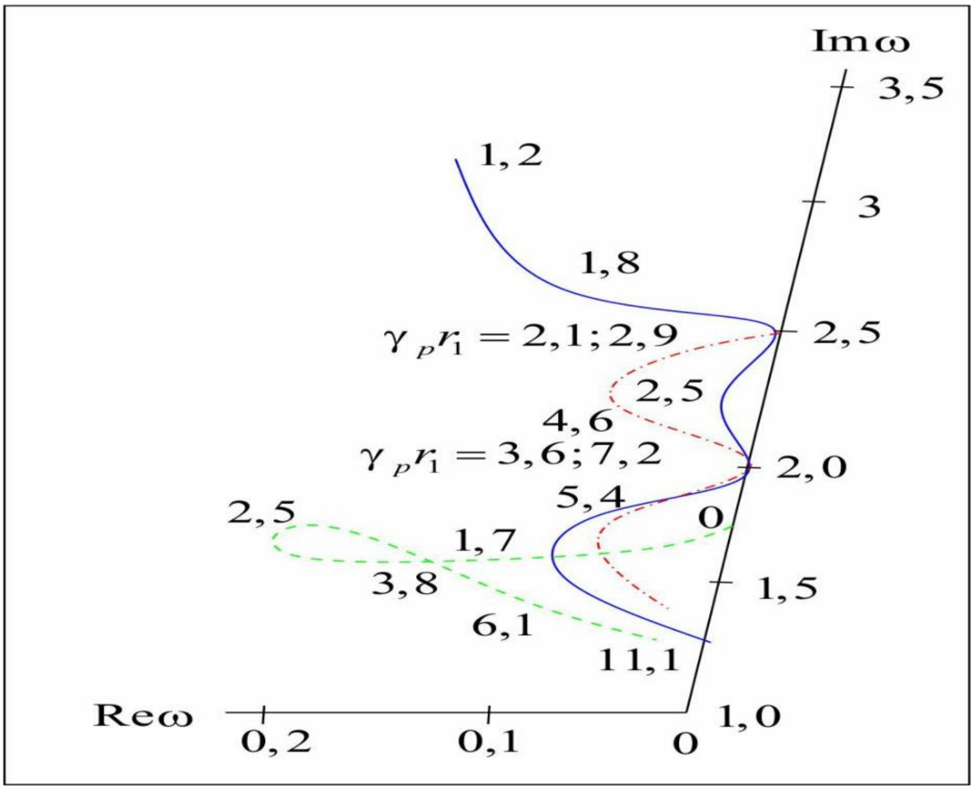

Figure 1. Lines of motion of the roots on the complex plane. 1 - zero mode, 2-first mode, and 3-second mode ($c_{p0} = 5000m/c, \quad \rho_0 = 3г/см^3, c_{p_1} = 4000, \quad c_{s_1} = 2000m/c, \quad \rho_1 = 3г/см^3$)

When studying the dispersion of the phase velocity of a wave over a cylindrical shell, the presence of a boundary frequency is characterized by ω_{gr} in the indicated region starting with which the phase velocity c_f decreases rapidly with increasing cut-off frequencies ω_{gr}. For the considered problem with a slight increase (up to 10%) of the longitudinal wave velocity in the pipe material, the boundary frequency ω_{gr} changes to 50%. As an example of a viscoelastic material, we consider Koltunov-Rzhanitsyn's three-parameter relaxation core [22-24]: $R_к(t) = A_к e^{-\beta_к t}/t^{1-\alpha_к}$. When the velocity of the longitudinal waves decreases from $c_{p2} = 5000m/sec$ to 2500 m/s, the phase velocity changes by no more than 20% ($A_к = 0{,}048; \quad \beta_к = 0{,}05; \quad \alpha_к = 0{,}1$) (figure 5). The group speed ϑ_{gr} in a viscoelastic environment is complex, the real part of which expresses the group velocity of the natural waves, and the imaginary part is the damping coefficient of the group waves.

 1921 (2021) 012127 doi:10.1088/1742-6596/1921/1/012127

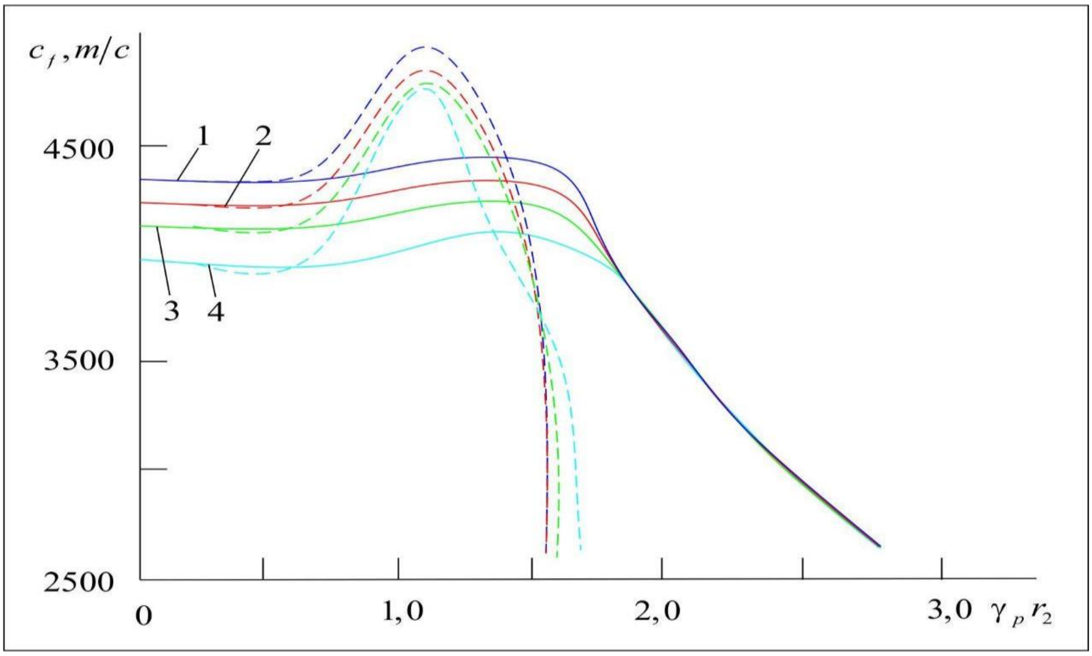

Figure 2. Dispersion of the phase (solid lines) and group (broken lines) velocities of the damped waves along the cylinder with the liquid in the case of a low-speed ($c_{p1} = 2250m$ / sec $c_{s1} = 1500m/sec$) $part1. -\Delta r_1/r_1 = 0{,}2. -0.05{,}3. -0.1{,}4. -03$

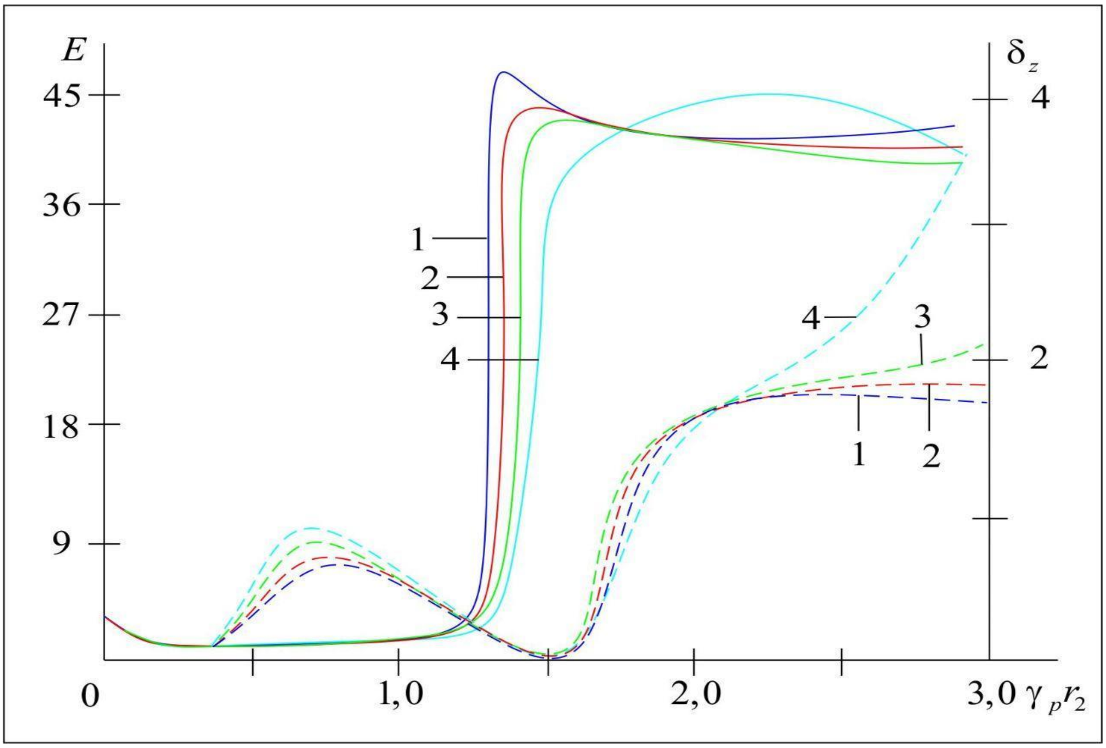

Figure 3. Dispersion of the phase (solid lines) and attenuation (broken lines) waves along the mechanical system in the case of a low-speed ($c_{p1} = 2250m$ / sec $c_{p1} = 1500m/sec$) $part1. -\Delta r_1/r_1 = 0{,}2. -0.05{,}3. -0.1{,}4. -03$

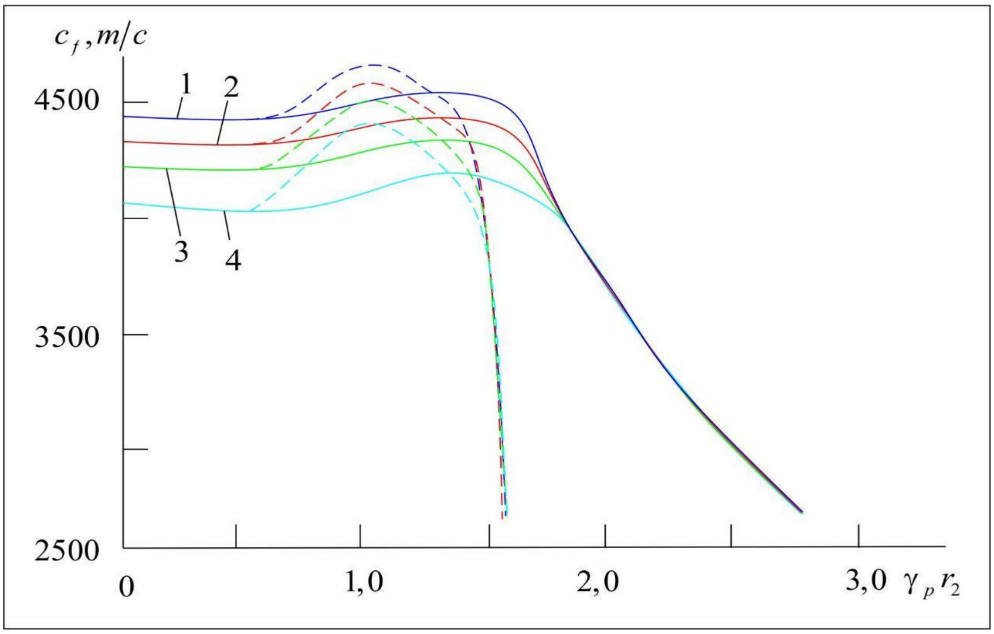

Figure 4. Dispersion of the phase (solid lines) and group (broken lines) velocities of the decaying wave along the cylinder with the liquid in the case of a low-speed
($c_{p1} = 3500 m/sec$ $c_{p1} = 2500 m/sec$) part1. $-\Delta r_1/r_1 = 0$,2. -0.05,3. -0.1,4. -0,3

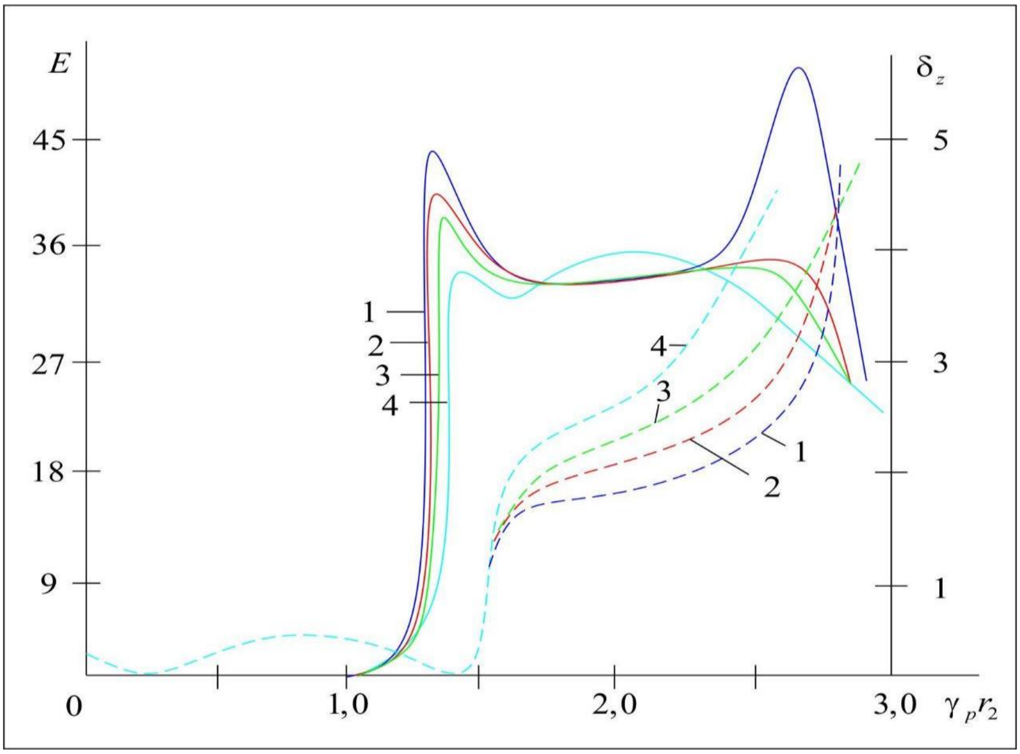

Figure 5. The dispersion of the phase (solid lines) and the attenuation (broken lines) of the wave along the mechanical system in the case of a low-speed ($c_{p1} = 3500 m/sec$, $c_{p1} = 2500 m/sec$), part1. $-\Delta r_1/r_1 = 0$,2. -0.05,3. -0.1,4. -03

At the WMC between the pipe and the outer layers, c_f is practically independent of Δr_1. Additionally, the phase velocity of the wave along the two-layered pipe decreases with increasing Δr_1 at low frequencies and is practically independent of Δr_1 at higher frequencies [25-29]. For example, ϑ_{gr} decreases with increasing $\Delta r_1/r \in [0,0.2]$.

When studying the wave motion of particles of dissipative mechanical systems, the concept of an excitation function is often used, which characterizes the distribution of the wave energy over various frequencies:

$$E(\gamma_p) = \left|\frac{\gamma_p \tilde{\Delta}_1}{\partial \tilde{\Delta}_1 / \partial c_f}\right|. \qquad (14)$$

Here, the determinant $\tilde{\Delta}_1$ is obtained from (12) by replacing the Bessel function in the first column with the Hankel function ($J_0(x)$ to $H_0^{(2)}(x)$ and $J_1(x)$ to $H_1^{(2)}(x)$). The excitation functions sharply increase in the vicinity of the point where the wavenumbers are $\gamma_p r_2$ =1 and 2 [30-34]. Delay energy begins to grow late: $\gamma_p r_2 \geq 1.5$. This represents the presence of a group of intense oscillations with phase velocities and a decrease in the corresponding attenuation coefficient in the interval $0 \leq \gamma_p r_2 \leq 1.4999$, the dynamics and kinematics of which have a weak environment.

The obtained numerical results were compared with the numerical results obtained from the analytical solution [35, 36] for the same values of parameters. The difference in results was up to 12%.

The research results can also be used in the development of a new design for drying cotton seeds [37,38], as well as in improving the energy efficiency and reliability of power supply [39,40].

4. Conclusions

1. A mathematical formulation has been developed for studying the damping properties of natural waves in multilayer cylindrical viscoelastic mechanical systems with weakened mechanical contacts (WMC).
2. An effective technique has been developed for studying the dispersion phenomenon and the damping capabilities of heterogeneous viscoelastic mechanical systems, and the complex roots of transcendent equations were determined by the Muller and Gauss methods.
3. The phase and group velocities of a structurally heterogeneous mechanical system were studied at various geometric and physical-mechanical parameters for the elements of the mechanical system.
4. The following conclusions were obtained:

- there are interference oscillations in viscoelastic mechanical systems with a weakened mechanical contact, which, according to their dispersion properties, have little dependence on the elastic parameters of the environment and are determined by the design features;
- phase wave velocities in heterogeneous viscoelastic mechanical systems with weakened mechanical contacts vary from the wave velocity along the cylinder to the shear wave velocity in the half-space;
- in viscoelastic mechanical systems with a weakened mechanical contact (WMC), the emerging waves have relatively large amplitudes and are characterized by large attenuation; therefore, as the distance between the source and receiver increases, their contribution to the total field decreases.

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Reference

[1] Vestyak A, Gorshkov A G and Tarlakovsky D V 1983 Unsteady interaction of deformable bodies with the environment. *Itogi Nauki i Tekhniki. Mechanics of a deformable solid.* **IIV, 4**, pp 69–148

[2] Davis R M 1961 Stress waves in solids. Publishing house of foreign countries (Moscow: literature)

[3] Safarov I I 1992 *Oscillations and waves in dissipatively heterogeneous media and structures*, (Tashkent: Fan) p 225

[4] Koltunov M A, Mayboroda V P and Zubchaninov V G 1983 *Strength calculations of products from polymeric materials* (Moscow: Engineering) p 447

[5] Bazarov M B, Safarov I I and Shokin Yu M 1996 *Numerical modeling of oscillations of dissipative-heterogeneous and homogeneous mechanical systems*. (Novosibirsk: Siberian Branch of the Russian Academy of Sciences) p

[6] White J E 1986 Excitation and propagation of seismic waves (Moscow: Nedra)

[7] Biot M A 1952 Propagation of elastic waves in cylindrical bore containing a fluid. *J Appl Phys* **23**, pp 997-1005

[8] Ionov A M and Maximov G A 1996 Propagation of tube waves generated by an external source in layered permeable rocks. *Geophys. J. Int.,* **124**, pp 888-906

[9] Ionov A M, Kozlov O B and Maksimov G A 1995 An algorithm for calculating the waves in a pipe generated by an external pulsed source in a well in an elastically layered environment *Acoustic Journal* **41 4** pp 603-612

[10] Ionov A M and Maksimov G A 1999 About excitation of a hydraulic wave in a well by an external seismic source *Acoustic Journal* **45** 3 pp 354-362

[11] Ziatdinov C P 2007 *Study of the geophysical properties of the near-wellbore space and the reservoir by the method of seismic sounding.* (St. Petersburg: St. Petersburg State University)

[12] Krauklis P V and Krauklis J I A 2002 Slow wave in a two-layer acoustic waveguide located in an elastic environment *Notes of the POMI scientific seminar* **285** pp 109-116

[13] Korneev V 2008 Slow waves in fractures filled with viscous fluid. *Geophysics* **73 1** pp 1-7

[14] Ionov A M 2007 Stanley wave generation by an incident P-wave propagating in the surrounding formation across a fluid-filled fracture *Geophysical prospecting* **55** pp 71-82

[15] Patzek T W and De A 2000 Lossy transmission line model of hydro fractured well dynamics *J. of Petrolium Science and Engineering* **25** pp 59-77

[16] Henry F 2005 *Characterization of borehole fractures by the body and interface waves. Section of Applied Geophysics, Faculty of Civil Engineering and Geotechnology*. (Delft University of Technology, Delft, The Netherlands)

[17] Maiboroda V P, Troyanovskii I E, Safarov I I, Vazagashvili M G and Katalymova I V 1992 Wave attenuation in an elastic medium. *Journal of Soviet Mathematics* **60(2)** pp 1379–1382

[18] Maiboroda V P, Safarov I I and Troyanovskii I E 1983 Free and forced oscillations of a system of rigid bodies on inhomogeneous viscoelastic snubbers. *Soviet Machine Science (English Translation of Mashinovedenie)* **3** pp 25–31

[19] Derov A V, Maximov G A, Lazarkov M Yu, Kashtan B M and Bakulin A N 2008 Verification of wave field excitation in thin fluid-filled fracture of finite size and its interaction with a borehole. *St. Petersburg, Days of Diffraction June* p 3-6

[20] Maksimov G A, Ortega E, Podyachev E B and Chillemi M R 2004 Rayleigh wave attenuation due to its scattering on two-dimensional irregularities of the walls of an empty well. *Akust. Journal*, **50 5** pp 682-692

[21] Mikhasev G I and Tovstik P E 2020 *Localized Dynamics of Thin-Walled Shells.* (New York: Chapman and Hall)

[22] Mikhasev G I and Altenbach H 2019 *Thin-walled Laminated Structures Buckling. Vibrations and their Suppression.* (Springer Cham Switzerland)
[23] Mikhasev G I, Mlechka I and Maevskaya S 2018 Effect of Magnetic Field on Free and Forced Vibrations of Laminated Cylindrical Shells Containing Magnetorheological Elastomers. In: Altenbach H., Carrera E., Kulikov G. (eds) Analysis and Modeling of Advanced Structures and Smart Systems. *Advanced Structured Materials, 81 Springer, Singapore* (DOI https://doi.org/10.1007/) pp 978-981 (10-6895-9_)
[24] Mikhasev G I 2017 Some Problems on Localized Vibrations and Waves in Thin Shells. In: Shell-like Structures *Advanced Theories and Applications, Springer* pp 149-209 (DOI 10.1007 / 978-3-319-42277-0_4).
[25] Mikhasev G, Bosiakov S, Petrova L, Maisyuk M and Yurkevich K 2016 Assessment of Eigenfrequencies of the Middle Ear Oscillating System: Effect of the Cartilage Transplant. In book: *Dynamical Systems: Theoretical and Experimental Analysis* (ed .: J. Awrejcewicz), **181** (Springer) pp 243-254
[26] Loginov P V, Salikhova Z R and Sultanov K S 2019 Experimental and Theoretical Method for Determining Mechanical Characteristics of Soils under Dynamic Loads *Mechanics of Solids* **54** pp 915–928 https://doi.org/10.3103/S0025654419060074
[27] Safarov I I, Homidov F F, Rakhmonov B S and Almuratov S N 2020 Seismic vibrations of complex relief of the surface of the naryn canyon (on the Norin river in Kyrgyzstan) during large-scale underground explosions. *Journal of Physics: Conference Series*, **1706(1),** 012125 (doi:10.1088/1742-6596/1706/1/012125)
[28] Safarov I I, Teshaev M Kh and Boltayev Z I 2020 Propagation of linear waves in multilayered structural - Inhomogeneous cylindrical shells *Journal of Critical Reviews* **7(12)** pp 893-904 (http://jcreview.com/)
[29] Teshaev M K, Safarov I I, Kuldashov N U, Ishmamatov M R and Ruziev T R 2019 On the Distribution of Free Waves on the Surface of a Viscoelastic Cylindrical Cavity *Journal of Vibrational Engineering and Technologies* **8(4)** pp 579-585 (https://link.springer.com/ article/ 10.1007/s42417-019-00160-x)
[30] Safarov I I,Teshaev M, Toshmatov E, Boltaev Z and Homidov F 2020 Torsional vibrations of a cylindrical shell in a linear viscoelastic medium *IOP Conference Series Materials Science and Engineering* **883**(1) 012190 (https://iopscience.iop.org/issue/1757-899X/883/1)
[31] Mirsaidov M M, Safarov I I, Teshaev M K and Boltayev Z I 2020 Dynamics of structural - Inhomogeneous coaxial-multi-layered systems cylinder-shells *Journal of Physics: Conference Series* 1706(1), 012033 (doi:10.1088/1742-6596/1706/1/012033)
[32] Teshaev M Kh, Safarov I I and Mirsaidov M M 2019 Oscillations of multilayer viscoelastic composite toroidal pipes *Journal of the Serbian Society for Computational Mechanics* **13 2** pp 104-115 (DOI: 10.24874 / jsscm.2019.13.02.02.08)
[33] Mirsaidov M, Safarov I I and Teshaev M 2020 Dynamic instability of vibrations of thin-wall composite curvorine viscoelastic tubes under the influence of pulse pressure. *E3S Web of Conferences* **164**, **14013** (DOI: 10.1051 / e3sconf / 202016414013)
[34] Boltaev Z, Safarov I I and Razokov T 2020 Natural vibrations of spherical inhomogeneity in a viscoela-stic medium *International Journal of Scientific and Technology Research* **9**(**1**) pp 3674–3680
[35] Durdiev D K and Totieva Z D 2020 The problem of determining the one-dimensional kernel of viscoelasticity equation with a source of explosive type *Journal of Inverse and Ill-Posed Problems* **28(1**) pp. 43-52.
[36] Durdiev D K and Rahmonov A A 2018 Inverse Problem for A System of Integro-Differential Equations for SH Waves in A Visco-Elastic Porous Medium: Global Solvability *Theoretical and Mathematical Physics(*Russian Federation*)* **195(3**) pp 923-937

[37] Raxmanov X K 2019 Improvement of Equipment and Technology of Drying of the Cotton Mass and its Technological Assessment on the Basis of its Thermal Properties *International Journal of Advanced Research in Science, Engineering and Technology* **6 5** (www.ijarset.com)

[38] Raxmanov X K 2020 Development of a New Design for Drying Cotton Seeds with Purpose of Efficient Use of Heat *International Journal of Advanced Research in Science, Engineering and Technology* **7 4** (www.ijarset.com)

[39] Sadullayev N N, Safarov A B and Nematov S N et al 2020 Opportunities and Prospects for the Using Renewable Energy Sources in Bukhara Region. *Appl Sol Energy* **56 pp**291–300 https://doi.org/10.3103/S0003701X20040106

[40] Sadullaev N N, Mukhamedkhanov U T, Nematov SH N and Sayliev F O 2020 Increasing Energy Efficiency and Reliability of Electric Supply of Low Power Consumers. *International Journal of Engineering Trends and Technology* **68(12)** pp 43-47. doi:10.14445/22315381/IJETT-V68I12P208