UDC 519.644.3

OPTIMAL QUADRATURE FORMULAS FOR HYPERSINGULAR INTEGRALS IN THE SOBOLEV SPACE

[1,2,3]***Akhmedov D.M.***, [3]***Avezov A.Kh.***

d.akhmedov@mathinst.uz

[1]V.I.Romanovskiy Institute of Mathematics, Uzbekistan Academy of Sciences, 9, University street, Tashkent 100174, Uzbekistan

[2]National University of Uzbekistan named after Mirzo Ulugbek, University street, 4, Tashkent 100174, Uzbekistan

[3]Bukhara State University, 11 Muhammad Iqbal Street, Bukhara 705018, Uzbekistan

The solution of the Cauchy problem for partial differential equations of hyperbolic type J. Hadamard led to singular integrals of a special form. Later they were called integrals in the sense of Hadamard, or Hadamard integrals. In addition to equations of the hyperbolic type, Hadamard integrals are widely used in the theory of elasticity, electrodynamics, aerodynamics, and a number of other important areas of mechanics and mathematical physics. The exact calculation of the Hadamard integrals is possible only in exceptional cases, so there is a need to develop approximate methods for calculating. In the present paper, we develop an optimal algorithm for the approximate calculation of the Hadamard integral for $p = 3$. Here we are engaged in finding the analytical form of the coefficients of the optimal quadrature formula.

Keywords: Optimal quadrature formulas,the extremal function, Sobolev space,optimal coefficients, Hadamard type singular integral.

Citation: Akhmedov D.M., Avezov A.Kh. 2023. Optimal Quadrature Formulas for Hypersingular Integrals in the Sobolev space. *Problems of Computational and Applied Mathematics.* 3(49): 1-10.

1 Introduction

There are a large number of problems, both in physics and technology, and directly in various sections of mathematics, the solution of which requires to calculate hypersingular integrals. Since direct computations of such integrals is possible only in exceptional cases, it becomes necessary to develop new approximate methods. This paper is devoted to construction of an approximate method for calculating hypersingular integrals. Particular attention is paid to investigation of connection between methods for calculating singular and hypersingular integrals.

We consider the following hypersingular integral

$$Hf = \int_a^b \frac{f(x)\mathrm{d}x}{(x-t)^p}, \quad a < t < b, p = 2, 3, ... \tag{1}$$

We construct an optimal quadrature formula for approximate integration of integral (1) for the case $a = 0, b = 1, p = 3$. In this case integral (1) becomes as follows

$$I(f,t) = \int_0^1 \frac{f(x)\mathrm{d}x}{(x-t)^3}, \quad 0 < t < 1, \tag{2}$$

where its kernel has a higher order singularity than the demension of the integral. A sufficient condition for f to be finite-part integrable is that the derivative of f is a Hölder continuous function.

2 Statement of the problem

We reduce integral (2) to the form integrating by parts

$$\int_0^1 \frac{f(x)\mathtt{d}x}{(x-t)^3} = -\frac{1}{2}\left(\frac{f(1)}{(1-t)^2} - \frac{f(0)}{t^2}\right) - \frac{1}{2}\left(\frac{f'(1)}{1-t} + \frac{f'(0)}{t}\right) + \int_0^1 \frac{f''(x)\mathtt{d}x}{2(x-t)}, \quad 0 < t < 1. \tag{3}$$

Now it is enough to construct an optimal quadrature formula for numerical integration of the Cauchy type singular integral which is in the right hand side of (2). For this first we introduce denotation $f''(x) = \varphi(x)$. Then we consider the following quadrature formula

$$\int_0^1 \frac{\varphi(x)}{x-t}\mathtt{d}x \cong \sum_{\beta=0}^{N} C[\beta]\varphi(x_\beta), 0 < t < 1, \tag{4}$$

in the Sobolev space $L_2^{(2)}(0,1)$. This space is a Hilbert space of classes of all real valued functions φ defined in the interval $[0,1]$ that differ by a polynomial of degree first and square integrable with derivative of order two. Here $C[\beta]$ are the coefficients, x_β are the nodes of the quadrature formula, N is a natural number.

The following difference is called *the error* of quadrature formula (4):

$$(\ell, \varphi) = \int_0^1 \frac{\varphi(x)}{x-t}\mathtt{d}x - \sum_{\beta=0}^{N} C[\beta]\varphi(x_\beta) = \int_{-\infty}^{\infty} \ell(x)\varphi(x)\mathtt{d}x,$$

where

$$\ell(x) = \frac{\varepsilon_{[0,1]}(x)}{x-t} - \sum_{\beta=0}^{N} C[\beta]\delta(x - x_\beta), \tag{5}$$

$\varepsilon_{[0,1]}(x)$ is the indicator of the interval $[0,1]$, δ is the Dirac delta-function, $\ell(x)$ is the error functional of quadrature formula (4).

Since the functional (5) defined in the space $L_2^{(2)}(0,1)$ [3], then we have

$$(\ell, x^\alpha) = 0, \qquad \alpha = 0, 1. \tag{6}$$

The main aim of the present paper is to construct optimal quadrature formulas in the sense of Sard of the form (4) in the space $L_2^{(2)}(0,1)$ by the Sobolev method for approximate integration of the Cauchy type singular integral. This means to find the coefficients $C[\beta]$ which satisfy the following equality

$$\|\mathring{\ell}|L_2^{(m)*}\| = \inf_{C[\beta]} \|\ell|L_2^{(m)*}\|. \tag{7}$$

Thus, in order to construct optimal quadrature formulas in the form (4) in the sense of Sard we have to consequently solve the following problems.

Problem 1. Find the norm of the error functional (5) of the quadrature formula (4) in the space $L_2^{(2)*}(0,1)$.

Problem 2 Find the coefficients $C[\beta]$ which satisfy the equality (7).

In the Hilbert spaces one can construct optimal quadrature formulas, optimal interpolation formulas, and splines using the Sobolev method. Applying this method in the different Hilbert spaces optimal quadrature formulas, interpolation formulas, and splines were constructed, for example, in the works [8–10]

In the works [1,2,4] for the norm of the error functional the following form was obtained

$$\|\ell\|^2 = \left[\sum_{\beta=0}^{N} \sum_{\gamma=0}^{N} C[\beta]C[\gamma] \frac{|x_\beta - x_\gamma|^3}{12} - 2 \sum_{\beta=0}^{N} C[\beta] \int_0^1 \frac{|x-x_\beta|^3}{12(x-t)} dx + \int_0^1 \int_0^1 \frac{|x-y|^3}{12(x-t)(y-t)} dx dy \right]. \quad (8)$$

Thus Problem 1 is solved for quadrature formulas of the form (4) in the space $L_2^{(2)}(0,1)$.

3 The main results

Assume that the nodes x_β of the quadrature formula (4) are fixed. The error functional (5) satisfies conditions (6). The norm of the error functional $\ell(x)$ is a multidimensional function with respect to the coefficients $C[\beta]$ $(\beta = \overline{0,N})$. For finding the point of the conditional minimum of the expression (8), under the conditions (6), we apply the Lagrange method.

We denote $\mathbf{C} = (C[0], C[1], ..., C[N])$ and $\lambda = (\lambda_0, \lambda_1)$. Consider the function

$$\Psi(\mathbf{C}, \lambda) = \|\ell\|^2 - 2\Big(\lambda_0(\ell(x), 1) + \lambda_1(\ell(x), x)\Big).$$

Equating to zero the partial derivatives of $\Psi(\mathbf{C}, \lambda)$ by $C[\beta]$ $(\beta = \overline{0,N})$ and λ_0, λ_1, we get the following system of linear equations

$$\sum_{\gamma=0}^{N} C[\gamma] \frac{|x_\beta - x_\gamma|^3}{12} + \lambda_0 + \lambda_1 \cdot x_\beta = f(x_\beta), \quad (9)$$

$$\beta = 0, 1, 2, ..., N,$$

$$\sum_{\gamma=0}^{N} C[\gamma] x_\gamma^\alpha = g_\alpha, \ \ \alpha = 0, 1, \quad (10)$$

where

$$f(x_\beta) = \int_0^1 \frac{|x - x_\beta|^3}{12(x-t)} dx, \quad (11)$$

$$g_\alpha = \int_0^1 \frac{x^\alpha}{x-t} dx, \quad (12)$$

and $C[\gamma]$, $\gamma = 0, 1, ..., N$ and λ_α, $\alpha = 0, 1$ are unknowns.

The system (9)–(10) has a unique solution and this solution gives the minimum to $\|\ell\|^2$ under the conditions (6). The uniqueness of the solution of such type of systems was discussed in [3, 10].

We give the algorithm for solution of system (9)-(10) when the nodes x_β are equally spaced, i.e., $x_\beta = h\beta$, $h = \frac{1}{N}$, $N \geqslant 1$. Here we use similar method suggested by S.L. Sobolev [3] for finding the coefficients of optimal quadrature formulas in the Sobolev space $L_2^{(2)}(0,1)$.

Suppose that $C[\beta]=0$ when $\beta<0$ and $\beta>N$. Using the definition of convolution, we rewrite system (9)-(10) in the following form:

$$G_2(h\beta)*C[\beta]+P_1(h\beta)=f(h\beta),\quad \beta=0,1,...,N, \tag{13}$$

$$\sum_{\beta=0}^{N}C[\beta]\cdot(h\beta)^{\alpha}=g_{\alpha},\quad \alpha=0,1, \tag{14}$$

where

$$G_2(h\beta)\;=\;\frac{h\beta^3\mathtt{sgn}(h\beta)}{12},$$

$P_1(x)$ is a polynomial of degree 1, $\mathtt{sgn}(h\beta)$ is the signum function.

Thus we have the following problem.

Problem 3. Find the discrete function $C[\beta]$ and polynomial $P_1(h\beta)$ of degree first which satisfy the system (13)-(14).

Further we investigate Problem 3. Instead of $C[\beta]$ we introduce the following functions

$$v(h\beta)\;=\;G_2(h\beta)*C[\beta], \tag{15}$$

$$u(h\beta)\;=\;v\,(h\beta)+p_0+p_1\cdot(h\beta). \tag{16}$$

In such statement the coefficients $C[\beta]$ are expressed by the function $u(h\beta)$, i.e. taking into account

$$hD_2(h\beta)*u(h\beta)=\delta(h\beta),\text{where}$$

$$D_2(h\beta)=\frac{3!}{h^4}\begin{cases}3!\sqrt{3}q^{|\beta|}, & |\beta|\geqslant 2,\\ 19-12\sqrt{3}, & |\beta|=1,\\ 6\sqrt{3}-8, & \beta=0,\end{cases} \tag{17}$$

here $q=\sqrt{3}-2$ and (16) , for the coefficients we have

$$C[\beta]=hD_2(h\beta)*u(h\beta). \tag{18}$$

Thus, if we find the function $u(h\beta)$, then the coefficients $C[\beta]$ will be found from equality (18). To calculate the convolution (18) it is required to find the representation of the function $u(h\beta)$ for all integer values of β. From equality (13) we get that $u(h\beta)= =f_2(h\beta)$ when $h\beta\in[0,1]$, i.e. $\beta=0,1,...,N$. Now we need to find the representation of the function $u(h\beta)$ when $\beta<0$ and $\beta>N$.

Since $C[\beta]=0$ when $h\beta\notin[0,1]$ then $C[\beta]=hD_2(h\beta)*u(h\beta)=0,\;\; h\beta\notin[0,1]$.

Now we calculate the convolution $v(h\beta)=G_2(h\beta)*C[\beta]$ when $\beta\leqslant 0$ and $\beta\geqslant N$. Suppose $\beta\leqslant 0$, then taking into account that $G_2(h\beta)=\frac{|h\beta|^3}{12}$ and equality (14), we have

$$v(h\beta)=-\frac{1}{12}(h\beta)^3g_0+\frac{1}{4}(h\beta)^2g_1-(h\beta)\cdot p_1^{(0)}-p_0^{(0)}, \tag{19}$$

Similarly, in the case $\beta\geqslant N$ for the convolution $v(h\beta)=G_2(h\beta)*C[\beta]$ we obtain

$$v(h\beta)=\frac{1}{12}(h\beta)^3g_0-\frac{1}{4}(h\beta)^2g_1+(h\beta)\cdot p_1^{(0)}+p_0^{(0)}. \tag{20}$$

We denote

$$p_1^- = p_1 - p_1^{(0)}, \qquad p_0^- = p_0 - p_0^{(0)} \tag{21}$$
$$p_1^+ = p_1 + p_1^{(0)}, \qquad p_0^+ = p_0 + p_0^{(0)}. \tag{22}$$

Taking into account (16), (19) and (20) we get the following problem

Problem 4. Find the solution of the equation

$$hD_2(h\beta) * u(h\beta) = 0, \qquad h\beta \notin [0,1] \tag{23}$$

having the form:

$$u(h\beta) = \begin{cases} -\frac{1}{12}(h\beta)^3 g_0 + \frac{1}{4}(h\beta)^2 g_1 + p_1^-(h\beta) + p_0^-, & \beta \leqslant 0, \\ f_m(h\beta), & 0 \leqslant \beta \leqslant N, \\ \frac{1}{12}(h\beta)^3 g_0 - \frac{1}{4}(h\beta)^2 g_1 + p_1^+(h\beta) + p_0^+, & \beta \geqslant N. \end{cases} \tag{24}$$

Here p_0^-, p_1^-, p_0^+ and p_1^+ are unknown polynomials of degree first with respect to $h\beta$.

If we find p_0^-, p_1^-, p_0^+ and p_1^+ then from (21), (22) we have

$$p_1 = \frac{1}{2}(p_1^- + p_1^+), \quad p_0 = \frac{1}{2}(p_0^- + p_0^+), \tag{25}$$
$$p_1^{(0)} = \frac{1}{2}(p_1^+ - p_1^-), \quad p_0^{(0)} = \frac{1}{2}(p_0^+ - p_0^-). \tag{26}$$

Unknowns p_0^-, p_1^-, p_0^+ and p_1^+ can be found from equation (23), using the function $D_2(h\beta)$ defined by (17). Then we obtain explicit form of the function $u(h\beta)$ and from (18) we find the coefficients $C[\beta]$. Furthermore from (25) we get $P_1(h\beta)$.

Thus Problem 4 and respectively Problems 3 will be solved.

Then, using the above algorithm, we obtain explicit formulas for coefficients of the optimal quadrature formula (4). It should be noted that the quadrature formula (4) is exact for linear function.

The following holds

Theorem 1. Coefficients of the optimal quadrature formulas (4), with equally spaced nodes in the space $L_2^{(2)}(0,1)$, have the following form

$$\begin{aligned} C[0] \;=\; & \frac{6}{h^3}\Bigg[\frac{g_0}{12}\left(h^3 - 3q^N(h^2 + h(q+2))\right) + \frac{g_1 q^N}{4}(h^2 + 2h(q+2)) + p_1^- h(q+1) + \\ & + f(0)(3q+2) - f(h)(12q+5) - q^N(3f(1)(q+1) + p_1^+ h(q+2)) + \\ & + 6(q+2)\sum_{\gamma=2}^{N} q^\gamma f(h\gamma)\Bigg], \end{aligned}$$

$$
\begin{aligned}
C[\beta] \;=\; & \frac{6}{h^3}\Bigg[6(q+2)\sum_{\gamma=0}^{\beta-2} q^{\beta-\gamma} f(h\gamma) - (12q+5)\Big(f(h(\beta-1)) + f(h(\beta+1))\Big) \\
& +(6q+4)f(h\beta) + 6(q+2)\sum_{\gamma=\beta+2}^{N} q^{\gamma-\beta} f(h\gamma) + \\
& +\frac{g_1}{4}\Big(q^{N-\beta}(2h(q+2)+h^2) - q^{\beta}h^2\Big) + q^{\beta}\Big(p_1^- h(q+2) - 3f(0)\times \\
& \times(q+1)\Big) - q^{N-\beta}\Big(3f(1)(q+1) + \frac{g_0}{4}(h(q+2)+h^2) + \\
& +p_1^+ h(q+2)\Big)\Bigg], \qquad \beta = \overline{1, N-1},
\end{aligned}
$$

$$
\begin{aligned}
C[N] \;=\; & \frac{6}{h^3}\Bigg[-\frac{g_0}{12}(3h(q+1) - h^3) + \frac{g_1}{4}(2h(q+1) - q^N h^2) + q^N\Big(p_1^- h(q+2) - \\
& -3f(0)(q+1)\Big) - p_1^+ h(q+1) + f(1)(3q+2) - f(1-h)\times \\
& \times(12q+5) + 6(q+2)\sum_{\gamma=0}^{N-2} q^{N-\gamma} f(h\gamma)\Bigg].
\end{aligned}
$$

where

$$p_1^- = \frac{\Delta_1}{\Delta}, \tag{27}$$

$$p_1^+ = \frac{\Delta_2}{\Delta}, \tag{28}$$

$$p_0^- = f(0,t), \tag{29}$$

$$p_0^+ = f(1,t) - \frac{1}{12}g_0 + \frac{1}{4}g_1 - \frac{\Delta_2}{\Delta}, \tag{30}$$

$$
\begin{aligned}
\Delta &= B_2^2 - A_2^2,\\
\Delta_1 &= A_2\Big[-F_1 - \tfrac{1}{12}g_0(B_1+3B_2+3B_3) + \tfrac{1}{4}g_1(B_1+2B_2+B_3-A_3)-\\
&\quad -A_1 f(0) - B_1\Big(f(1) - \tfrac{1}{12}g_0 + \tfrac{1}{4}g_1\Big)\Big]-\\
&\quad -B_2\Big[-F_2 - \tfrac{1}{12}g_0(A_1+3A_2+3A_3) + \tfrac{1}{4}g_1(A_1+2A_2+A_3-B_3)-\\
&\quad -B_1 f(0) - A_1\Big(f(1) - \tfrac{1}{12}g_0 + \tfrac{1}{4}g_1\Big)\Big],\\
\Delta_2 &= -A_2\Big[-F_2 - \tfrac{1}{12}g_0(A_1+3A_2+3A_3) + \tfrac{1}{4}g_1(A_1+2A_2+A_3-B_3)-\\
&\quad -B_1 f(0) - A_1\Big(f(1) - \tfrac{1}{12}g_0 + \tfrac{1}{4}g_1\Big)\Big]+\\
&\quad +B_2\Big[-F_1 - \tfrac{1}{12}g_0(B_1+3B_2+3B_3) + \tfrac{1}{4}g_1(B_1+2B_2+B_3-A_3)-\\
&\quad -A_1 f(0) - B_1\Big(f(1) - \tfrac{1}{12}g_0 + \tfrac{1}{4}g_1\Big)\Big],\\
F_1 &= 6(q+2)\sum_{\gamma=1}^{N} q^{\gamma+1} f(h\gamma) - (12q+5)f(0),\\
F_2 &= 6(q+2)\sum_{\gamma=0}^{N-1} q^{N+1-\gamma} f(h\gamma) - (12q+5)f(1),
\end{aligned}
$$

$$
\begin{aligned}
f(h\beta) &= \frac{1}{12}\Big(-\frac{11}{3}(h\beta)^3 + (5t+3)(h\beta)^2 - (2t^2+3t+1,5)(h\beta) +\\
&\quad +(t^2+\frac{t}{2}+\frac{1}{3}) + (t-h\beta)^3(-2\ln|h\beta-t| + \ln(t-t^2))\Big),\\
g_0 &= \int_0^1 \frac{dx}{x-t} = \ln\frac{1-t}{t},\\
g_1 &= \int_0^1 \frac{x}{x-t}dx = 1 + t\ln\frac{1-t}{t},
\end{aligned}
$$

$$
\begin{aligned}
A_1 &= 3q+2, & B_1 &= 3q^N(3q+1)\\
A_2 &= h(2q+1), & B_2 &= hq^N(2q+1)\\
A_3 &= h^2 q, & B_3 &= -h^2 q^{N+1},
\end{aligned}
$$

$q = \sqrt{3}-2$.

Proof. From (24) with $\beta = 0$ and $\beta = N$ we immediately obtain (29) and (30), i.e.

$$p_0^- = f(0), \tag{31}$$

$$p_0^+ = f(1) - \frac{1}{12}g_0 + \frac{1}{4}g_1 - p_1^+. \tag{32}$$

From (23), using (17) and (24) for $\beta = -1$ and $\beta = N+1$, we have

$$-p_1^-\sum_{\gamma=1}^{\infty}D(h\gamma-h)(h\gamma)+p_1^+\sum_{\gamma=1}^{\infty}D(h(N+\gamma)+h)(h\gamma)=$$
$$=-\sum_{\gamma=0}^{N}D(h\gamma+h)f(h\gamma,t)-\frac{1}{12}g_0\sum_{\gamma=1}^{\infty}D(h\gamma-h)(h\gamma)^3-$$
$$-\frac{1}{4}g_1\sum_{\gamma=1}^{\infty}D(h\gamma-h)(h\gamma)^2-\frac{1}{12}g_0\sum_{\gamma=1}^{\infty}D(h(N+\gamma)+h)(1+h\gamma)^3+$$
$$+\frac{1}{4}g_1\sum_{\gamma=1}^{\infty}D(h(N+\gamma)+h)(1+h\gamma)^2-\left(f(1)-\frac{1}{12}g_0+\frac{1}{4}g_1\right)\times$$
$$\times\sum_{\gamma=1}^{\infty}D(h(N+\gamma)+h)-f(0)\sum_{\gamma=1}^{\infty}D(h\gamma-h), \tag{33}$$

$$-p_1^-\sum_{\gamma=1}^{\infty}D(h(N+\gamma)+h)(h\gamma)+p_1^+\sum_{\gamma=1}^{\infty}D(h\gamma-h)(h\gamma)=$$
$$=-\sum_{\gamma=0}^{N}D(h(N+1)-h\gamma)f(h\gamma,t)-\frac{1}{12}g_0\sum_{\gamma=1}^{\infty}D(h(N+\gamma)+h)(h\gamma)^3-$$
$$-\frac{1}{4}g_1\sum_{\gamma=1}^{\infty}D(h(N+\gamma)+h)(h\gamma)^2-\frac{1}{12}g_0\sum_{\gamma=1}^{\infty}D(h\gamma-h)(1+h\gamma)^3+$$
$$+\frac{1}{4}g_1\sum_{\gamma=1}^{\infty}D(h\gamma-h)(1+h\gamma)^2-\left(f(1)-\frac{1}{12}g_0+\frac{1}{4}g_1\right)\times$$
$$\times\sum_{\gamma=1}^{\infty}D(h\gamma-h)-f(0)\sum_{\gamma=1}^{\infty}D(h(N+\gamma)+h). \tag{34}$$

Thus, for the unknowns p_1^-, p_1^+, p_0^-, p_0^+ we have obtained a system of linear equations (31)-(34). Solving this system, we have (27)-(30). This means that we have obtained an explicit form of the function $u(h\beta)$.

Further, using (17) and (24), from (18) calculating the convolution $hD_2(h\beta)*u(h\beta)$ for $\beta=\overline{0,N}$, respectively, we obtain results of the theorem. Theorem 1 is proved. □

Remark 1. So, the approximate calculation of equality (3) is as follows

$$I(f,t)\cong-\frac{1}{2}\left(\frac{f(1)}{(1-t)^2}-\frac{f(0)}{t^2}\right)-\frac{1}{2}\left(\frac{f'(1)}{1-t}+\frac{f'(0)}{t}\right)+\frac{1}{2}\sum_{\beta=0}^{N}C[\beta]f''(x_\beta).$$

4 Conclusion

In the present paper, in the Sobolev space $L_2^{(2)}(0,1)$ we constructed the optimal quadrature formula for approximate solution of hypersingular integrals with Cauchy kernel. Here we found analytical forms for coefficients of the constructed optimal quadrature formulas. We applied these coefficients to approximate calculation of the Hadamard type singular integral. We showed that hypersingular integrals can be calculated with higher accuracy using the optimal quadrature formulas which are constructed based on Sobolev method.

5 Acknowledgments

The author is very thankfull to professor Abdullo R. Hayotov for discussion the results of the present work.

References

[1] Akhmedov D. M., Hayotov A. R. Shadimetov Kh. M.*Optimal quadrature formulas with derivatives for Cauchy type singular integrals* Applied Mathematics and Computation (2018) vol.317, pp.150-159.

[2] Shadimetov Kh.M. *On an extremal function of quadrature formulas* Reports of Uzbekistan Academy of Sciences. (1995) no.9-10, pp.3-5.

[3] Sobolev S.L. *Introduction to the Theory of Cubature Formulas.* Nauka, Moscow, (1974).

[4] Shadimetov Kh.M.*Reducing the construction of composite optimal quadrature formulas to the solution of difference schemes* Reports of Uzbekistan Academy of Sciences.(1998) no.7, pp.3-6.

[5] Shadimetov Kh.M. *Weight optimal quadrature formulas* Problem of Computational and applied mathematics (1978) no. 51, pp. 169-177.

[6] Shadimetov Kh.M. *On an explicit form of a discrete analog of the differential operator of order* $2m$. Reports of Uzbekistan Academy of Sciences (1996) no. 9, pp.5-7.

[7] Shadimetov Kh.M. *Optimal quadrature formulas of Euler-Maclaurin type.* Reports of Uzbekistan Academy of Sciences. (1999) no.10, pp.6-9.

[8] Boltaev N. D., Hayotov A. R., Shadimetov Kh. M. *Construction of optimal quadrature formula for numerical calculation of Fourier coefficients in Sobolev space* $L_2^{(1)}$ American Journal of Numerical Analysis (2016) no. 4, pp.1-7.

[9] Babaev S. S., Hayotov A. R., Khayriev U. N. *On an optimal quadrature formula for approximation of Fourier integrals in the space* $W_2^{(1,0)}$ Uzbek Mathematical Journal (2020) no.2, pp. 23-36.

[10] Shadimetov Kh. M. *Optimal lattice quadrature and cubature formulas in Sobolev spaces.* Tashkent, Uzbekistan (2019).

[11] Shadimetov Kh. M., Hayotov A. R. Akhmedov D. M. *Optimal quadrature formulas for Cauchy type singular integrals in Sobolev space.* Applied Mathematics and Computation (2015) vol.263, pp.302-314.

Received May 3, 2024

УДК 519.644.3

ОПТИМАЛЬНЫЕ КВАДРАТУРНЫЕ ФОРМУЛЫ ДЛЯ ГИПЕРСИНГУЛЯРНЫХ ИНТЕГРАЛОВ В ПРОСТРАНСТВЕ СОБОЛЕВА

[1,2,3]***Ахмедов Д.М.***, [3]***Авезов А.Х.***

d.akhmedov@mathinst.uz

[1]Институт математики им. В.И.Романовского Академии наук Республики Узбекистана, ул. Университетская, 9, Ташкент 100174, Узбекистан

[2]Национальный университет Узбекистана имени Мирзо Улугбека, улица Университетская, 4, Ташкент 100174, Узбекистан
[3]Бухарский государственный университет, ул. Мухаммада Икбала, 11, Бухара 705018, Узбекистан

Решение задачи Коши для дифференциальных уравнений в частных производных гиперболического типа привело Ж.Адамара к введению сингулярных интегралов особого вида. Позднее они получили название интегралов в смысле Адамара, или интегралов Адамара. Кроме уравнений гиперболического типа, интегралы Адамара находят широкое применение в теории упругости, электродинамике, аэродинамике и ряде других важных областей механики и математической физики. Точное вычисление интегралов Адамара возможно только в исключительных случаях, поэтому возникает необходимость в разработке приближенных методов вычислени. В настоящей работе мы разработаем оптимальный алгоритм приближенного вычисления интегралов Адамара при $p = 3$. Здесь мы занимаемся нахождением аналитического вида коэффициентов оптимальной квадратурной формулы.

Ключевые слова: Оптимальные квадратурные формулы, экстремальная функция, пространство Соболева, оптимальные коэффициенты, сингулярный интеграл типа Адамара.

Цитирование: *Ахмедов Д.М., Авезов А.Х.* Оптимальные квадратурные формулы для гиперсингулярных интегралов в пространстве Соболева // Проблемы вычислительной и прикладной математики. – 2023. – № 3(49). – С. 1-10.