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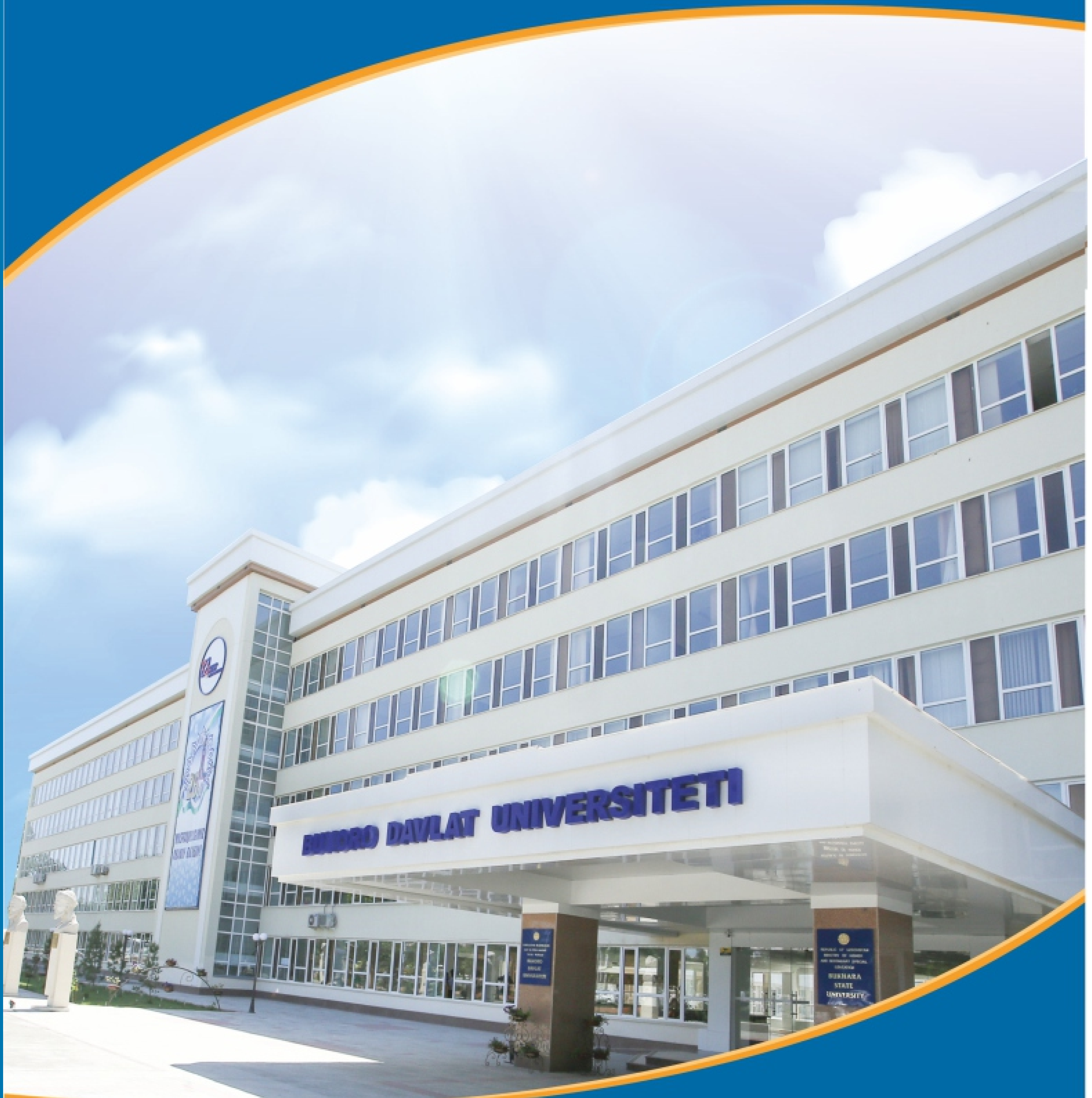
ILMIY AXBOROTI



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MUNDARIJA *** СОДЕРЖАНИЕ *** CONTENTS

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KOSHI MASALASINI TAQRIBIY YECHISH

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Annotatsiya. *Matematika, mexanika, fizika, kimyo, iqtisodiyot fanlarining bir qator masalalarini yechishda funksiya, uning argumenti va hosilasining o‘zaro bog‘liqligini ifodalovchi tenglamalarga asoslanadi. Bunday tenglamalar differensial tenglamalar deyiladi. Koshi masalasi - differensial tenglamalar nazariyasining asosiy masalalaridan biri. Differensial tenglamaning ma’lum boshlang‘ich shartlarni qanoatlantiruvchi yechimi (integrali)ni izlashdan iborat. Maqolada Koshi masalasini taqribiy yechish usullari Eyler, Eyler - Koshi usullari, Runge-Kutta metodlari haqida ma’lumot berilgan.*

Kalit so‘zlar: *oddiy dufferensial tenglama, iteratsion metod, Koshi masalasi, taqribiy yechim, Eyler usuli, Eyler - Koshi usullari****,*** *Runge-Kutta metodi, boshlang‘ich shart, xatolik, Runge qoidasi.*

ПРИБЛИЖЁННОЕ РЕШЕНИЕ ЗАДАЧИ КОШИ

Аннотация. *Решение ряда задач математики, механики, физики, химии и экономики основано на уравнениях, отражающих взаимозависимость функции, её аргумента и её производной. Такие уравнения называются дифференциальными уравнениями. Задача Коши - одна из основных задач теории дифференциальных уравнений. Он заключается в поиске решения (интеграла) дифференциального уравнения, удовлетворяющего определённым начальным условиям. В статье приведены сведения о методах приближённого решения задачи Коши: методах Эйлера, методах Эйлера-Коши и методах Рунге-Кутты.*

Ключевые слова: *обыкновенное дифференциальное уравнение, итерационный метод, задача Коши, приближённое решение, метод Эйлера, методы Эйлера-Коши, метод Рунге-Кутты, начальное условие, погрешность, правило Рунге.*

APPROXIMATE SOLUTION OF THE CAUCHY PROBLEM

Abstract. *The solution of a number of problems in mathematics, mechanics, physics, chemistry and economics is based on equations that reflect the interdependence of a function, its argument and its derivative. Such equations are called differential equations. The Cauchy problem is one of the main problems in the theory of differential equations. It consists of finding a solution (integral) of a differential equation that satisfies certain initial conditions. The article provides information on methods for approximate solution of the Cauchy problem: Euler methods, Euler-Cauchy methods and Runge-Kutta methods.*

Keywords: *ordinary differential equation, iteration method, Cauchy problem, approximate solution, Euler's method, Euler-Cauchy methods, Runge-Kutta method, initial condition, error, Runge's rule.*

Kirish. Ilmiy va tatbiqiy masalalarda ko‘pincha shunday oddiy differensial tenglamalar uchraydiki, ularning umumiy yechimini oshkor ko‘rinishda ifodalash mumkin emas. Yechimi oshkor ko‘rinishda topiladigan differensial tenglamalar sinfi ancha tor. Masalan, ko‘rinishi soddagina bo‘lgan

$$y' = x + x^2 + y^2$$

differensial tenglamaning umumiy yechimini elementar funksiyalar orqali ifodalab bo‘lmaydi. Bu yechim ancha murakkab tarzda kasr tartibli Bessel funksiyalari orqali ifodalanadi. Ko‘p hollarda yechimning hatto shunaqa ifodasini ham bilmaymiz. Shuning uchun ham bunday tenglamalami u yoki bu taqribiy metod bilan yechishga to‘g‘ri keladi.

Taqribiy yechim analitik ko‘rinishda yoki jadval shaklida izlanishiga qarab taqribiy metodlar *analitik* va *sonli metodlarga* bo‘linadi.

Oddiy differensial tenglama uchun Koshi masalasi va chegaraviy masala qo‘yiladi. Koshi masalasini yechish chegaraviy masalani yechishga nisbatan ancha yengildir. Hozirda EHMlarning rivojlanishi bilan aniqlik tartibi yuqori bo‘lgan sonli metodlarga e’tibor kuchaydi.

Asosiy qism. Faraz qilaylik, bizga quyidagi Koshi masalasi berilgan bo‘lsin

$$y' = f(x, y), \tag{1}$$

$$\mathrm{y}(\mathrm{x}_0) = \mathrm{y}_0 \tag{2}$$

Bu masalani $[x_0, X\ ]$ oraliqda yechish talab etilgan bo‘lsin.

Berilgan kesmani N ta teng bo‘lakka bo‘lamiz:

$$x_i = x_0 + ih,\ \ i = 1,2, \dots, N, \quad N = \frac{X - x_0}{h}, \qquad h > 0.$$

Adams ekstrapolyatsion metodiga ko‘ra

$$y_{n+1} = y(x_n) + h\left[y^1(x_n) + \tfrac{1}{2}\Delta y'(x_{n-1}) + \tfrac{5}{12}\Delta^2 y'(x_{n-2}) + \tfrac{3}{8}\Delta^3 y'(x_{n-3}) + \cdots \right.$$

$$\left. + C_k \Delta^k y'(x_{n-k})\right] + R_k \tag{3}$$

ga ega bo‘lamiz. Bunda

$$C_k = \int_0^1 \frac{u(u+1)\cdots(u+k)}{k!} du \tag{4}$$

$$R_k = h\int_0^! z_k(u)du = h^{k+2}\int_0^1 \frac{u(u+1)\cdots(u+k)}{(k+1)!} y^{(k+2)}(\xi)du$$

Eyler va Eyler - Koshi usullari

(3) formulada $k = 0$ bo‘lgan holda hisoblash jarayoni

$$y_{n+1} = y_n + hf_n \tag{5}$$

formula bilan tashkil etiladi. Buni Eyler taklif qilgan, shuning uchun bunday hisoblash jarayoni *Eyler metodi* deb yuritiladi.

Eyler usulining quyidagicha modifikatsiyasi mavjud.

Avval quyidagi yaqinlashish

$$y_{n+1/2} = y_n + \frac{h}{2}f_n = y_n + \frac{h}{2}f(x_n, y_n)$$

hisoblanadi, so‘ng

$$y_{n+1} = y_n + hf_{n+1/2} \tag{6}$$

formula yordamida x_{n+1} nuqtadagi yechimning taqribiy qiymati topiladi. Bu yerda

$y_{n+1/2} = y\left(x_n + \frac{h}{2}\right), \qquad f_{n+1/2} = f\left(x_n + \frac{h}{2}, y_n + \frac{1}{2}\right)$

(6) ifoda *Eyler usulining modifikatsiyasidir.*

Eyler-Koshining modifikatsiyalangan metodi quyidagicha aniqlanadi.

Avval $\bar{y}_{n+1} = y_n + hf_n$

hisoblanadi, so‘ng x_{n+1} nuqtadagi yechimning taqribiy qiymati

$$y_{n+1} = y_n + \frac{h}{2}(f_n + \bar{f}_{n+1}) \tag{7}$$

formula bilan aniqlanadi. Bu yerda

$$\bar{f}_{n+1} = f(x_{n+1} + \bar{y}_{n+1})$$

Agar (1), (2) masalaning taqribiy yechimini $y_{n+1}^{(0)} = y_n + hf(x_n, y_n)$ deb, uni quyidagi

$$y_{n+1}^{(k+1)} = y_k + \frac{h}{2}(f(x_n, y_n) + f\left(x_{n+1}, y_{n+1}^{(k)}\right)) \tag{8}$$

iteratsion jarayon bilan berilgan aniqlik miqdorida topilsa, (8) ifoda *Eyler-Koshining iteratsion metodi* deyiladi.

Runge-Kutta metodi

(1), (2) masalani yechish uchun

$y(x_{n+1}) = \ y_n + h\int_0^1 y'\,(x_n + uh)du$ formulani qulaylik uchun indekssiz quyidagicha

$$y(x + h)\ =\ y(x) + h\int_0^1 f\,(x + uh, y(x\ +\ uh))\,du \tag{9}$$

ko‘rinishda yozib olamiz.

Bu yerdagi integralni kvadratur formulalar yordamida taqribiy hisoblash mumkin emas, chunki integral ostida noma’lum funksiya qatnashyapti. Uni quyidagicha hisoblaymiz. Buning uchun uch guruhdan iborat

$$\alpha_2, \alpha_3, \dots, \alpha_r; \tag{α}$$

$$\beta_{21,}$$

$$\beta_{31,}, \beta_{32,}$$

$$\dots\dots\dots\dots \tag{β}$$

$$\beta_{r1,}, \beta_{r2,} \dots, \beta_{rr-1,}$$

$$p_{1,}p_2, \dots, p_r \tag{p}$$

hozircha noma'lum parametrlami kiritamiz va bular yordamida (9) dagi integralni kvadratur summaga o'xshash chekli summaga almashtiramiz

$$y(x+h) - y(x) \cong \sum_{i=1}^{r} p_i K_i \tag{10}$$

bu yerda

$K_1 = hf(x, y)$,
$K_2 = hf(x + \alpha_2 h, y \; + \; \beta_{21} K_1)$,
$K_3 = hf(x + \alpha_3 h, y \; + \; \beta_{31} K_1 + \beta_{32} K_2)$,
…………………………………………
$K_r = hf(x + \alpha_r h, y \; + \; \beta_{r1} K_1 + \beta_{r2} K_2 + \dots + \beta_{r-1} K_{r-1})$,

Agar (α) va (β) guruh parametrlar tanlangan bo'lsa, $K_1, K_2, \dots, K_r$, miqdorlar ketma-ket hisoblanadi.

(10) taqribiy tenglikning xatoligi

$$\varphi_r(h) = \Delta y - \sum_{i=1}^{r} p_i K_i$$

bo'lsin. (1) ning o'ng tomoni yetarlicha silliq bo'lsa, u holda $\varphi_r(h)$ funksiya ham yetarlicha tartibgacha uzluksiz hosilalarga ega bo'ladi. Demak, quyidagi Makloren formulasini yoza olamiz:

$$\varphi_r(h) = \sum_{j=0}^{s} \frac{h^j}{j!} \varphi_r^{(j)}(0) + \frac{h^{s+1}}{(s+1)!} \varphi_r^{(s+1)}(\theta h), 0 < \theta < 1. \tag{11}$$

Agar (α), (β), *(p)* guruh parametrlarini tanlash evaziga $\varphi_r^{(j)}(0) = 0, \;\; j = 0,1, \dots, s$ va $\varphi_r^{(s+1)}(0) \neq 0$ bo'lishini ta'minlasak, (10) formulaning xatoligi

$$\varphi_r(h) = \frac{h^{s+1}}{(s+1)!} \varphi_r^{(s+1)}(\theta h)$$

ko'rinishda bo'lib, uning qadami h ga nisbatan tartibi $s+1$ ga teng. Odatda, s ni Runge-Kutta metodining aniqlik darajasi deyiladi. Noma'lum $(\alpha), (\beta), (p)$ guruh elementlarini quyidagi talablardan topamiz. (10) ning chap va o'ng tomonini h ning darajalari bo'yicha yoyilmasida ixtiyoriy $f(x, y)$ va h uchun imkoni boricha h ning yuqori darajasigacha hadlar bir xil bo'lishini talab etamiz.

Boshqacha aytganda,

$$\varphi_r(h) = y(x+h) - y(x) - \sum_{i=1}^{r} p_i K_i(h)$$

funksiya

$$\varphi_r(0) = \varphi_r^{(1)}(\theta) = \dots = \varphi_r^{(s)}(0) = 0, \qquad \varphi_r^{(s+1)}(0) \neq 0$$

xossalarga ega bo'lib, $(\alpha), (\beta), (p)$ guruh elementlari shunday tanlanishi kerakki, ixtiyoriy h va $f(x, y)$ uchun s mumkin qadar katta bo'lsin.

Umumiy holda $(\alpha), (\beta), (p)$ guruh parametrlarni aniqlaydigan tenglamalar sistemasini yozish ancha murakkab bo'lganligi uchun Runge-Kutta metodi bilan bir qadamli qoidalaming r ning to'rtta qiymatidagisini topish masalasini ko'ramiz.

$r \; = \; 1$ bo'lsin, unda (10)

$$y(x+h) - y(x) = p_1 hf(x, y)$$

ko'rinishda bo'ladi. Bundan

$$\varphi_1(h) = y(x+h) - y(x) - p_1 hf(x, y)$$

ekanligini ko'ramiz. Bundan hosilalar olamiz

$$\varphi_1'(h) = y'(x+h) - p_1 f(x, y)$$

$$\varphi_1''(h) = y'(x+h)$$

$\varphi_1''(0) = y''(x)$ miqdor p_1 parametrga bog'liq emas va p_1 ni tanlash hisobiga nolga aylanmaydi. $\varphi_1''(0) = y''(x) - p_1 f(x, y) = (1 - p_1) f(x, y)$ miqdor ixtiyoriy $f(x, y)$ uchun $p_1 = 1$ bo'lganda nolga teng bo'ladi.

Demak, (10) $r \; = \; 1$ da $p_1 = 1$ bo'lsa

$$y(x+h)-y(x)\cong p_1 hf(x,y)$$

ko‘rinishda bo‘lib, uning xatoligi esa (11) ga asosan

$$\varphi_1(h)=\frac{h^2}{2}\varphi_1''(\theta h)=\frac{h^2}{2}h''(x+\theta h),\quad 0<\theta<1$$

bo‘ladi. Birinchi tartibli Runge-Kutta metodi Eyler metodi bilan ustma-ust tushar ekan.

Endi $r=\ 2$ bo‘lsin. (10) formula

$y(x+h)-y(x)\cong p_1K_1+p_2K_2=hp_1f(x,y)+hp_2f(x+a_2h,y+\beta_{21}f(x,y))$ ko‘rinishda bo‘ladi.

$p_1,p_2,\alpha_2,\beta_{21}$ parametrlami aniqlash uchun $y(x+h)-y(x)$ va $p_1K_1+p_2K_2$ larni h ning darajalari bo‘yicha yoyilmasini topamiz:

$$y(x+h)-y(x)=\frac{h}{1!}y'(x)+\frac{h^2}{2!}+y''(x)+\frac{h^3}{3!}y''(x)+o(h^4)=hf++\frac{h^2}{2!}\left(f_x+f\cdot f_y\right)+\frac{h^3}{3!}\left(f_{xx}+2f\cdot f_{xy}+f_y\left(f_x+f\cdot f_y\right)\right)+o(h^4)\qquad(12)$$

$$p_1K_1+p_2K_2=hp_1f(x,y)+hp_1f\left(x+\alpha_2h,y+\alpha_2hf(x,y)\right)=h(p_1+p_2)+h^2p_2(\alpha_2f_x+\beta_{21}f\cdot f_x)+\frac{h^3}{2}p_2\left(\alpha_2^2f_{xx}+2\alpha_2\beta_{21}f\cdot f_{xy}+\beta_{21}^2f^2\cdot f_{yy}\right)+o(h^4)$$

(12) va (13) yoyilmalarda ixtiyoriy $f(x,y)$ uchun hadlar h ning iloji boricha yuqori darajalarigacha ustma-ust tushsin, deb talab qilamiz. Bu ikki yoyilmani taqqoslasak, $p_1,p_2,\alpha_2,\beta_{21}$ larni topish uchun quyidagiga ega bo‘lamiz:

$$hf\ :\ p_1+p_2=1,$$
$$\mathrm{h}^2\mathrm{f}_x\ :\ \mathrm{p}_2+\alpha_2=\frac{1}{2},$$
$$\mathrm{h}^2\mathrm{f}\cdot\mathrm{f}_x\ :\ \mathrm{p}_2\beta_{21}=\frac{1}{2}.$$

Yuqoridagi (12), (13) yoyilmalarning ko‘rinishidan ma’lumki, $r=2$ da kiritilgan $p_1,p_2,\alpha_2,\beta_{21}$parametrlarni tanlash evaziga ixtiyoriy $f(x,y)$ uchun h^3 li hadlaming bir xil bo‘lmasligini ko‘rish mumkin. Demak, yuqoridagi $p_1,p_2,\alpha_2,\beta_{21}$ larga bog‘liq uchta lenglamadan $p_2\neq 0$ deb

$$p_1=1-p_2,\alpha_2=\beta_{21}=\frac{1}{2p_2}$$

larni aniqlaymiz. Odatda, p_2 ni shunday tanlanadiki, hosil bo‘ladigan formula hisoblashlarda qulaylikka ega bo‘lishi kerak.

Misol uchun $p_2=\frac{1}{2}$ bo‘lsin, unda $p_2=\frac{1}{2}\ \alpha_2=\beta_{21}=1$ bo‘ladi va quyidagi ikkinchi tartibli

$$y(x+h)=y(x)+\frac{h}{2}(f(x,y)+f(x+h,y+K_1))$$

formula hosil bo‘ladi.

Agar $p_2=1$ desak, unda $p_1=0$, $\alpha_2=\beta_{21}=\frac{1}{2}$ bo‘ladi va

$$y(x+h)=y(x)+hf(x\ +\frac{h}{2},y+\frac{h}{2}f(x,y))$$

ikkinchi tartibli hisoblash formulasiga ega bo‘lamiz.

$r=3$ bo‘lganda kiritilgan $\alpha_2,\alpha_3,p_1,p_2,p_3,\beta_{21},\beta_{31},\beta_{32}$ parametrlar uchun bajarilishi kerak bo‘lgan shartlarni keltirish bilan chegaralanamiz, chunki (12) va (13) tipidagi yoyilmalar $r=3$ da bundan ham katta ko‘rinishda bo‘ladi. Ular quyidagicha

$$p_1+p_2+p_3=1,$$
$$p_2\alpha_2+p_3\alpha_3=\frac{1}{2},$$
$$p_2\alpha_2^2+p_3\alpha_3^2=\frac{1}{3},$$
$$p_2\alpha_2\beta_{32}=\frac{1}{6},$$
$$\beta_{31}+\beta_{32}=\alpha_3,$$
$$\beta_{21}=\alpha_2$$

ko‘rinishga ega. Bu tenglamalar sistemasini yechimlaridan biri quyidagilar:

$$\alpha_2=\frac{1}{2},\alpha_3=1,\beta_{21}=\frac{1}{2},\beta_{31}=-1,\beta_{32}=2,$$

$$p_1 = \frac{1}{6}, \qquad p_2 = \frac{2}{3}, \qquad p_3 = \frac{1}{6}.$$

Bu parametrlar bilan aniqlangan uchinchi tartibli Runge-Kutta metodi

$$y(x+h) = y(x) + \frac{1}{6}(K_1 + 4K_2 + K_3)$$

ko‘rinishda bo‘lib, bu yerda

$$K_1 = hf(x,y), \qquad K_2 = hf\left(x + \frac{h}{2}, y + \frac{1}{2}K_1\right)$$
$$K_3 = hf(x+h, y - K_1 + 2K_2).$$

Bundan boshqa yana bir formulani keltiramiz:

$$y(x+h) = y(x) + \frac{1}{4}(K_! + 3K_3)$$
$$K_1 = hf(x,y), \qquad K_2 = hf\left(x + \frac{h}{3}, \qquad y + \frac{1}{3}K_1\right)$$
$$K_3 = hf\left(x + \frac{2}{3}h, y + \frac{2}{3}K_2\right).$$

Endi $r = 4$ bo‘lganda eng ko‘p ishlatiladigan to‘rtinchi tartibli metodni keltiramiz:

$$y(x+h) = y(x) + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4),$$

bu yerda

$$K_1 = hf(x,y), K_2 = hf\left(x + \frac{h}{2}, y + \frac{1}{2}K_1\right)$$
$$K_3 = hf\left(x + \frac{h}{2}, y + \frac{1}{2}K_2\right) \qquad K_4 = hf(x+h, y+K_3)$$

Yuqorida chiqarilgan birinchi, ikkinchi, uchinchi va to‘rtinchi tartibli metodlarni birinchi tartibli differensial tenglamalar sistemasi uchun Koshi masalasini taqribiy yechishda qo‘llash mumkin. Masalan, quyidagi Koshi masalasi berilgan bo‘lsin:

$$y_i' = f_i(x, y_1, y_2, \dots, y_n),$$
$$y_i(x_0) = y_{i0}, \quad i = 1,2, \dots n$$

$r = 1$ da hisoblash jarayoni quyidagicha amalga oshiriladi:

$$y_i(x+h) = y_i(x) + hf_i(x, y_1, y_2, \dots, y_n), \ \ i = 1,2, \dots, n.$$

$r = 2$ da esa formulalar

$$y_i(x+h) = y_i(x) + \frac{1}{2}(K_{i1} + K_{i2}),$$
$$K_{i1} = hf(x, y_1, y_2, \dots, y_n),$$
$$K_{i2} = hf(x+h, y_1 + K_{11}, y_2 + K_{21}, \dots, y_n + K_{n1}), \ \ i = 1,2, \dots, n$$

ko‘rinishga ega bo‘ladi.

Runge-Kutta metodi bilan qurilgan metodlarda yechimning bitta nuqtadagi qiymati ishtirok etishi bois ular bir qadamli metodlar deyiladi.

Runge-Kutta metodi xatoligining bosh hadi va uni baholashda Runge qoidasi

Tartibi S ga teng Runge-Kutta metodi bilan Koshi masalasini yechishda har bir qadamdagi xatolik miqdori

$$\frac{h^{S+1}}{(S+1)!} \cdot \varphi_r^{(S+1)}(\theta h), \qquad 0 < \theta < 1$$

ga teng bo‘lib, tartibi h^{S+1} edi. Agar differensial tenglamaning o‘ng tomoni ancha murakkab bo‘lsa, bu xatolikni baholash amaliyotda ancha qiyinchiliklarni tug‘diradi. Metodning jami xatoligi tartibi esa h^S ga teng bo‘ladi [8], ya’ni

$$y(x_n) = y_n + p(x_n)h^S + o(h^{s+1}) \tag{14}$$

Agar $p(x_n) \neq 0$ bo‘lsa, yetarlicha kichik h uchun $\rho(x_n)h^S$ miqdor metod xatoligiga ancha yaxshi yaqinlashgan bo‘ladi. Bu miqdor, odatda, xatolikning bosh hadi deb ataladi. Xatolikning bosh hadi xatolikni to‘liq xarakterlamaydi, lekin uning miqdori metod xatoligi miqdorini yaxshi tasavvur etishga imkon beradi.

(14) formula yordamida xatolikning bosh hadini aposterior baholash qoidasi mavjud, uni *Runge qoidasi* deb ataladi.

Faraz qilaylik, $[x_0, X]$ oraliqning ξ, nuqtasida tartibi S ga teng metod bilan $h - h_1$ va $h = h_2$ qadamlarda yechimning y_{h1} va y_{h2} taqribiy qiymatlari topilgan bo‘lsin.

Agar h_1 j va h_2 yetarlicha kichik bo‘lsa, unda (14) ga asosan

$$y(\xi) - y_{h1} \cong \rho(\xi) h_1^S,$$
$$y(\xi) - y_{h2} \cong \rho(\xi) h_2^S,$$

taqribiy tengliklarning xatoliklari ancha kichik bo‘lishligini kutish mumkin. Bundan

$$\rho(\xi) \cong \frac{y_{h1} - y_{h2}}{h_2^S - h_1^S} \qquad (15)$$

ligini aniqlaymiz. Demak, hisoblashlami $h \sim h_x$, $h = h_2$ qadam bilan £, nuqtagacha bajarsak, xatolikning bosh hadining taqribiy qiymatlarini ikkala holda ham aniqlagan bo‘lamiz:

$$\mathrm{h}_1^S \cong \frac{\mathrm{y}_{\mathrm{h}1} - \mathrm{y}_{\mathrm{h}2}}{\mathrm{h}_2^S - \mathrm{h}_1^S}, \quad \mathrm{h}_2^S \cong \frac{\mathrm{y}_{\mathrm{h}1} - \mathrm{y}_{\mathrm{h}2}}{\mathrm{h}_2^S - \mathrm{h}_1^S},$$

Agar bu miqdorlar berilgan aniqlik ε dan katta bo‘lsa, quyidagi taqribiy tenglik

$$\varepsilon \approx |\rho(\xi) h_\varepsilon^S \approx \left| \frac{y_{h1} - y_{h2}}{h_2^S - h_1^S} \right| h_\varepsilon^S$$

dan tegishli shartlarni bajaradigan qadamni ko‘rsatish mumkin:

$$h_\varepsilon \approx \sqrt[S]{\varepsilon \left| \frac{h_2^S - h_1^S}{y_{h1} - y_{h2}} \right|}.$$

Agar $h_1 = h, \quad h_2 = 2h$ bo‘lsa, (15) formula

$$\rho(\xi) = \frac{y_h - y_{2h}}{h^2(2^S - 1)}$$

ko‘rinishda bo‘lib, metod xatoligi uchun

$$y - y_h \approx \frac{y_h - y_{2h}}{2^S - 1} \qquad (16)$$

taqribiy ifodaga ega bo‘lamiz. Bu formula asimptotik xarakterga ega bo‘lib, kafolatlangan bahoni bermaydi. Shunga qaramasdan, u amaliy nuqtayi nazardan ko‘pincha yaxshi natijalarni beradi.

Shu narsani ta’kidlash lozimki, (16) formulada yaxlitlash xatoligi inobatga olinmagan. Shuning uchun, hisoblash jarayonida yaxlitlash xatoligi salmoqli bo‘lsa, xatolikning bosh hadini baholashda (16) ni qo‘llab bo‘lmaydi.

Agar (16) ni quyidagicha

$$y \approx y_h + \frac{y_h - y_{2h}}{2^S - 1} \qquad (17)$$

yozsak, (17) ning o‘ng tomoni yechimning aniq qiymati y ga, y_h va y_{2h} dan ko‘ra ancha yaqinroq bo‘lishligiga umid qilish mumkin.

Natijalar.

Misol 1. $\boldsymbol{y' = x + y}$

$y(0) = 1$

Koshi masalasining [0;0,4] oraliqda $h = 0{,}1$ qadam bilan Eyler usulida yechimini toping. Taqribiy yechim va aniq yechim orasidagi farqni hisoblang. Analitik yechim $y(x) = 2e^x - x - 1$.

Yechish. Differensial tenglamaning o‘ng tomoni $f(x, y) = x + y$;

boshlang‘ich ma’lumotlar: $x_0 = 0, \; y_0 = 1$;

yechim izlanayotgan $[a, b]$ oraliq: $a = 0, \quad b = 0{,}4$;

oraliqning bo‘linish soni: $n = 4, \quad h = 0{,}1$.

Quyidagi

$x_{i+1} = x_i + 0{,}1;\ \Delta y_i = 0{,}1(x_i + y_i,);\ y_{i+1} = y_i + \Delta y_i;\ (i = 0{,}1{,}2{,}3)$ rekurrent formulalardan foydalanib ketma-ket quyidagilami topamiz:

1-qadamda $(i = 0)$: $x_1 = O,1;\ y_1 = 1 + 0{,}1(0 + 1) = 1{,}1,$

2-qadamda $(i = 0)$: $x_2 = O,2;\ y_2 = 1{,}1 + 0{,}1(0{,}1 + 1{,}1) = 1{,}22,$

3-qadamda $(i = 0)$: $x_3 = O,3;\ y_3 = 1{,}22 + 0{,}1(0{,}2 + 1{,}22) = 1{,}362,$

4-qadamda $(i = 0)$: $x_4 = O,4;\ y_4 = 1{,}362 + 0{,}1(0{,}3 + 1{,}362) = 1{,}52821.$

$d_i = |y(x_i) - y_i|$ deb belgilaymiz va hisoblashlarimiz natijalarini jadval ko‘rinishida keltiramiz:

i	x_i	y_i	$y(x_i)$	d_i
1	0,1	1,1	1,110342	0,010342
2	0,2	1,22	1,242806	0,022806
3	0,3	1,362	1,399718	0,037718
4	0,4	1,5282	1,583649	0,055449

Misol 2.

$$y' = x + y$$
$$\mathrm{y}(0) = 1$$

Koshi masalasini [0;0,4] oraliqda $h = 0{,}1$ qadam bilan Eyler-Koshi usuli bilan yeching.

Yechish. $x_0 = 0$, $y_0 = 1$ deb, Eyler-Koshi metodini indeks i uchun quyidagicha yozamiz:

$$x_{i+1} = x_i + h; \quad y_{i+1} = y_i + \Delta y_i, \quad (i = 0{,}1{,}2{,}3)$$
$$\Delta y_i = \frac{1}{2}\left(K_i^{(1)} + K_i^{(2)}\right), \quad K_i^{(1)} = h \cdot f(x_i, y_i), \quad K_i^{(2)} = h \cdot f(x_i + h, y_i + K_i^{(1)})$$

Yechilyotgan masala uchun

$$i = 0;\ x_1, = x_0 + h = 0 + 0{,}1; \quad K_0^{(1)} = h(x_0 + y_0) = 0{,}1(0 + 1) = 0{,}1;$$
$$K_0^{(2)} = h(x_0 + h + y_0 + K_0^{(1)}) = 0{,}1 \cdot (0 + 0{,}1 + 1 + 0{,}1) = 0{,}12;$$
$$\Delta y_0 = \frac{1}{2}(K_0^{(1)} + K_0^{(2)}) = \frac{1}{2}(0{,}1\ +\ 0{,}12) = 0{,}11;$$
$$y_1 = y_0, + \Delta y_0 = 1 + o, 11 = 1{,}11.$$

Xuddi shunday hisoblashlarni qolgan qadamlar uchun bajaramiz, xatolik $d_i = |y(x_i) - y_i|$ va hisoblashlar natijasini quyidagi jadvalda keltiramiz:

i	x_i	$K_{i=1}^{(1)}$	$K_{i=1}^{(2)}$	$\Delta y_{i=1}$	y_i	$y(x_i)$	d_i
1	0,1	0,1	0,12	0,11	1,11	1,110342	0,000342
2	0,2	0,121	0,1431	0,13205	1,24205	1,242805	0,000756
3	0,3	0,144205	0,168626	0,156415	1,398466	1,399718	0,001252
4	0,4	0,169847	0,196831	0,183339	1,581804	1,583649	0,001845

Topilgan taqribiy yechim xatoligi $|y_4 - y(x_4)| \approx 0{,}002$ dan ortmaydi.

Misol 3. Yuqoridagi misolni to‘rtinchi tartibli Runge-Kutta usuli bilan yeching.

Yechish.

$$x_{i+1} = x_i + h, \quad K_i^{(1)} = hf(x_i, y_i), \quad K_i^{(2)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{K_i^{(1)}}{2}\right)$$
$$K_i^{(3)} = hf\left(x_i + \frac{h}{2}, y_i + \frac{K_i^{(2)}}{2}\right), \quad K_i^{(4)} = hf\left(x_i + h, y_i + K_i^{(3)}\right)$$
$$\Delta y_i = \frac{1}{2}\left(K_0^{(1)} + 2K_i^{(2)} + 2K_i^{(3)} + K_i^{(4)}\right), \quad y_{i+1} = y_i + \Delta y_i\ (i = 0{,}1{,}2{,}3$$

$x_0 = 0$, $y_0 = 1$ deb y_1 nitopamiz:

$$i = 0;\ x_1 = x_0 + h = 0 + 0{,}1; \quad K_0^{(1)} = h(x_0\ +\ y_0) = 0{,}1(0 + 1) = 0{,}1$$
$$K_i^{(2)} = h\left(x_0 + \frac{1}{2}h + \frac{1}{2}K_0^{(1)}\right) = 0{,}1(0 + 0{,}05 + 1 + 0{,}05) = 0{,}11;$$
$$K_i^{(3)} = h\left(x_0 + \frac{1}{2}h + y_0 + \frac{1}{2}K_0^{(2)}\right) = 0{,}1(0 + 0{,}05 + 1 + 0{,}055) = 0{,}1105;$$
$$K_i^{(4)} = h\left(x_0 + h, y_0 + K_0^{(3)}\right) = 0{,}1(0 + 0{,}1 + 1 + 0{,}1105) = 0{,}12105;$$
$$\Delta y_0 = \frac{1}{6}\left(K_0^{(1)} + 2K_0^{(2)} + 2K_0^{(3)} + K_0^{(4)}\right) = \frac{1}{6}(0{,}1 + 2 \cdot 0{,}11 + 2 \cdot 0{,}1105 + 0{,}12105) \cong 0{,}11;$$
$$y_1 = y_0 + \Delta y_0 \approx 1 + 0{,}110342\ =\ 1{,}110342.$$

Shunday hisoblashlarni keyingi qadamlar uchun ham bajarib hisoblash natijalarini esa quyidagi jadvalda keltiramiz:

i	$x_i,$	$K_{i-1}^{(1)}$	$K_{i-1}^{(2)}$	$K_{i-1}^{(3)}$	$K_{i-1}^{(4)}$	Δy_{i-1}	y_i
1	0,1	0,100000	0,110000	0,110500	0,121050	0,110342	1,110342
2	0,2	0,121034	0,132086	0,132638	0,144298	0,132468	1,242805
3	0,3	0,144281	0,156495	0,157105	0,169991	0,156912	1,399717
4	0,4	0,169972	0,183470	0,184145	0,198386	0,183931	1,583648

Topilgan taqribiy yechim xatoligi 0,000001 dan ortmaydi.

Taqqoslash uchun uchta metod bilan yechilgan bitta masalani tugun nuqtalardagi yechimning taqribiy qiymatlarini va aniq yechimning qiymatlarini quyidagi jadvalda keltiramiz.

i	x_i	Usullar bo‘yicha topilgan y_i qiymati			Aniq yechimi $y(x_i) = 2e^x - x - 1$
		Eyler	Eyler- Koshi	Runge- Kutta	
1	0,1	1,1	1,11	1,110342	1,110342
2	0,2	1,22	1,24205	1,242805	1,242805
3	0,3	1,362	1,398466	1,399717	1,399718
4	0,4	1,5282	1,581804	1,583648	1,583649

Xulosa. Koshi masalasi yordamida fizika va texnikaning juda ko‘plab muhim masalalarini hal qilish mumkin. Koshi masalasini yechish bu biror bir tabiiy jarayonni ifodalovchi differensial tenglamani va qo‘shimcha (boshlang‘ich) shartni qanoatlantiruvchi noma’lum funksiyani aniqlashdan iborat. Maqolada Koshi masalasini taqribiy yechish usullari Eyler, Eyler - Koshi usullari, Runge-Kutta metodlari haqida ma’lumot berilgan. Bu usullar misollar orqali batafsil yoritilgan.

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