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Algorithm for Hydraulic and Thermal Calculation of a Two-Pipe Heating Network Based on Quasi-One-Dimensional Modeling

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Abstract: Algorithms for hydraulic and thermal calculation of a two-pipe heating network with a limited number of heat exchangers have been developed. When modeling the functioning of the network, a single-term representation of the resistance coefficient, analogues of Kirchhoff's laws taking into account the variability of the leveling height of the network route, as well as Shukhov's formula and the average flow temperature of the coolant at the confluence of flows were used. Formulas were obtained for determining arc flow rates, nodal pressure and temperature, taking into account the characteristics of individual heat exchangers. Based on the proposed calculation method, a software product was compiled and a computational experiment was carried out within the framework of the quadratic resistance law. Summary calculation results related to the network and individual heat exchangers are presented. The pressure and heat losses determined during the calculation make it possible to select the power of the heat source and pump.

Keywords: two-pipe heating network, multi-section heat exchanger, working fluid flow, quadratic resistance law, temperature, environment, Shukhov's formula, analogues of Kirchhoff's laws, heat and pressure loss in the network, computational experiment.

1. Introduction

To ensure the necessary conditions for temperature and humidity in working and living spaces, various methods are used. Depending on the coolant, various devices have been developed and are being developed. If the ambient temperature is high, as is observed in tropical latitudes, then cooling is organized. If the ambient temperature is low for living and activity, then the room is insulated. This is required not only for people, but also for fauna and flora, which are part of the human economy.

The main source of heat for the earth is the Sun. The accepted international solar constant is 1.36 kW/m^2 . This means that 1 square meter of the surface of the upper atmosphere perpendicular to the rays perceives 1.36 kW of solar energy. This energy on the entire surface of the Earth from the Sun increases the power generated by humanity thousands of times. In this regard, the use of solar energy is one of the main areas of green energy. The use of wind and water energy is also associated with solar energy [1].

Among natural energy resources, heat occupies a certain place in the bowels of the Earth. Like a dying star, the Earth transfers its heat to the surface boundary layer of the atmosphere. The power of this heat transfer process is thousands of times less than solar radiation, but it can also be used for the benefit of humanity. Work is underway to use the Earth's temperature gradient [2-3], including the energy of geysers and groundwater [4] for heating or cooling rooms.

Studying these and other energy sources and developing effective ways to extract them are major challenges in the world's energy sector. At the same time, in the energy sector, methods and devices for heating and cooling rooms, which are the subject of research into heat and mass transfer processes, play an important role [5-13]. Mathematical models and methods for solving heat transfer problems for various geometric and physical forms have been developed [14-16]. Research continues in the area of resource and energy saving when organizing heat and mass transfer.

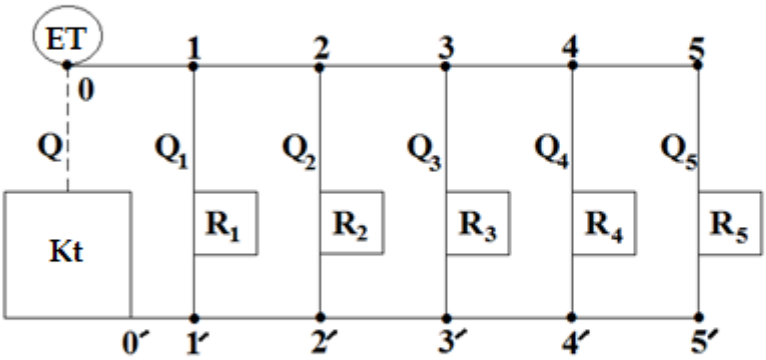
In post-Soviet territories there is a transition to individual heating and cooling systems for premises. Those. Instead of a single centralized heating system, thermal power plants and others began to widely use more compact and economical heating systems, including modern boilers and heat exchangers. The calculation of these installations, necessary for the design of a heating system, is carried out within the framework of empirical dependencies, which are based on the thermal pressure of the source and drain [17-18]. In particular, the calculation of a multi-section heat exchanger implies the distribution of coolant among sections, which determines the intensity of heat exchange with the environment.

Work [19] presents a simplified model of a heat exchanger connected to a single-strand heating network. The hypothetical basis of this model is the flow of coolant along the outer contour of the heat exchanger, and the liquid is idle in the internal vertical pipes. The problem of heat exchange in the working fluid-pipe-environment system is solved within the framework of boundary conditions of the third kind. In general, analogues of Kirchhoff's laws, which are important components in the theory of flow distribution , were used for hydraulic calculations [20-24].

A similar problem for a heat exchanger connected to a two-strand heating network was formed by the authors of the work earlier and solved analytically. This solution is used below in constructing algorithms for hydraulic and thermal calculations of a two-pipe heating network when a specific hydrodynamic flow regime has been established in the network. For thermal calculations, Shukhov's formula was used [12,23], which takes into account the integral characteristics of the flow and pipeline.

2. Formulation of the problem

A simple two-pipe heating network consists of a boiler, riser, expansion tank, distribution (upper) and collecting (lower) lines. In Fig. 1 shows a schematic representation of a network with five heat exchangers.



***Kt*– boiler; *ET*- expansion tank; Q – total flow rate on the riser; Q_1, Q_2,… – costs by link; 1, 2,… – points of connection of links to the upper highway; $1'$, $2'$,… – connection points to the lower highway; R_1, R_2,… – heat exchangers**

Figure 1. Schematic representation of a simple two-pipe heating network with five heat exchangers

Each link (in the figure $(n-1)n(n'(n-1)')$) of a two-pipe network consists of a supply part along the upper main line, a segment and an outlet part along the lower main line. The segment with the heat exchanger nn' consists of an input jumper with a valve-regulator, the heat exchanger itself and an output jumper. They are connected from above to the upper line, and from below to the lower line. The total difference in the leveling height n of the th segment is $\dot{h}_n - h_n$. Here $\dot{h}_n$, h_n are the leveling heights n of the th nodes of the upper and lower highways.

The distribution (upper) line (012345 in the figure) starts from the expansion tank and reaches the upper node of the final N segment. The (lower) line collecting spent working fluid (in the figure $0'1'2'3'4'5'$) starts from the lower node N of the th segment and ends at the bottom of the container in the furnace.

The distance from the bottom of the boiler to the expansion tank is equal to the sum of the height of the boiler with the length of the riser and is $H_s = \dot{h}_0 - h_0$. Each n link $(n-1)n$ has its own link lengths with lengths L_{n-1} in the upper and lower lines. Parts $n(n+1)$ of the upper line have internal D_n and external $\bar{D}_n$ diameters, resistance coefficient λ_n^{gv} and length L_n, leveling height h_n, measured from the zero level $0'$. Same designations D'_n, $\bar{D}'_n$, λ_n^{gn}, L'_n, h'_n are entered for parts $n'(n+1)'$ for the lower main.

The costs by segments are Q_1, Q_2,… Q_N, the sum of which, according to the law of conservation of mass, is equal to the total water consumption Q in the network.

Each segment (in the figure $(n-1)nn'(n-1)'$) consists of upper and lower jumpers with lengths l_n^v and $l_n^{v'}$. They are made from the same pipe with a diameter D_v and thickness δ_v. The resistance coefficient of the vertical component segments is λ_v. The upper and lower jumpers are connected to each other n by a heat exchanger (in the figure R_n), the parameters of which, except for the number of sections and ambient temperature, are the same.

We need to find the nodal pressures and temperatures of the working agent, for the calculation of which we also need the values of local flow rates. Those. the problem of flow distribution in a two-pipe network is solved.

3. Materials, methods and object of study

The mathematical apparatus involved is built taking into account the force of gravity. The heated coolant rises from the boiler to the expansion tank. The low density of water in the boiler serves as a driving force and gravity and friction work against it. Next, the heated coolant enters the upper line. It has a negative slope and possibly telescopic : from a large diameter to a smaller diameter.

The flow is distributed into segments along which the working unit descends down to the lower line. The lower main also has a negative slope in the direction of flow and is possibly telescopic .

With this schematization, the main problem of the theory of flow distribution is solved in advance [20-24,26] - the flow directions in the arcs of a complex network are determined. Despite this, it is quite difficult to use the provisions of the theory of flow distribution , so below we use formulas for hydraulic calculation of the network using analogues of Kirchhoff's laws, as well as the formula for the pressure drop in a heat exchanger with M sections.

The pressure drop in an elementary section from the beginning (H) to the end (K) takes into account changes in the leveling height and friction force of the pipeline:

$$p_H + \rho g h_H = p_K + \rho g h_K - \frac{\lambda}{2D\rho f^2} Q^2.$$

Such a quasi-one-dimensional representation of the law of conservation of momentum for a pipeline of circular cross-section is universal in nature from the point of view of the hydrodynamic flow regime, because drag coefficient

$$\lambda = \zeta \left(k / D\right)^{\theta} \mathrm{Re}^{n}$$

takes into account all flow regimes. For given values of the equivalent roughness coefficient k and Reynolds number $\mathrm{Re}$the formula covers all five flow regimes and is called the generalized Leibenzon formula .

According to [25-26], in the laminar flow regime, constant formulas can be used according to the Stokes formula $\left(\zeta = 64,\ \theta = 0,\ \varphi = -1\right)$, in the transition regime - the Zaichenko formula , in the smooth regime of turbulent flow around roughness - $\left(\zeta = 0.0025,\ \theta = 0,\ \varphi = 1/3\right)$ the Blasius formula $\left(\zeta = 0.3164,\ \theta = 0,\ \varphi = -1/4\right)$, in mixed – Leibenzon formula $\left(\zeta = 10^{-0.627},\ \theta = 0{,}127,\ \varphi = -0.123\right)$, and in the developed form – Shifrinson's formula $\left(\zeta = 0.11,\ \theta = 0.25,\ \varphi = 0\right)$.

The law of conservation of mass takes into account the mutual equality of input and output costs. The volumetric flow rates in the sections of the arcs n and n' are mutually equal, because in the direction of flow they have positive knowledge. In addition, as already noted, there is equality

$$Q = \sum_{n=1}^{M+1} Q_n .$$

Analogs of Kirchhoff's first and second laws were used in hydraulic calculations [ ] .

The law of heat conservation in an elementary area is described by Shukhov's formula [12,23]:

$$T_K = T_{oc} + \left(T_H - T_{oc}\right)\exp\left(-Sh\, l\right).$$

Here T_κ is the temperature of the coolant at the outlet of the section; T_H, T_{oc} – inlet temperature and ambient temperature; $Sh = \dfrac{k_{cp}\pi D_{op}}{\rho Q c_B}$ – Shukhov parameter; k_{cp} – the average value of the heat transfer coefficient between the liquid and the environment through the walls of the pipeline; πD_{op} – external area of the pipeline per linear meter; Q – volumetric flow rate of liquid; c_B – specific heat capacity of the pipeline material.

Thermal calculations take into account the formula for the average temperature of mutually combining flows.

Let's start with the hydraulic calculation of a two-pipe network.

In the jumpers, the fluid flow is Q. At a given inlet pressure $p_н = p_0$ at node 1, the pressure is determined from the formula:

$$p_0 + \rho g\left(h_0^v - h_1^v\right) - p_1 = K_1^g Q^{\alpha}.$$

Here $K_1^{gn} = \dfrac{\lambda_0^{gn} \rho L_0}{2 D_g f_g^2}$; $h_0^v - h_1^v$ – change in leveling height in part of the link from the upper and lower pipes.

The pressure drop across the elementary link $1'0'$is determined from the formula

$$p_{1'}+\rho g(h_1^{v'}-h_0^{v'})-p_{0'}=K_1^{gn}Q^{\alpha}.$$

Note that the diameter D_g for the upper and lower lines was taken to be the same value. Also, the lengths of the links in the upper and lower mains are the same and amount to L_0.

If $n=1$ those. **the network consists of one segment** , then the pressure drop formula for a segment with a heat exchanger is:

$$p_1+\rho g\left(h_1^v-h_1^{v'}\right)-p_1'=\left(\frac{\lambda_v\rho l_1^v}{2D_vf_v^2}+K_vr_{M_1}^{\alpha}+\frac{\lambda_v\rho l_1^{v'}}{2D_vf_v^2}\right)Q^{\alpha}.$$

This r_{M_1} takes into account the number of sections M_1, height and length of individual sections of the 1st heat exchanger:

$$r_m=\frac{\sqrt[\alpha]{2K_g+r_{m-1}K_v}}{\sqrt[\alpha]{K_v}+\sqrt[\alpha]{2K_g+r_{m-1}K_v}}.$$

Calculation r_m is carried out from $r_0=1$ recurrently for $K_g=\frac{\lambda_g\rho l_c}{2D_gf_g^2}$, $K_v=\frac{\lambda_v\rho h_c}{2D_vf_v^2}$.

That for a network consisting of one segment, summing the last three equalities term by term , we obtain the formula

$$p_0+\rho g(h_0^v-h_0^{v'})-p_{0'}=\left(K_0^{gv}+\frac{\lambda_v\rho l_1^v}{2D_vf_v^2}+K_vr_{M_1}^{\alpha}+\frac{\lambda_v\rho l_1^{v'}}{2D_vf_v^2}+K_0^{gn}\right)Q^{\alpha}.$$

Let's proceed to the hydraulic calculation of a network with N segments .

We divide the network into links 1, 2, …, N. Each n link includes an upper arc $(n-1)n$, n a segment with heat exchanger M_n sections and a lower arc $n'(n-1)'$.

The total difference in leveling height between the upper and lower points of the link is h_n-h_n'. The pressure loss in part of the link in the upper line is calculated from the flow rate

$$q_n=Q_n=Q-\sum_{i=1}^{n}Q_i\ \left(q_0=Q,\ q_1=Q-Q_1,\ q_2=Q-Q_1-Q_2...\right),$$

which is the same as the flow rate $q_{n'}$ of the lower arc $(n+1)'\,n'$.

Accordingly, the total pressure loss in n the th segment is determined by the dependence

$$p_n+\rho g(h_n-h_n')-p_{n'}=\frac{\lambda_v\rho l_n^v}{2D_vf_v^2}Q_n^{\alpha}+K_vr_{M_n}^{\alpha}Q_n^{\alpha}+\frac{\lambda_v\rho l_n^{v'}}{2D_vf_v^2}Q_n^{\alpha}=\Lambda_nQ_n^{\alpha}.$$

In the composition, Λ_n through the term, $K_vr_{M_n}^{\alpha}Q_n^{\alpha}$ the pressure drop across n the th heat exchanger with M_n sections at flow rate is taken into account Q_n. The first and third terms Λ_n reflect the pressure difference in the upper and lower bridges of the segment.

To determine the pressure values at the connection nodes of links and link flow rates for a fixed number of connected network segments, we use the method of mathematical induction. Those. with an increase in the number of segments, as was done for calculating a multi-section heat exchanger, starting from two segments. In this case, the calculation of values p_1 is p_0' carried out separately from this algorithm.

Consider **a network** consisting **of two segments** (Fig. 2).

Let's consider the contour separately $122'1$. This corresponds to a two-segment system in which the remaining segments 3, 4,... are disabled. The meanings of p_1 and $p_{0'}$ are defined above.

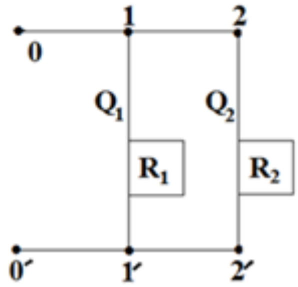


Figure 2. A network consisting of two segments.

Let's calculate the pressure drop along an arc $11'$ (in a segment) consisting of three parts,

$$p_1 + \rho g\left(h_1 - h_1'\right) - p_{1'} =$$

$$= \left(\frac{\lambda_v \rho l_1^v}{2D_v f_v^2} + K_v r_{M_1}^\alpha + \frac{\lambda_v \rho l_1^{v'}}{2D_v f_v^2}\right) Q_1^\alpha = \bar{\Lambda}_1 Q_1^\alpha$$

and along an arc $122'1'$ consisting of five parts:

$$p_1 + \rho g\left(h_1 - h_1'\right) - p_1' =$$

$$= \left(\frac{\lambda_1^{gv} \rho L_1}{2D_g f_g^2} + \frac{\lambda_v \rho l_2^v}{2D_v f_v^2} + K_v r_{M_2}^\alpha + \frac{\lambda_v \rho l_2^{v'}}{2D_v f_v^2} + \frac{\lambda_1^{gn} \rho L_1}{2D_g f_g^2}\right) Q_2^\alpha = \Lambda_2 Q_2^\alpha .$$

Here, the first and last terms Λ_2 take into account pressure losses in parts of the upper and lower lines, which have the same length L_1. The left sides of the equalities are mutually equal, then their right sides will also be mutually equal:

$$\bar{\Lambda}_1 Q_1^\alpha = \Lambda_2 Q_2^\alpha .$$

From here we find

$$Q_1 = S_{21} Q_2 ,$$

where

$$S_{21} = \sqrt[\alpha]{\frac{\Lambda_2}{\bar{\Lambda}_1}} = \sqrt[\alpha]{\frac{\dfrac{\lambda_1^{gv} \rho L_1}{2D_g f_g^2} + \dfrac{\lambda_v \rho l_2^v}{2D_v f_v^2} + K_v r_{M_2}^\alpha + \dfrac{\lambda_v \rho l_2^{v'}}{2D_v f_v^2} + \dfrac{\lambda_1^{gn} \rho L_1}{2D_g f_g^2}}{\dfrac{\lambda_v \rho l_1^v}{2D_v f_v^2} + K_v r_{M_1}^\alpha + \dfrac{\lambda_v \rho l_1^{v'}}{2D_v f_v^2}}} .$$

Here the index „ 21 " refers the result to the first segment of the two-segment network.

Because

$$Q = Q_1 + Q_2 = S_{21} Q_2 + Q_2 = (1 + S_{21}) Q_2 ,$$

then we have

$$Q_2 = \frac{1}{1+S_{21}} Q, \quad Q_1 = \frac{S_{21}}{1+S_{21}} Q.$$

All that remains is to find p_1':

$$p_1' = p_1 + \rho g\left(h_1 - h_1'\right) + \bar{\Lambda}_1 \left(\frac{S_{21}}{1+S_{21}}\right)^{\alpha} Q^{\alpha}.$$

Let's consider **a network of three segments** (Fig. 3).

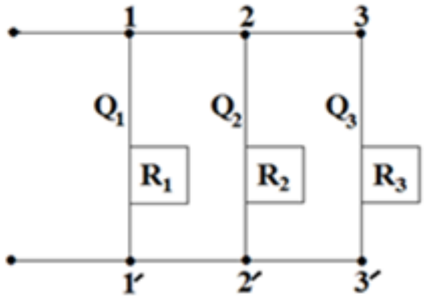


Figure 3. A network consisting of two segments. Let's repeat.

Let's repeat the procedure from the first algorithm for calculating a multi-section heat exchanger. The difference is that instead of identical sections, in this case each link has its own indicators.

Let's calculate the pressure loss on the arcs $22'$ and $233'2'$:

$$p_2 + \rho g\left(h_2 + h_2'\right) - p_2' =$$

$$= \left(\frac{\lambda_v \rho l_2^v}{2D_v f_v^2} + K_v r_{M_2}^{\alpha} + \frac{\lambda_v \rho l_2^{v'}}{2D_v f_v^2}\right) Q_2^{\alpha} = \bar{\Lambda}_2 Q_2^{\alpha},$$

$$p_2 + \rho g\left(h_2 + h_2'\right) - p_2' =$$

$$= \left(\frac{\lambda_2^{gv} \rho L_2}{2D_g f_g^2} + \frac{\lambda_v \rho l_3^v}{2D_v f_v^2} + K_v r_{M_3}^{\alpha} + \frac{\lambda_v \rho l_3^{v'}}{2D_v f_v^2} + \frac{\lambda_2^{gn} \rho L_2}{2D_g f_g^2}\right) Q_3^{\alpha} = \Lambda_3 Q_3^{\alpha}.$$

From the last parts of the equalities we get

$$Q_2 = S_{32} Q_3,$$

where

$$S_{32} = \sqrt[\alpha]{\frac{\Lambda_3}{\bar{\Lambda}_2}} = \sqrt[\alpha]{\frac{\frac{\lambda_2^{gv} \rho L_2}{2D_g f_g^2} + \frac{\lambda_v \rho l_3^v}{2D_v f_v^2} + K_v r_{M_3}^{\alpha} + \frac{\lambda_v \rho l_3^{v'}}{2D_v f_v^2} + \frac{\lambda_2^{gn} \rho L_2}{2D_g f_g^2}}{\frac{\lambda_v \rho l_2^v}{2D_v f_v^2} + K_v r_{M_2}^{\alpha} + \frac{\lambda_v \rho l_2^{v'}}{2D_v f_v^2}}}.$$

Now let's turn to the contour $122'1'$.

The flow rate along the arcs 12 and $2'1$ is equal to $Q - Q_1$, and along the arc $22' - Q_2$. That's why

$$p_1+\rho g\left(h_1+h_1'\right)-p_{1'}=\left(\frac{\lambda_v\rho l_1^v}{2D_v f_v^2}+K_v r_{M_1}^{\alpha}+\frac{\lambda_v\rho l_1^{v'}}{2D_v f_v^2}\right)Q_1^{\alpha}=\left(\bar{\Lambda}_1 Q_1^{\alpha}=\right)$$

$$=\left(\frac{\lambda_1^{gv}\rho L_1}{2D_g f_g^2}+\frac{\lambda_2\rho l_2^v}{2D_2 f_2^2}+K_v r_{M_2}^{\alpha}+\frac{\lambda_2'\rho l_2^{v'}}{2D_2' f_2'^2}+\frac{\lambda_1^{gn}\rho L_1}{2D_g f_g^2}\right)Q_2^{\alpha}\left(=\Lambda_2 Q_2^{\alpha}\right).$$

From here we find

$$Q_1=S_{31}Q_2,$$

where $S_{31}=S_{21}$.

According to the model,

$$Q=Q_1+Q_2+Q_3=S_{31}Q_2+Q_2+Q_3=S_{32}\left(1+S_{31}\right)Q_3+Q_3=\left[1+S_{32}\left(1+S_{31}\right)\right]Q_3.$$

It follows that

$$Q_3=\frac{1}{1+S_{32}\left(1+S_{31}\right)}Q,$$

$$Q_2=\frac{S_{32}}{1+S_{32}\left(1+S_{31}\right)}Q,$$

$$Q_1=\frac{S_{32}S_{31}}{1+S_{32}\left(1+S_{31}\right)}Q.$$

In this case, the pressure loss in the heat exchanger is

$$p_{1'}'=p_1+\rho g\left(h_1-h_1'\right)+\bar{\Lambda}_1 Q_1^{\alpha}=$$

$$=p_1+\rho g\left(h_1-h_1'\right)+\bar{\Lambda}_1\left(\frac{S_{32}S_{31}}{1+S_{32}\left(1+S_{31}\right)}\right)^{\alpha}Q^{\alpha}.$$

Consider **a network of four segments** (see Fig. 2 without the left part) .

Following the steps to calculate a network of three segments, we have

$$Q_3=S_{43}Q_4,$$

where

$$S_{43}=\sqrt[\alpha]{\frac{\dfrac{\lambda_3^{gv}\rho L_3}{2D_g f_g^2}+\dfrac{\lambda_v\rho l_4^v}{2D_v f_v^2}+K_v r_{M_4}^{\alpha}+\dfrac{\lambda_v\rho l_4^{v'}}{2D_v f_v^2}+\dfrac{\lambda_3^{gn}\rho L_3}{2D_g f_g^2}}{\dfrac{\lambda_v\rho l_3^v}{2D_v f_v^2}+K_v r_{M_3}^{\alpha}+\dfrac{\lambda_v\rho l_3^{v'}}{2D_v f_v^2}}};$$

$$Q_2=S_{42}Q_3,$$

where $S_{42}=S_{32}$;

$$Q_1=S_{41}Q_2,$$

where $S_{41}=S_{21}$.

Here

$$Q = Q_1 + Q_2 + Q_3 + Q_4 = \left[1 + S_{42}\left(1 + S_{41}\right)\right]Q_3 + Q_4 =$$

$$= \left\{1 + S_{43}\left[1 + S_{42}\left(1 + S_{41}\right)\right]\right\}Q_4.$$

From here we find

$$Q_4 = \frac{Q}{1 + S_{43}\left[1 + S_{42}\left(1 + S_{41}\right)\right]}.$$

Then

$$Q_3 = \frac{S_{43}}{1 + S_{43}\left[1 + S_{42}\left(1 + S_{41}\right)\right]}Q,$$

$$Q_2 = \frac{S_{43}S_{42}}{1 + S_{43}\left[1 + S_{42}\left(1 + S_{41}\right)\right]}Q,$$

$$Q_1 = \frac{S_{43}S_{42}S_{41}}{1 + S_{43}\left[1 + S_{42}\left(1 + S_{41}\right)\right]}Q.$$

In this case, when $\bar{\Lambda}_1 = \frac{\lambda_v \rho l_1^v}{2D_v f_v^2} + K_v r_{M_1}^\alpha + \frac{\lambda_v \rho l_1^{v'}}{2D_v f_v^2}$ we have

$$p'_{1'} = p_1 + \rho g\left(h_1 - h'_1\right) - \bar{\Lambda}_1\left(\frac{S_{43}S_{42}S_{41}}{1 + S_{43}\left[1 + S_{42}\left(1 + S_{41}\right)\right]}\right)^\alpha Q^\alpha.$$

The result is a visual algorithm for calculating a network of N links.

If the number of links (segments) is equal to N, then for the flow rate of the first segment we have the formula

$$Q_1 = \frac{S_{N(N-1)}S_{N(N-2)}\ldots S_{N3}S_{N2}S_{N1}}{1 + S_{N(N-1)}\left[1 + S_{N(N-2)}\left(1 + \ldots S_{N2}\left(1 + S_{N1}\right)\right)\right]}Q,\ldots,$$

for the flow rate n of the th segment –

$$Q_n = \frac{S_{N(N-1)}S_{N(N-2)}\ldots S_{Nn}}{1 + S_{N(N-1)}\left[1 + S_{N(N-2)}\left(1 + \ldots S_{N2}\left(1 + S_{N1}\right)\right)\right]}Q,\ldots,$$

for the flow rate $N-1$ of the th segment –

$$Q_{N-1} = \frac{S_{N(N-1)}}{1 + S_{N(N-1)}\left[1 + S_{N(N-2)}\left(1 + \ldots S_{N2}\left(1 + S_{N1}\right)\right)\right]}Q$$

and for the flow rate N of the th segment –

$$Q_N = \frac{1}{1 + S_{N(N-1)}\left[1 + S_{N(N-2)}\left(1 + \ldots S_{N2}\left(1 + S_{N1}\right)\right)\right]}Q.$$

Using these values, arc flow rates are calculated:

along the arcs 01 and $0'1'$ the flow rate is $q_0 = Q$;

along the arcs 12 and $2'1'$ the flow rate is $q_1 = Q - Q_1$;

along arcs 23 and $3'2'$ the flow rate is $q_2 = Q - Q_1 - Q_2; \dots$

along arcs $n(n+1)$ and $(n+1)' n'$ the flow rate is $q_n = Q - \sum_{i=1}^{n} Q_i; \dots$

along arcs $N(N-1)$, NN' and $N'(N-1)'$ the flow rate is $q_N = Q - \sum_{i=1}^{N} Q_i$.

With known values of flow rate and local difference in leveling height, it is not difficult to calculate the values of all nodal pressures. But of the nodal pressures we are only interested in $p_{0'}$:

$$p_{0'} = p_0 + \rho g \left(h_0 + h_0'\right) - \left(\frac{\lambda_0^{gv} \rho L_0}{2 D_g f_g^2} + \frac{\lambda_1 \rho l_1^v}{2 D_1 f_1^2} + K_v r_{M_1}^{\alpha} + \frac{\lambda_1' \rho l_1^{v'}}{2 D_1' f_1'^2} + \frac{\lambda_0^{gn} \rho L_0}{2 D_g f_g^2} \right) Q^{\alpha}.$$

All arc costs that are necessary for thermal calculations, as well as the pressure loss on the network, have been determined.

Thermal calculation of the network within the framework of the Shukhov formula is carried out for all arcs that are taken into account when calculating individual heat exchangers.

At the outlet of the expansion tank, the temperature of the heated water is T_H. We need to find the temperature of the exhaust liquid T_K at the inlet to the boiler, i.e. at the end of the two-strand part of the pipeline.

First, we determine the nodal water temperatures in the upper main, where the flow rate decreases along the way. Next, we will determine the temperature differences in the links between the upper and lower lines, where the flow rate is Q_i. At the nodes of the lower main line we find the average temperature of the working agent depending on the flow rates of the mutually connecting flows.

In order not to pile up the volume of calculations, we assume that in the zone n of the th link the ambient temperature is T_{ocn}. Since the proportions of the temperature difference in parts of the upper and lower mains are small, then with this assumption the error will be quite small.

***Arc* 01**. At the end of the arc, the coolant temperature is

$$T_1 = T_{oc0} + \left(T_H - T_{oc0}\right) \exp\left(-Sh_1 \; L_0\right),$$

where $Sh_1 = \dfrac{k_{cp} \pi \left(D_0 + 2\delta_0\right)}{\rho c_B Q}$.

***Arc* 02**. At the end of this segment we have

$$T_2 = T_{oc1} + \left(T_1 - T_{oc1}\right) \exp\left(-Sh_2 \; L_1\right),$$

where $Sh_2 = \dfrac{k_{cp} \pi \left(D_1 + 2\delta_1\right)}{\rho c_B q_1}$. Etc…

Arcs n(n+* 1 *) for n<N. The final temperature at the exit of these arcs, with the designation $Sh_{n+1} = \dfrac{k_{cp} \pi \left(D_{n+1} + 2\delta_{n+1}\right)}{\rho c_B q_{n+1}}$, will be

$$T_{n+1} = T_{ocn} + \left(T_n - T_{ocn}\right) \exp\left(-Sh_{n+1} \; L_n\right) \dots$$

Arc $(N-1)NN'(N-1)'$. The fluid flow through this three-link arc is constant and amounts to Q_N. Let's carry out the calculation along the arcs.

Arc $(N-1)N$:

$$T_N = T_{ocn} + (T_{N-1} - T_{ocN-1})\exp(-Sh_N \ L_{N-1}).$$

Arc NN'. This is a segment with N a heat exchanger.

For each n segment with a heat exchanger, the temperature is calculated in three steps. In the first step, the temperature value at the end of the upper jumper (i.e. at the inlet to the heat exchanger) is calculated:

$$T_{n1} = T_{oc} + (T_n - T_{ocn})\exp(-Sh_{pn1} \ \dot{h}_n)$$

at $Sh_{pn1} = \dfrac{k_{cp}\pi(D_v + 2\delta_v)}{\rho c_B Q_n}$.

For the heat exchanger, the calculation is carried out using our formulas. As a result, the value of the water temperature T_{n2} at the outlet of the heat exchanger at the inlet water temperature is determined T_{n1}. The value T_{n2} is used in the formula for the temperature at the lower end of the downstream pipe:

$$T'_{n'} = T_{oc} + (T_{n2} - T_{ocn})\exp(-Sh_{pn2} \ h'_n)$$

at $Sh_{pn1} = \dfrac{k_{cp}\pi(D_v + 2\delta_v)}{\rho c_B Q_n}$.

Here, according to the model, the vertical pipes have the same diameters D_v and thicknesses δ_v.

This entire three-step process can be expressed through the relationship $T'_{n'} = F(n, T_n, Q_n)$, which is facilitated by the materials for the heat exchanger. And for the arc NN' we use this algorithm and as a result we find the value $T'_{n'}$.

Arc $N'(N-1)'$. Here we have

$$T_{(N-1)'} = T_{oc} + (T_{N'} - T_{ocN'})\exp(-Sh_{N'} \ L_{N'})$$

at $Sh_{N'} = \dfrac{k_{cp}\pi(D_{N'} + 2\delta_{N'})}{\rho c_B Q_N}$.

The node $(N-1)'$ is characterized by the fact that the flows along the arcs NN'(with flow rate Q_{N-1} and temperature T_{N-1}) and $(N-1)NN'(N-1)'$(with flow rate Q_N and temperature $T_{(N-1)'}$) will merge into a common flow flowing along the arc $(N-1)'(N-2)'$. In this regard, we determine the average flow temperature of water at the outlet of the unit $(N-1)'$:

$$T_{(N-1)'} = \frac{Q_{N-1}T'_{(N-1)'} + Q_N T_{(N-1)'}}{Q_{N-2}},$$

where was taken into account $Q_{N-2} = Q_{N-1} + Q_N$, and the value $T_{(N-1)'}$ was calculated using the formula $T'_{N-1} = F\left(N-1, T_{N-1}, Q_{N-1}\right)$.

The average temperature of the liquid is calculated similarly **at**

$n' = \left(N-2\right)',\ \left(N-3\right)',\ \ldots\ 3',\ 2',\ 1'$:

$$T_{n'} = \frac{Q_n T'_n + Q_{n+1} T_{(n-1)'}}{Q_{n-1}}.$$

The final calculated temperature value $T_{1'}$ corresponds to the temperature of the coolant at the outlet of the heat exchanger. Using it, we find the temperature value at the outlet from the network (i.e. at the entrance to the furnace):

$$T_K = T_{0'} = T_{oc0} + \left(T_{1'} - T_{oc0}\right)\exp\left(-Sh_{1'}\ L_0\right),$$

Where $Sh_{1'} = \dfrac{k_{cp}\pi\left(D'_0 + 2\delta'_0\right)}{\rho c_B Q}$.

This is the proposed algorithm for calculating the temperature difference of the liquid working agent in a two-pipe network with N heat exchangers. Note that the calculation for a separate link with a heat exchanger was carried out above with the corresponding elements.

4. Results

As an example, we calculated a two-strand heating network with five radiators.

The radiator parameters were: $l = 0.07\ m$, $h = 0.60\ m$, $h_p = 0.15\ m$, $D_g = 0.03\ m$, $D_v = 0.01\ m$, $\delta_g = 0.03\ m$, $\delta_v = 0.08\ m$, $\lambda_g = 0.002$, $\lambda_v = 0.002$.

The diameters of the jumpers (pipelines in segments) were $D_p = 0.03\ m$, their thickness was $\delta_p = 0.05\ m$, the resistance coefficient – $\lambda_p = 0.002$. In all other arcs, the value of the drag coefficient was calculated for the quadratic flow regime: $\lambda = 0.11\left(k / D\right)^{0.25}$ at $k = 0.005\ m$.

The inlet fluid velocity was $w = 0.8000\ m/s$, which corresponds to the volumetric flow rate $Q = 1.0053\ m^3/s$.

The values of other network indicators are presented in the following table. 1

Table 1. Summary table of initial data for calculation two-strand heating network with five radiators

Indicators	Arc or node numbers					
	0	1	2	3	4	5
Length of the upper and lower arcs, m	10.0	8.0	10.0	5.0	10.0	10.0
Diameters of the upper and lower arcs, m	0.40	0.40	0.40	0.35	0.35	0.35
Wall thicknesses of the upper and lower arches, m	0.005	0.005	0.005	0.004	0.004	0.004
Leveling height of the upper nodes, m	3.50	3.49	3.47	3.45	3.42	3.40
Leveling height of lower nodes m	0.000	0.010	0.030	0.050	0.080	0.100

Leveling height of the upper arc assembly, m	3.50	3.49	3.47	3.45	3.42	3.40
Leveling height of the lower arc unit, m	0.0	0.01	0.03	0.05	0.08	0.1
Length of the upper vertical pipe, m	–	2.78	2.74	2.70	2.64	2.60
Length of the lower vertical pipe, m	–	0.10	0.10	0.10	0.10	0.10
Number of sections in radiators	–	12	10	15	8	7
Ambient temperature, in Celsius	15	20	17	15	17	20

Let us present the calculation results.

The developed software product allows you to obtain complete information about each individual heat exchanger. To control the progress of the calculation, the values of the same indicators were additionally printed for each heat exchanger. In particular , for the 4th heat exchanger with 8 fins, the following results were obtained (Table 2).

Table 2. Summary table of calculated data two-strand heating network with five radiators

Indicators	Numbers of arcs or network nodes					
	0	1	2	3	4	5
Area costs of the upper and lower arcs, l/s	–	1.00531	0.77143	0.55163	0.34843	0.16048
Precinct costs of segments, l/ s	–	0.23388	0.21980	0.20320	0.18795	0.16048
Nodal pressure of the upper arc, Pa	120000	114865.56	112596.83	111217.50	110878.15	110805.50
Pressure at the lower end of the segment, Pa	–	145197.92	142937.12	141593.07	141034.02	141106.68
Nodal pressure of the lower arc, Pa	131912.17	137046.62	139315.34	140694.67	141034.02	141106.68
Nodal temperatures of the upper arc, C	70	69.04	68.05	66.31	65.13	60.08

Coolant temperature at the lower end of the segments, C	–	68.38	67.35	65.57	64.36	59.19
Nodal temperatures of the lower arc, C	51.82	51.92	49.03	53.07	44.17	48.99

Table 3. Calculation results for the second heat exchanger with eight sections

Radiator performance	Numbers of arcs or heat exchanger units							
	1	2	3	4	5	6	7	8
Upper arc flow, l/ s	0.18795	0.16339	0.13936	0.11573	0.09236	0.06917	0.04608	0.02303
Vertical arc flow, l/ s	0.02456	0.02403	0.02364	0.02337	0.02319	0.02309	0.02305	0.02302
Upper nodal pressures, Pa	141034.02	141033.89	141028.37	141033.74	141033.70	141033.68	141033.67	141033.66
Lower nodal pressures, Pa	141028.15	141028.28	141028.37	141028.43	141028.47	141028.49	141028.50	141028.51
Temperatures in the upper nodes, ^{0}C	64.36	64.09	63.78	63.40	62.93	62.30	61.35	59.47
Temperature at the lower end of the vertical pipes, ^{0}C	53.85	53.54	53.19	52.80	52.33	51.71	50.79	51.29
Temperatures in the lower nodes, ^{0}C	54.20	53.92	53.62	53.29	52.93	52.50	51.96	51.29

5. Conclusion

A model and algorithm for calculating a modern heat exchanger and a two-strand heating network within the framework of a quasi-one-dimensional approach are proposed.

When describing mass transfer, the factor of leveling height was taken into account, a single-term representation of the law of friction resistance and analogues of Kirchhoff's laws were used. When modeling heat transfer, the Shukhov and average flow temperature formulas were used.

Analytical formulas are obtained for calculating the flow rates for the elements of the upper and lower arcs, as well as for the vertical segments, and the nodal pressure and temperature of the working fluid in a network with a limited number of segments and its multi-section heat exchangers.

Using the proposed algorithms for calculating the network and its active element, a software product was developed and a computational experiment was carried out to determine the pressure loss and heat of the coolant.

It was revealed that the lowest value of the coolant temperature is observed in the lower part of the most finished element (heat exchanger and segment).

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