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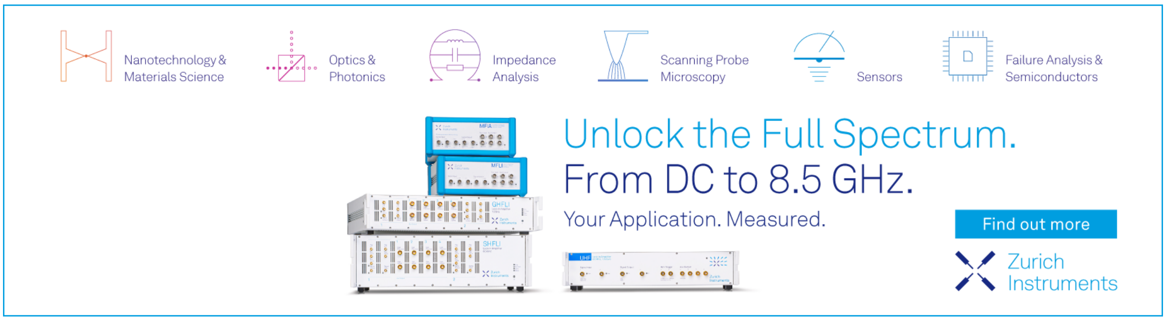

# Mathematical Model and Calculation Algorithm Modern Heat Exchanger Connected to a Two-Pipe Heating Network

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**Abstract.** Using friction and gravity resistance forces, a model of coolant movement through a multi-section heat exchanger has been developed. The friction drag coefficient is taken in the form of a one-term Leibenzon representation. The heat transfer of elementary arcs is taken into account by the averaged heat transfer coefficient through the finned outer surface of the pipelines. To solve the hydraulic problem, analogues of Kirchhoff's laws and the method of mathematical induction were used, and to solve the thermal problem, Shukhov's formulas and the concept of the average flow temperature of the coolant at the confluence of flows were used. Some results of a computational experiment carried out using a newly developed software product based on a mathematical model and calculation algorithm are presented.



## INTRODUCTION

Large-scale work on green energy is underway around the world. Nuclear power plants and hydroelectric power stations are being built, and energy based on solar, wind and geothermal energy is being developed. Along with this, the development and widespread use of resource-saving technologies in relation to sectors of the economy and in everyday life require great attention. In particular, the transition from incandescent lamps to diode lamps contributed to a significant reduction in the electricity used for consecration. Various household devices with maximum energy savings have been developed and are being developed.

A similar picture is observed in the area of heating residential and industrial premises. The trend of abandoning a centralized integrated heating system in favor of separate, compact, multi-circuit and economical heating systems has made it possible to significantly reduce the volume of domestic gas consumption in production.

Design work on heating systems is carried out in accordance with existing standards, which in an integral form determine the volume of heat loss in the heating system. Depending on the methods of using heat and the needs of production shops, two- and multi-line heating networks are organized [1,2]. In practice, for example, thermal power plants were widely used to heat the working unit, which could provide heat to the city or its individual areas [2-4]. To ensure the reliability of the heating network, a ring structure was used [5, 6].

The methodology for hydraulic and thermal calculation of the heat supply network developed gradually. The first steps in this direction related to the solutions of linearized quasi-one-dimensional equations of conservation of mass, momentum and energy in individual sections of pipelines [3, 5, 7]. Subsequently, the theory of flow distribution was formed [5, 6. 8-10]. This theory is based on information about the hydrodynamic regime of friction and analogues of Kirchhoff's laws. Depending on the law of resistance, which is determined by the flow regime, and the averaging methods used, a system of equations is formed from linear and nonlinear ordinary differential equations or partial differential equations. In addition, the direction of flow in individual arcs of the network is not known in advance. The Lobachev-Cross, Vyhandu methods and their modifications for solved such systems have been developed [8, 9, 11-15].

Below is a mathematical model of a heat exchanger that is connected to a two-strand network. The friction resistance law was adopted according to Leibenzon, which is a one-term representation [11-12]. Using the method

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of mathematical induction and analogues of Kirchhoff's laws, formulas for hydraulic calculations were obtained taking into account changes in the leveling height of the route.

Thermal calculation of a multi-section heat exchanger was carried out using the Kirchhoff formula and the formula for the average coolant temperature along the flow.

For the quadratic flow regime, numerical results of the exact solution of the problem of the state of the working fluid in the heat exchanger are presented.

## STATEMENT AND MATHEMATICAL MODEL OF THE PROBLEM

Heat exchangers are made up of separate sections that have the same shapes and dimensions: height $h_c$, width $l_c$, internal diameters: vertical pipes $D_v$ and horizontal pipes $D_g$ To calculate the heat exchanger, the transverse areas of the pipes $f_v$ and $f_g$, their resistance coefficients $\lambda_v$ and , are also required $\lambda_g$. In addition, we need the diameter and outer surface area of the sections and jumpers, the average heat transfer value $k_{cp}$ of the heat exchanger and other parameters that are used in the process of calculating the pressure and heat loss of the working fluid when passing through the heat exchanger.

Hot water descends from above - from the upper main line (Fig. 1).

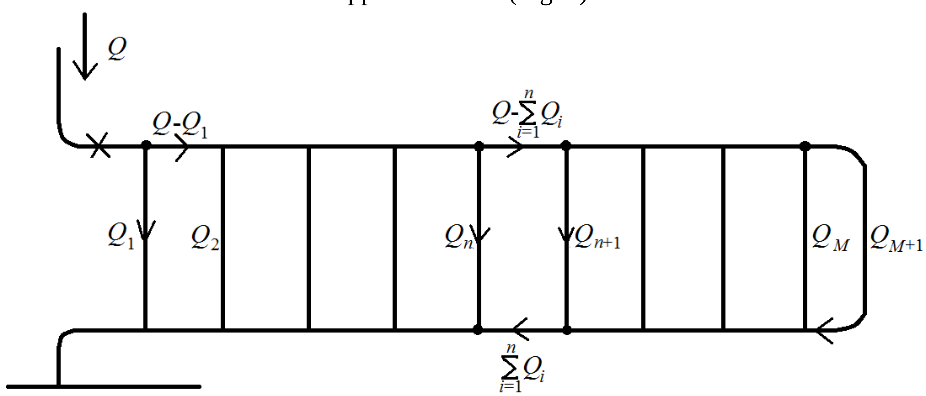


**FIGURE 1.** Schematic representation of the heat exchanger, connected to a two-pipe network on one side

The heat exchanger is connected to the main line through a bend and a valve regulator. Depending on the position of the tap, a different cross-sectional clearance of the jumper is formed, which regulates the water flow $Q$ through the heat exchanger. The total length of the input jumper, taking into account the equivalent $\Delta\dot{l}_p$ of local resistance, is $\dot{l}_p$, the height difference in it $\dot{h}_p$, the average diameter and cross-sectional area determined for the jumper are $\dot{D}_p$ and $\dot{f}_p$. The resistance coefficient of the bend (elbow) and crane is $\dot{\lambda}_p$. The spent working agent, through the final section of the heat exchanger, enters the lower line through the outlet pipe, which has indicators $l_p$, $h_p$, $D_p$, $f_p$ and $\Delta l_p$. The last indicator is responsible for the local bending resistance of the output jumper.

We assume that there is no loss of water in the heat exchanger, i.e. the input and output water flow rates are the same and amount to $Q$.

It is required to determine the pressure and temperature losses of the liquid in the heat exchanger if the inlet temperature of the heated water is $T_H$, the inlet pressure is $p_H$, and the average room temperature is $T_{oc}$.

First, the hydraulics problem is solved: arc flow rates and node pressures are determined, and then the water temperature values in the design nodes of the heat exchanger are calculated.

## METHODS

Hydraulic calculation of the network is carried out taking into account changes in the leveling height of nodes $h$ and pressure drop according to the Darcy-Weisbach law:

$$\frac{dp}{dx}+\rho g\frac{dh}{dx}+\frac{\lambda}{2D}\rho w^2=0 , \quad (1)$$

where traditional notations are used [3, 5, 7, 24].

When calculating the pressure drop, we implement a one-term representation of the law of viscous friction resistance, generalizing Nikuradze's experimental results for the resistance coefficient of a round pipe at various flow rates, diameters and equivalent pipe roughness [11-12]:

$$(2)\ \lambda=\zeta(k/D)^{\theta}\,\mathrm{Re}^{n} . \quad (2)$$

Here, in the laminar flow regime, you can use the Stokes formulas ($\zeta=64$, $\theta=0$, $\varphi=-1$), in the transitional regime - the Zaichenko formulas ($\zeta=0.0025$, $\theta=0$, $\varphi=1/3$), in the smooth regime of turbulent flow around roughness - the Blasius formulas ($\zeta=0.3164$, $\theta=0$, $\varphi=-1/4$), in the mixed regime - the Leibenzon formulas ($\zeta=10^{-0.627}$, $\theta=0.127$, $\varphi=-0.123$), and in the developed regime - the Shifrinson formulas ($\zeta=0.11$, $\theta=0.25$, $\varphi=0$).

The upper units of the heat exchanger with *M+1* sections as *1, 2, 3, ... , M, M+1* and the lower ones – through $1'$, $2'$, $3'$, … , $M'$, $(M+1)'$. We denote the costs through vertical pipes as $Q_1$, $Q_2$,…, $Q_m$,…, $Q_M$. If their values are known, then the fluid flow through the last pipe on the right is $Q_{M+1}=Q-\sum_{i=1}^{M}Q_i$. The arc flow rates through the upper nozzle are $q_1=Q-Q_1$, $q_2=Q-Q_1-Q_2$, …, $q_m=Q-Q_1-Q_2-...-Q_m$, …, $q_{M-1}=Q-Q_1-...-Q_m-...-Q_{M-2}-Q_{M-1}$ and $q_M=Q-Q_1-...-Q_m-...-Q_{M-1}$. The arc flow rates along the lower part are also determined.

As you can see, the total water flow through the vertical pipes is $Q$. By this we ensured the fulfillment of an analogue of Kirchhoff's first law.

We denote the nodal pressures by $p_1$, $p_2$, …, $p_m$, …, $p_M$, $p_{M+1}$, $p_{1'}$, $p_{2'}$, …, $p_{m'}$, …, $p_{M'}$, $p_{(M+1)'}$. To determine the values of these quantities, we use an analogue of Kirchhoff's second law.

The calculation begins by determining the pressure drop $p$ and leveling height $h$ at the inlet jumper according to the solution of the above equation[13-25].

The input jumper has the shape of a bend (elbow). Its resistance consists of friction resistance along the length of the jumper and local bending and valve-regulator resistances, expressed in total through $\Delta\dot{l}_p$. The total length of the input jumper is $l_p$. If we denote the inlet pressure by $p_H$, then we have

$$p_H+\rho g\dot{h}_p-p_1=\frac{\lambda_p\rho(\dot{l}_p+\Delta\dot{l}_p)}{2\dot{D}_p\dot{f}_p^2}Q^{\alpha}=K_1Q^{\alpha} . \quad (3)$$

The density $\rho$ of the liquid is taken for the average temperature of the coolant.

In the lower jumper, the pressure drop is calculated using the formula

$$p_{1'}+\rho g h_p-p_K=\frac{\lambda_p\rho l_p}{2D_p f_p^2}Q^{\alpha}=K_KQ^{\alpha} , \quad (4)$$

where $p_K$ is the value of water pressure at the point of connection of the heat exchanger to the lower line. Here the values of $l_p$, $\lambda_p$ and $D_p$ as noted above, are taken without a valve-regulator.

The main difficulty of hydraulic calculations is related to the circularity of the network under consideration, which is due to the unknown direction of flow in specific arcs. To solve this problem, we turn to the provisions of mathematical induction, taking the number of rings *M=1, M=2* etc., and determine the values of arc flow rates.

Number of circuits *M=1*. In this case, a contour $122'1$ of two sections is formed (Fig. 3.2). The total flow is distributed over the arcs $11'$ and $122'1'$. Let us denote them by $Q_1$ and $Q_2$, moreover $Q_1+Q_2=Q$. The pressure drop along the arcs $122'1'$ will $11'$ be the same:

$$\begin{cases} p_1 + \rho g h_c - p_{1'} = K_v Q_2^\alpha \\ p_1 + \rho g h_c - p_{1'} = \left(2K_g + K_v\right) Q_2^\alpha \end{cases} \tag{5}$$

Here and below we use the following notation:

$$K_g = \frac{\lambda_g \rho l_c}{2 D_g f_g^2}, \; K_v = \frac{\lambda_v \rho h_c}{2 D_v f_v^2}, \tag{6}$$

which relate to the horizontal and vertical pipes of the heat exchanger sections [26].

System (1) implies the equality

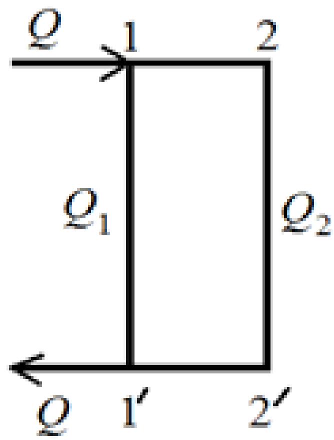


**FIGURE 2.** Diagram of a heat exchanger with two sections

$$K_v Q_1^\alpha = \left(2K_g + K_v\right) Q_2^\alpha . \tag{7}$$

The advantages of our mathematical model over well-known models [5, 6, 8, 10] of a multi-circuit network are that the direction of flow in the arcs is determined in advance so that all costs $Q_i$ have positive values. Because coefficients $K_g$ and $K_v$ are also positive quantities, then we can take:

$$\sqrt[\alpha]{K_v} Q_1 = \sqrt[\alpha]{2K_g + K_v} \left(Q - Q_1\right) \tag{8}$$

This form of notation in mathematics is accepted for the case $\alpha = 2$, i.e. in quadratic resistance mode. But in other cases ($\alpha \neq 2$) the designation is also accepted $\sqrt[\alpha]{x} = x^{1/\alpha}$. In this case, from equation (3) we find

$$Q_1 = \frac{\sqrt[\alpha]{2K_g + K_v}}{\sqrt[\alpha]{K_v} + \sqrt[\alpha]{2K_g + K_v}} Q = r_1 Q, \; Q_2 = \left(1 - r_1\right) Q \tag{9}$$

Then from the first equation of system (1) we can find

$$p_1' = p + \rho g h_c - K_v Q_1^\alpha \tag{10}$$

Formulas for *M=2, 3, …*

For the occasion *M* parallel contours problem is solved as follows.

Respecting the generality with the previous options, we proceeded from the equality

$$Q_2 = r_{M-1} \left(Q - Q_1\right) \tag{11}$$

Here $r_{M-1} = \dfrac{\sqrt[\alpha]{2K_g + r_{M-2}^\alpha K_v}}{\sqrt[\alpha]{K_v} + \sqrt[\alpha]{2K_g + r_{M-2}^\alpha K_v}}$.

At $r_0 = 1$ and further the values $r_m$ are calculated recursively.

Let us turn to the contour , where the flows $122'1'$, $Q_2$, $Q - Q_1$ and $Q_1$, have been established along arcs along the clock hand $Q - Q_1$.

Let's calculate the pressure drop along the arcs $11'$ and $122'1'$:

$$p_1 + \rho g h_c - p_{1'} = K_v Q_1^\alpha = K_g \left(Q - Q_1\right)^\alpha + K_v Q_2^\alpha + K_g \left(Q - Q_1\right)^\alpha . \tag{12}$$

Taking into account the new value, $Q_2$ we obtain the equation

$$K_v Q_1^{\alpha} = \left(2K_g + K_v r_{M-1}^{\alpha}\right)\left(Q - Q_1\right)^{\alpha} . \tag{13}$$

From here we find

$$Q_1 = \frac{\sqrt[\alpha]{2K_g + r_{M-1}^{\alpha} K_v}}{\sqrt[\alpha]{K_v} + \sqrt{2K_g + r_{M-1}^{\alpha} K_v}} Q = r_M Q . \tag{14}$$

Let us determine the arc flow rates for the remaining vertical pipes: $Q_2 = r_{M-1}(1 - r_M)Q$, $Q_3 = r_{M-2}(1 - r_{M-1})(1 - r_M)Q$, …, $Q_m = r_{M-m+1}(1 - r_{M-m+2})...(1 - r_M)Q$ , $Q_M = r_1(1 - r_2)(1 - r_3)...(1 - r_M)Q$, $Q_{M+1} = (1 - r_1)(1 - r_2)(1 - r_3)...(1 - r_M)Q$.

Let us determine the costs for the upper horizontal arcs: $q_1 = (1 - r_M)Q$, $q_2 = (1 - r_{M-1})(1 - r_M)Q$, …, $q_m = (1 - r_{M-m+1})(1 - r_{M-m+2})...(1 - r_M)Q$ , $q_M = (1 - r_1)(1 - r_2)...(1 - r_M)Q$.

Similar dependencies for the lower horizontal arcs of the network: $q_1' = Q_M + Q_{M-1} + ... + Q_2$, $q_2' = Q_M + Q_{M-1} + ... + Q_3$ ,…, $q_m' = Q_M + Q_{M-1} + ... + Q_m$ ,…, $q_{M-1}' = Q_M + Q_{M-1}$, $q_M' = Q_{M+1}$.

These values could be expressed in terms of $r_M$, $Q$, but here is an option that is easily implemented in programming.

The same algorithm was compiled to calculate the degree (square) pressure difference along an arbitrary arc.

The pressure value $p_1$ at the 1st node of the upper pipe is known. Then for subsequent nodes $i - 2..M + 1$ of the upper arc the pressure values are calculated using the recurrent formula:

$$p_i = p_{i-1} - K_g q_{i-1}^{\alpha} , \tag{15}$$

in the lower nodes of vertical pipes at known values $K_v$:

$$p_i' = p_i - K_v Q_i^{\alpha} , \tag{16}$$

at the end of the lower arc, the pressure value at the nodes of the lower arc can also be calculated using another formula:

$$p_{i'}' = p_{(i+1)'}' - K_g q_{i'}^{\alpha} . \tag{17}$$

We are only interested in the change in pressure along the arc $11'$. The pressure value at the outlet of the heat exchanger (to the input of the output jumper) is:

$$p_{1'} = p_1 + \rho g h_c - K_v Q_1^{\alpha} = p_1 + \rho g h_c - K_v r_M^{\alpha} Q^{\alpha} . \tag{18}$$

Thus, the flow rates for the vertical pipes $Q_i$, the upper pipes $q_i$, and the lower $q_i'$ pipes, as well as the pressure drop along the arc , are determined $11'$, which corresponds to the total resistance of the heat exchanger.

Taking into account the upper and lower jumpers, the relationship between the inlet and outlet pressures of the heat exchanger is

$$p_H + \rho g\left(h_c + \dot{h}_p + h_p\right) - p_K = \left(K_1 + K_v r_M^{\alpha} + K_K\right)Q^{\alpha} . \tag{19}$$

## MATERIALS

Based on the above material, a program was compiled in the Pascal ABC environment.

The main indicators were: $\rho = 1000.0\ kg\,m^{-3}$, $l = 0.07\ m$, $h = 0.6\ m$, $h_p = 0.15\ m$, $\dot{h}_p = 0.15\ m$, $D_g = 0.03\ m$, $D_v = 0.03\ m$, $D_p = 0.03\ m$, $D_m = 0.03\ m$, $\dot{D}_p = 0.03\ m$, $\lambda_g = 0.002$, $\lambda_v = 0.002$, $\dot{\lambda}_p = 0.2$, $\lambda_p = 0.2$, $\lambda_m = 0.002$, $p_H = 110000.0\ Pa$, $g = 9.80665\ m\,s^{-2}$.

The reaction of the final results to changes in a number of initial indicators was checked. Changes in the values of internal and external diameters, length and height of sections showed adequate changes in results according to physical concepts.

Let us present the results of calculating the coefficient $r_i$ Since the geometric dimensions of the heat exchanger did not change in the calculations, they applied to all calculation options.

**TABLE 1.** Coefficient values $r_i$ for given geometric parameters heat exchanger sections

| $i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r_i$ | 1 | 0.50012 | 0.33381 | 0.25108 | 0.2019 | 0.16962 | 0.14707 | 0.13066 | 0.11839 | 0.10903 | 0.10181 | 0.09618 | 0.09178 | 0.08831 | 0.08558 | 0.08343 | 0.08172 | 0.08038 | 0.07931 | 0.07847 | 0.0778 |

Below we present some of the comparison results in the form of graphs. Here and further in the graphs, number 1 corresponds to the speed of the total flow in the heat exchanger $w = 0.3\ m\,s^{-1}$, number 2 – velocity $0.4\ m\,s^{-1}$. Calculations were carried out for variants of ambient temperature $T_{oc} = 293.15\ K$.

Let us imagine the values of arc volumetric flow rates (liters per second) at $T_{oc} = 293.15\ K$, $w = 0.3\ m\,s^{-1}$ for radiators with five, ten and fifteen sections.

Here, in three options, the flow rate through the first arc is the total flow rate (0.21206 l/s) of the working agent through the heat exchanger [21]. In the first vertical pipes, the flow rates were 0.04282, 0.02312 and 0.01815 l/s, and in the final vertical pipes - 0.04219, 0.02040 and 0.01254 l/s. It is not difficult to make sure that it is distributed between the vertical pipes in descending order and the sum of the flow rate through all vertical pipes is equal to the total flow rate through the heat exchanger (Fig. 3). In the results for the upper and lower arcs there were differences ( $\pm 1$) in the last digit.

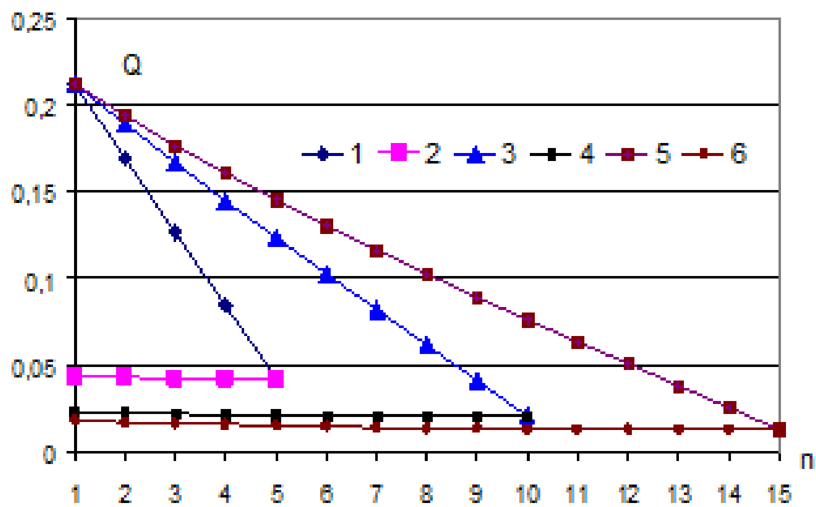


**FIGURE 3.** Distribution of volumetric flow rates in the upper (1,3,5) and vertical pipes (2,4,6) in heat exchangers with five, ten and fifteen sections

With an increase in the number of sections, the costs for the vertical pipes decrease, because, as already noted, the sum of the flow rate for all vertical pipes is equal to the total flow rate for the heat exchanger.

Calculations of nodal pressures were carried out according to the quadratic law of resistance. The results, in particular from table. 2 showed that without taking into account changes in leveling height, the main pressure drops correspond to the vertical pipes of the heat exchanger, since the diameters of the vertical pipes ( $D_v = 0.01\ m$) are significantly smaller than the other pipes. In the calculation algorithm, the pressures in the upper arc were first calculated, then the pressure drop along the vertical pipes was calculated, and finally a number of values of the nodes of the lower arc were calculated.

**TABLE 2.** Values of nodal pressures (Pa) in a heat exchanger with ten sections

| Nodal pressure (Pa) | Arc numbers | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Upper arc nodes | 111470 | 111469.8 | 111469.7 | 111469.6 | 111469.5 | 111469.4 | 111469.4 | 111469.4 | 111469.4 | 111469.4 |
| The lower ends of the vertical pipe branch | 111464.8 | 111464.9 | 111465 | 111465.1 | 111465.2 | 111465.3 | 111465.3 | 111465.3 | 111465.3 | 111465.3 |
| Knots of the bottom arcs | 111464.8 | 111464.9 | 111465.1 | 111465.2 | 111465.2 | 111465.3 | 111465.3 | 111465.3 | 111465.3 | 111465.3 |

The results of nodal pressures for a heat exchanger with ten sections are presented. Similar results were obtained for a heat exchanger with five (output 111452.13 Pa) and fifteen (output 111466.75 Pa) sections (output of a heat exchanger with five sections 111464.8 Pa). A comparison of the results showed that with an increase in the number of sections, the output pressure of the heat exchanger increases. This fact is consistent with the laws of hydraulics [7]: an increase in the number of parallel threads contributes to a nonlinear decrease in pressure loss.

The upper curves of Fig.4 correspond to $w = 0.4\ m\,s^{-1}$, and the lower ones correspond to $w = 0.3\ m\,s^{-1}$. In practice, there is a linear change in the flow rate along the upper and lower arcs (Fig. 4), since the flow rate at the beginning of the upper arc is an order of magnitude greater than the flow rate through the vertical pipes (Fig. 5). With the removal of the inlet section, the flow rates through the vertical pipes decrease more slowly and this decrease is close to the exponential law.

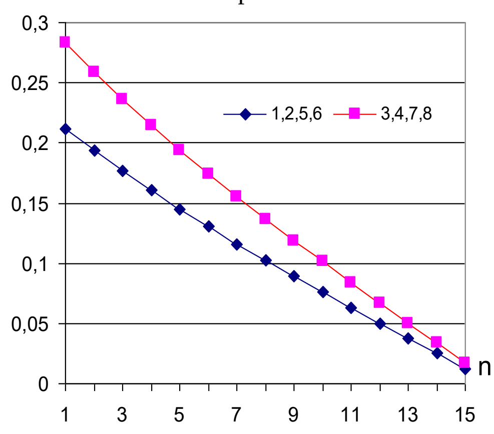


**FIGURE 4.** Comparison of the results of local flow rates of the upper and lower arcs for two calculation options (see text for symbols)

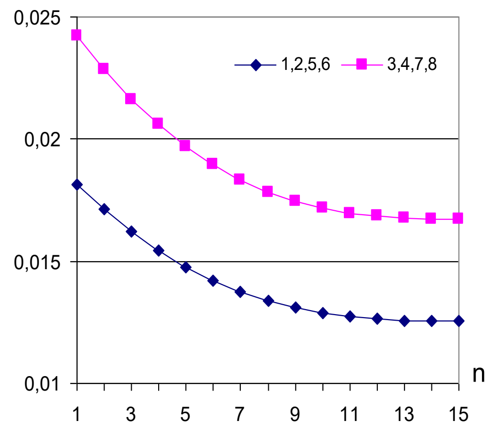


**FIGURE 5.** Comparison of the results of local flow rates for vertical pipes for two calculation options (see text for symbols)

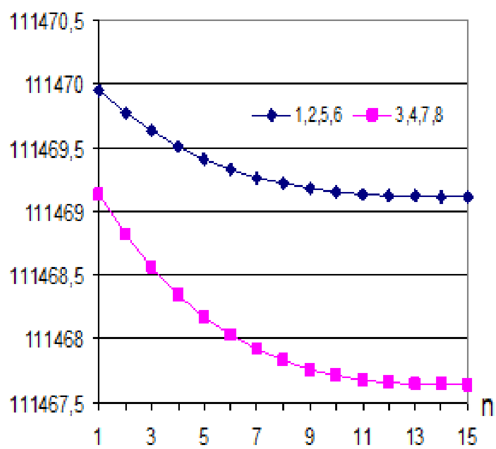


**FIGURE 6.** Comparison of the results of nodal pressures of the upper arc for two calculation options (see text for symbols)

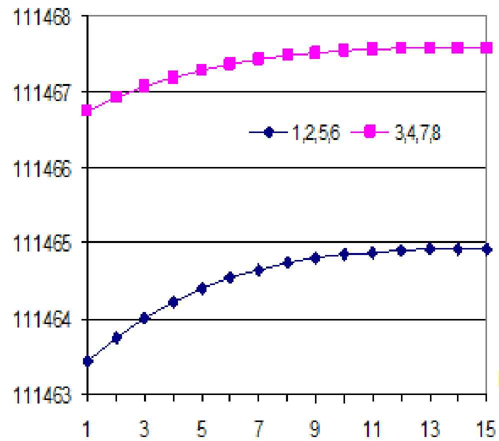


**FIGURE 7.** Comparison of the results of nodal pressures of the lower arc for two calculation options (see text for symbols)

In Figure 8 shows the pressure values at the outlet of the heat exchanger with 2, 3,... and 15 sections. The upper final pressure curves are related to the velocity $w = 0.3 \ m s^{-1}$, and the lower ones are related to $w = 0.4 \ m s^{-1}$. As the number of sections increases, the pressure loss decreases, which is explained by Kirchhoff's second law: with an increase in the number of parallel threads, the pressure loss decreases.

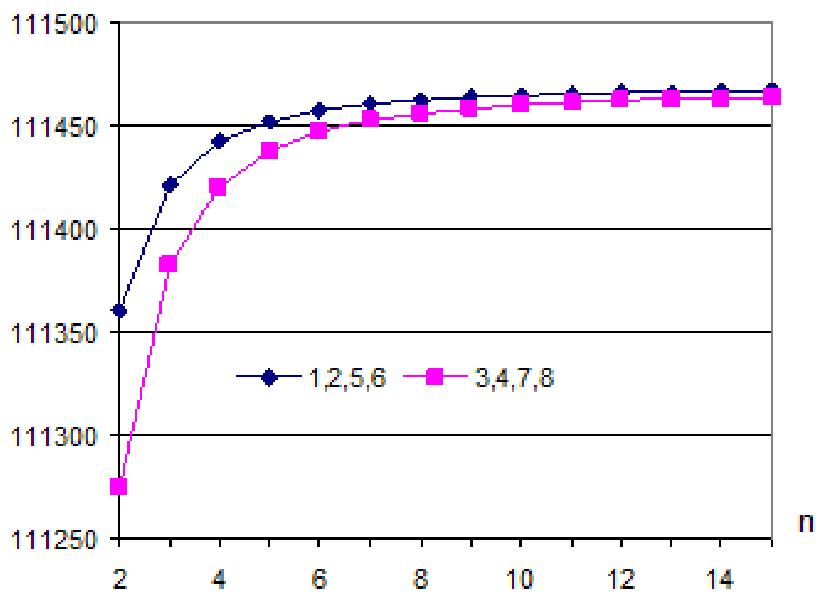


**FIGURE 8.** Pressure values at the outlet of the heat exchanger for 8 variants of the initial data, depending on the number of sections in the heat exchanger (see text for symbols)

## CONCLUSION

Within the framework of a quasi-one-dimensional approach, a mathematical model of a modern heat exchanger connected to a two-strand heating network is proposed.

Using analogues of Kirchhoff's two laws and the provisions of mathematical induction, a method for hydraulic calculation of a modern heat exchanger in a limited number of sections has been developed. The law of resistance takes into account all five hydrodynamic flow regimes, since it is used in the formula of its one-term representation. When describing mass transfer, the factor of leveling height is taken into account.

Analytical formulas are obtained for calculating local flow rates of incompressible fluid and pressure values in nodes. With known values of sectional calculations, the values of nodal pressures of the working fluid in a multi-section heat exchanger were determined.

It was revealed that the costs through the vertical radiator pipes are almost uniform: with an increase in the number of sections, as well as with distance from the entrance to the heat exchanger, the costs through the vertical pipes decrease.

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