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Thermal calculation of a modern multi-section heat exchanger

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Thermal Calculation of a Modern Multi-Section Heat Exchanger

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Abstract. A mathematical model and algorithm for calculating a heat exchanger consisting of similar sections and connected to a two-line heating network are proposed. The upper and lower, as well as vertical heat exchange pipes are presented with their own lengths and internal diameters. The outer diameters of the nozzles are presented taking into account the ribs. For known values of arc flow rates, input temperature and ambient temperature, the path heat loss is described by Shukhov's formula. When flows merge in the lower nodes, the coolant temperature is defined as the average flow rate. The results of calculations for various numbers of sections in the heat exchanger are given.

INTRODUCTION

The growth of industrial and housing construction, improvement of amenities and improvement of sanitary and living conditions of the population lead to a significant increase in the volume of consumption of water, gas and electricity for economic and productive needs. These needs are met through the construction of utility networks, as well as through the modernization of existing systems [1-9]. The latter option contributes to a significant reduction in capital investment and requires a search for system reserves that make it possible to provide the population and infrastructure of the economy with the necessary product [10].

One of these reserves is to improve the management of technological processes for the supply and distribution of the target product (water, gas and electricity) based on the use of modern methods of mathematical and computer modeling. At the same time, the main attention must be paid to the reliability and controllability of utility networks in order to guarantee a reserve of their capacity and the possibility of their prompt reconstruction. Computer modeling and automation must accompany the entire life cycle of engineering networks: the design stage, work monitoring (operational management) and reconstruction [12-15].

Engineering networks belong to the class of continuously evolving systems, the development of which occurs in time and space. When operational management capabilities are exhausted, the reconstruction mechanism is activated. Here, the main role is played by capacity capabilities and consumption volume, as well as controlled and uncontrollable internal and external factors.

In terms of their structure, utility networks can be linear, looped or combined. The widespread use of a looped or multi-ring network is explained by the fact that the flow in it, observing the laws of nature, self-generates with minimal energy consumption. Another advantage of multi-circuit networks is to ensure the reliability of functions in the event of failure of certain arcs of the utility network [13-16]. These failures are corrected by increasing the consumption of target products along other routes (arcs). And in networks with a linear structure, the failure of a certain arc leads to the failure of the entire system.

As an example of the positive features of the ring organization, let us cite the presence of parallel threads of main gas pipelines. They communicate with each other through jumpers, which are installed at certain distances [17-19].

If the parallel threads have different diameters, the bridges contribute to the same pressure loss along the parallel threads. If a certain section of the mainline fails, then disconnecting this section leads to a redistribution of the flow along the existing individual parallel threads. Those capital investments in installing jumpers are justified by ensuring the reliability of the utility network.

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Let's introduce the main consumers of the theory of flow distribution. In addition to the gas transportation system, this can include water supply networks, oil and petroleum product pipelines, pneumatic drives and hydraulic drives [20].

The provisions of the theory of flow distribution are also successfully implemented when modeling space heating, heat supply for a microdistrict or an entire city, as well as traffic flow on a road network [25, 26].

Below we propose an algorithm for thermal calculation of a separate heat exchanger with a limited number of sections, which is connected to a two-strand heat supply network.

The basics of the theory of heat transfer and its types are presented in works [1-6]. Variants of the heat transfer equation are proposed for normal conditions, when the heat flow is proportional to the temperature gradient and the equation is of parabolic type, and for extreme conditions. That occur under the influence of powerful sources: chemical laser, electric or gas welding and are described by hyperbolic type equations taking into account the relaxation properties of the process. Analytical solutions to heat transfer problems have been obtained for various variants of boundary conditions in various orthogonal spatial coordinates.

A special place in thermal calculations is occupied by the problems of the heat transfer process from flowing coolants, which can represent free or forced convection [21-23]. Unfortunately, taking into account convective heat transfer requires turning to a system of nonlinear Navier-Stokes equations, which are practically not amenable to analytical solution, and existing solutions differ significantly from the phenomenon being studied, at least in that in nature the flow is turbulent. In this regard, an engineering version of the calculation has been developed - a quasi-one-dimensional approach to describing the hydrodynamics of the flow, accompanied by the heat transfer process [20-23].

A frequently used result of this engineering approach is the Shukhov formula [1, 10]. It was obtained taking into account the average value of the heat transfer coefficient in the coolant-pipeline-thermal insulation-environment system and the average value of the outer diameter of the pipeline. In literary sources one can also find modifications of the Shukhov formula [1, 10].

Along with Shukhov's formula, when developing an algorithm for calculating the heat exchanger, a formula was used that determines the average flow temperature of the currents combining into one flow.

Some numerical results are presented that demonstrate the features of the proposed mathematical model and calculation algorithm [24].

STATEMENT AND MATHEMATICAL MODEL OF THE PROBLEM

In Fig. 1 schematically shows a heat exchanger with $M+1$ identical sections, which is connected to the upper and lower lines through jumpers. The upper jumper is equipped with a valve-regulator. Considering the input temperature, ambient temperature, geometric parameters and arc flow rates of the coolant by volume as given, it is necessary to determine the outlet temperature of the coolant.

The object is represented as a system consisting of finned tubes connected in series and parallel (Fig. 1). For the calculation, we need geometric quantities (mainly arc flow rates) and thermophysical indicators of the system.

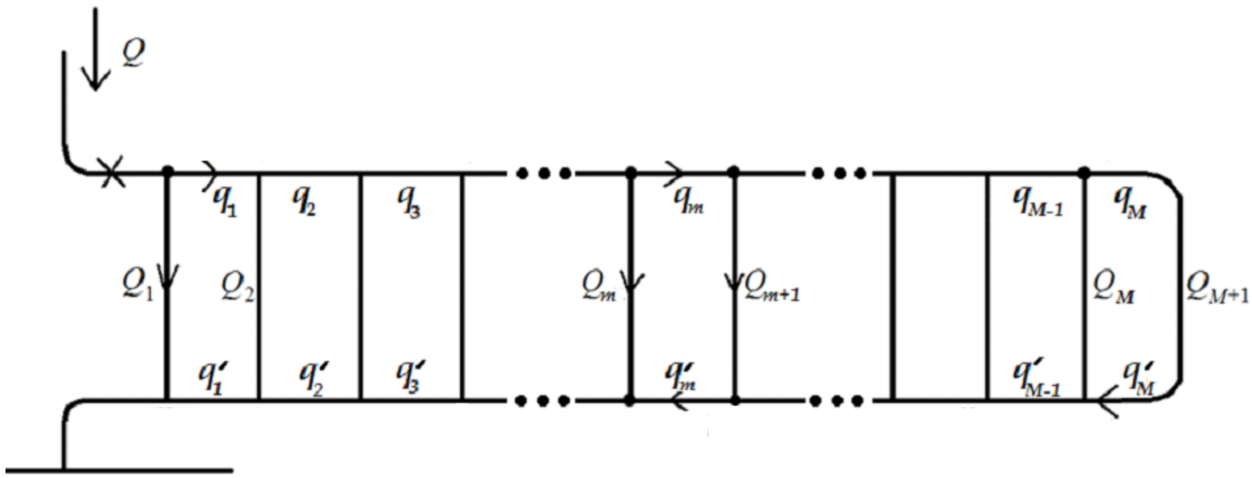

FIGURE 1. Schematic representation of the heat exchanger, connected to a two-pipe network on one side

The upper and lower left parts of the Fig.1 attach the input and output jumpers connecting the heat exchanger with the upper (coolant distributing) and lower (coolant collecting) lines.

The input jumper, taking into account local resistance, has a length of $\dot{l}_p$ and the output jumper has a length of l_p. The leveling height difference in them is $\dot{h}_p$ and h_p. The internal diameters are $\dot{D}_p$ and D_p and the cross-sectional areas are $\dot{f}_p$ and f_p. Because the thickness of their walls, taking into account the ribs, is $\dot{\delta}_p$ and δ_p then the outer diameters are $\dot{D}_p + 2\dot{\delta}_p$ and $D_p + 2\delta_p$.

New sections can be added to the heat exchanger or the number of sections can be reduced. In general, we believe that the number of sections is $M+1$. The height of the sections is h_c, length l_c, internal diameter of vertical pipes D_v, and horizontal ones D_g, averaging of wall thickness taking into account the ribs - δ_v and δ_g. Depending on these data, the cross-sectional areas f_v and are determined f_g, as well as the outer diameters of the vertical and horizontal pipes of the heat exchanger.

From the hydrodynamic indicators we need: input (and output) flow, Q arc (or section) flow of the upper q_i and lower $q_i' = q_i$, as well as vertical Q_i pipes. We consider that they are defined in m^3/s.

The thermophysical indicators of the system consist of the heat capacity c_p of the working fluid, the average heat transfer value k_{cp} of the coolant elements, the temperature at the inlet to the heat exchanger is T_H, and at the outlet T_K, ρ, Q is the mass flow rate of the coolant. The coolant temperature changes smoothly along the length of the arcs, which is described by Shukhov's formula [1, 10]:

$$T(x) = T_{oc} + (T_H - T_{oc})\exp\left(-\frac{kD_{op}}{c_\rho \rho Q}x\right). \tag{1}$$

An abrupt change in temperature is observed at the points of confluence of two flows with flow rates Q_1 and Q_2 and temperature T_1 and T_2. Then the average flow temperature of the coolant after merging is determined by the formula

$$T_K = \frac{T_1 Q_1 + T_2 Q_2}{Q_1 + Q_2}. \tag{2}$$

The total loss of thermal energy in the heat exchanger is calculated by the formula:

$$Q_{nom} = \rho Q c_p (T_H - T_K). \tag{3}$$

METHODS

Let us number the nodes of the upper arc through *1, 2, 3, ... , M, M+1* and the nodes of the lower arc through $1'$, $2'$, $3'$, ..., M', $(M+1)'$. First, we determine the nodal temperatures in the upper arc (T_1, T_2, ..., T_M), then - in the lower nodes of the vertical pipes (for them, the input data are the nodal temperatures of the upper arc), then - the temperatures in the lower arcs and at the end - the output temperature T_K.

Node 1. Calculation of the temperature of the working agent at the end of the input jumper. In it, the water flow rate is Q, the average diameter of the outer surface $\dot{D}_p + 2\dot{\delta}_p$ and the inlet temperature T_K.

Let us determine the value of the Shukhov coefficient for the input jumper:

$$Sh_{\dot{p}} = \frac{k_{cp}\pi(\dot{D}_p + 2\dot{\delta}_p)}{\rho Q c_B}. \tag{4}$$

Then in node 1 the temperature of the heated water is

$$T_1 = T_{oc} + (T_H - T_{oc})\exp(-Sh_{\dot{p}}\dot{l}_p), \tag{5}$$

where $\dot{l}_p$ is the total length of the input jumper without taking into account local resistances.

Node 2. The arc consumption 12 is q_1. Due to this

$$T_2 = T_{oc} + (T_1 - T_{oc})\exp(-Sh_g l_c), \tag{6}$$

where l_c is the length of one section; here and further

$$Sh_g = \frac{k_{cp}\pi(D_g + 2\delta_g)}{\rho q_i c_B}.$$

Nodes *3…M+1*. At these we use a similar recurrent formula for temperature:

$$T_m = T_{oc} + (T_{m-1} - T_{oc})\exp(-Sh_g l_c). \tag{7}$$

To calculate the temperature of heated water at the lower ends of vertical pipes *1, 2, ..., m, ..., M+1* a single formula is used

$$\overline{T}_{m'} = T_{oc} + (T_m - T_{oc})\exp(-Sh_{vm} h_c), \tag{8}$$

where h_c is the height of the section (the distance between the axes of the upper and lower nozzles);

$$Sh_{vm} = \frac{k_{cp}\pi(D_v + 2\delta_v)}{\rho Q_m c_B}.$$

But here different flow rates Q_m through the vertical pipes are used.

Let's move on to calculating the nodal temperatures in the lower arcs. Note that in links M(M+1), $(M+1)(M+1)'$, $(M+1)'M$ the coolant flow rate is the same: $Q_{M+1} = q_{(M+1)'} = q_{M+1}$.

Heated water flows Q_{M+1} to the node on the right with flow rate M' and temperature

$$\dot{T}_{M'} = T_{oc} + \left(T_{(M+1)'} - T_{oc}\right)\exp\left(-Sh_{v(M+1)'} h_g\right). \tag{9}$$

Here and below we used the notation

$$Sh_{vm'} = \frac{k_{cp}\pi(D_g + 2\delta_g)}{\rho q_{m'} c_B}.$$

Due to the merging of flows with flow rates Q_M, Q_{M+1} the average temperature of the heated agent at the beginning of the arc $M'(M-1)'$ is

$$\dot{T}_{M'} = \frac{Q_M \overline{T}_{M'} + Q_{M+1}\dot{T}_{M'}}{Q_M + Q_{M+1}} = \frac{Q_M \overline{T}_{M'} + Q_{M+1}\dot{T}_{M'}}{q'_M}. \tag{10}$$

A similar formula is obtained for the nodal temperatures of water at the nodes $m = (M-1)', \ldots, 1'$.

$$T_{m'} = \frac{Q_m \overline{T}_{m'} + q_m \dot{T}_{m'}}{q_{m-1}}, \tag{11}$$

to use which you first need to find $\overline{T}_{m'}$.

In particular, at the outlet of the node $1'$ the average water temperature is

$$T_{1'} = \frac{Q_1 \overline{T}_{1'} + q_1 \dot{T}_{1'}}{Q}. \tag{12}$$

It remains to determine the temperature of the water that enters the “return” - the lower line:

$$T_K = T_{oc} + (T_{1'} - T_{oc})\exp(-Sh_p l_p), \tag{13}$$

where l_p is the length of the output jumper;

$$Sh_p = \frac{k_{cp}\pi(D_p + 2\delta_p)}{\rho Q c_B}.$$

The presented algorithm is easily implemented in the form of a calculation program where known local costs are used.

The main interest is the nodal temperatures of the water and, especially, the temperature at the outlet of the heat exchanger.

As an example, below we present the values of nodal temperatures for a heat exchanger with ten sections (Table 1), obtained at $T_H = 343.15\ K$ and $T_{oc} = 340.76\ K$.

The nature of the temperature change of the working agent along the upper arc is decreasing. But here a feature of Shukhov's formula manifests itself: with a decrease in liquid flow, a change in the temperature of the heated water becomes noticeable. Also, noticeable changes in water temperature are observed along the vertical pipes.

Despite the fact that the fluid velocity is higher than in horizontal pipes, large values of the outer surface per linear meter and their length led to this result. As a result, in the lower extreme node of the heat exchanger, the coolant temperature takes on its lowest value. In the lower arc, in the direction of flow, the temperature increases almost uniformly, which is explained by the merging of flows from the vertical pipes to the flow of the lower arc.

TABLE 1. Values of nodal temperatures (K) in a heat exchanger with ten sections

Nodal temperature (K)	**Arc numbers**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Upper arc	343.12	343.08	343.03	342.97	342.9	342.82	342.72	342.58	342.38	341.97
At the lower end of the vertical pipe	341.33	341.23	341.13	341.04	340.94	340.84	340.72	340.58	340.38	339.99
Lower arcs	340.76	340.7	340.64	340.58	340.51	340.44	340.36	340.27	340.15	339.99

Below we present some of the comparison results in the form of graphs. Here and further in the graphs, numbers 1, 2, 5, 6 correspond to the speed of the total flow in the heat exchanger $w = 0.3\ m\,s^{-1}$, numbers 3, 4, 7, 8 – speed $0.4\ m\,s^{-1}$; numbers 1, 3, 5, 7 – ambient temperature $T_{oc} = 293.15\ K$ and numbers 2, 4, 6, 8 – $298.15\ K$ numbers 1-4 heat transfer coefficient $k_{cp} = 7.5\ W\,m^{-2}\,s^{-1}$, and numbers 5-8 – $17.5\ W\,m^{-2}\,s^{-1}$.

The following three figures provide a summary of the eight calculation options.

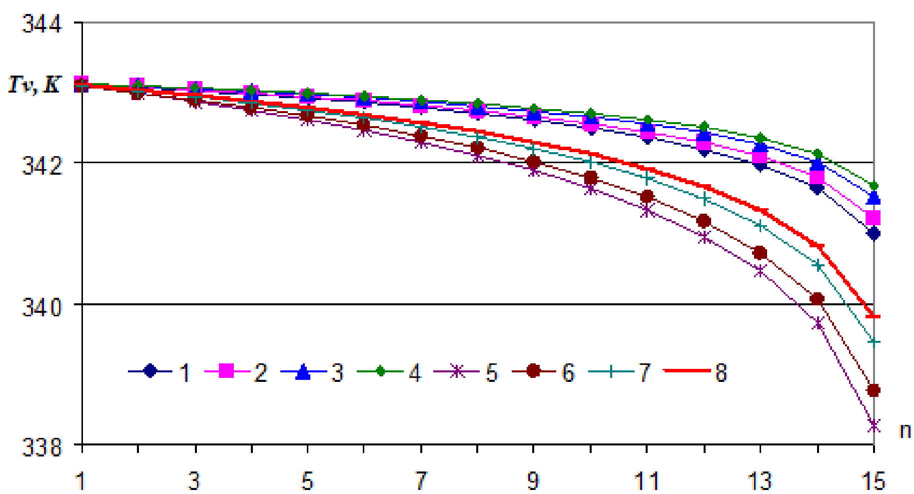

FIGURE 2. Coolant temperature values in the nodes of the upper arc of a heat exchanger with 15 sections (see text for designations)

The temperature value at the nodes of the upper arc (Fig. 2) serves as input (boundary) conditions in calculating the temperature difference of the coolant through the vertical pipes. The coolant temperature values at the lower ends of the vertical pipes depending on the number of heat exchanger sections are presented in Fig. 3. The more intense temperature drop at the end of the heat exchanger is also explained by the lower coolant flow rate in the last sections.

A more uniform increase in coolant temperature in the direction of flow is observed in the nodes of the lower arc (Fig. 4). The temperature values *n=15* for the considered options repeat the results in Fig. 3 *n=14*. The coolant enters the unit both from the left and from the top. At the outlet of the unit, *n=14* the coolant temperature represents the average flow rate of these flows, etc. The temperature value at *n=1* represents the final temperature of the coolant in a heat exchanger with 15 sections.

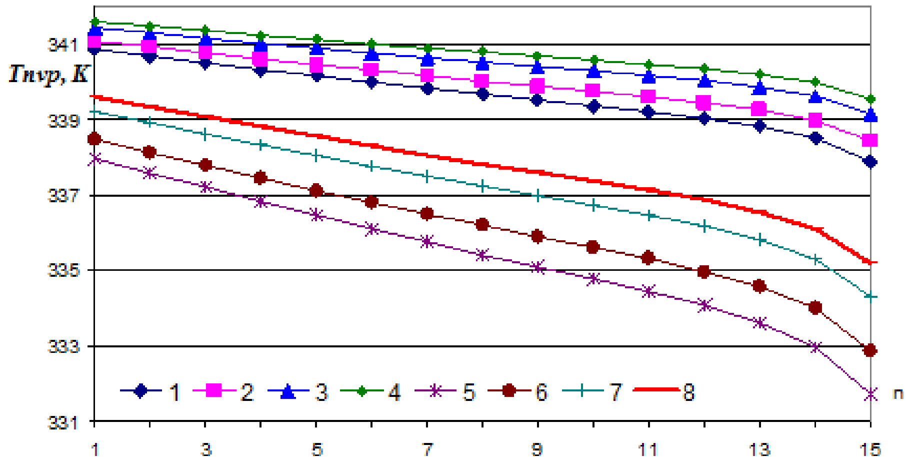

FIGURE 3. Values of the coolant temperature at the approach to the lower end of the vertical pipes of the heat exchanger with 15 sections (for designations, see the text and Fig. 2)

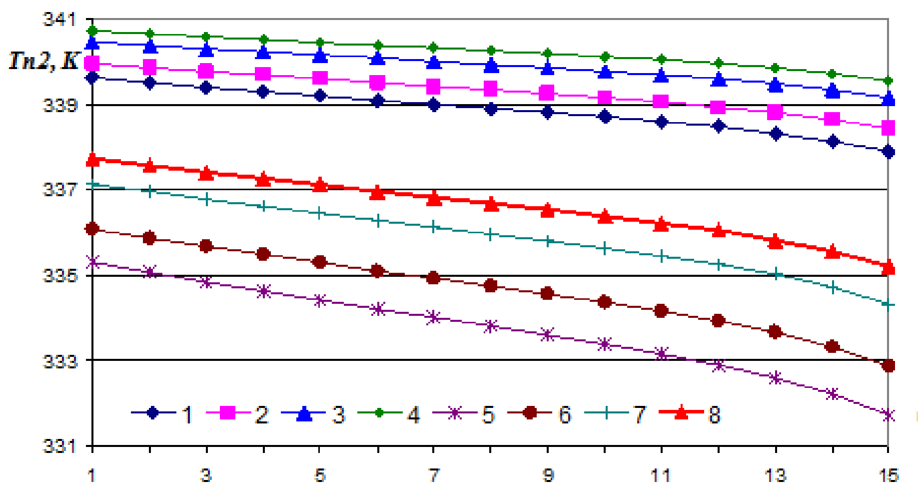

FIGURE 4. Values of nodal temperatures in the lower arc of the heat exchanger

In Fig. 4 shows changes in the coolant outlet temperature depending on the number of heat exchanger sections for options 1-8. They indicate that:

a) with an increase in the value of the average heat transfer coefficient, the heat removal process intensifies (this can also be achieved by increasing the area of contact with the coolant, heat exchanger and air);

b) at lower flow rates, an increase in heat removal is also observed;

c) the heating process is self-regulating: as the ambient temperature increases, the heat transfer of the coolant decreases.

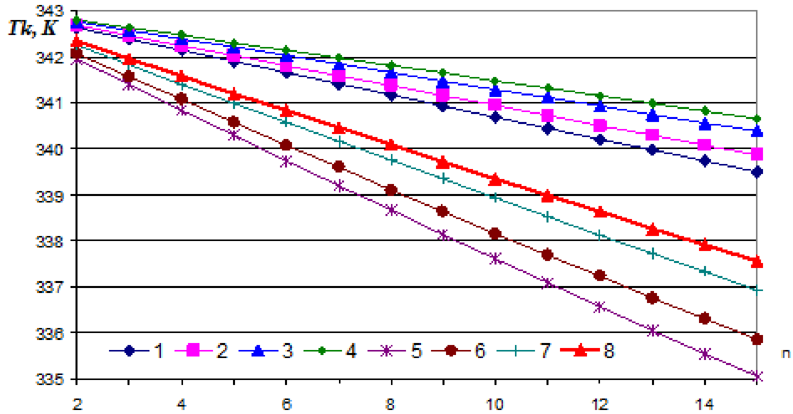

FIGURE 5. Temperature values of the coolant at the outlet of the heat exchanger depending on the number of sections in the heat exchanger

These judgments are also confirmed by the graphs in Fig. 6, which presents the results of a computational experiment on the total heat loss through a heat exchanger with *n* sections for the eight options of initial data listed above.

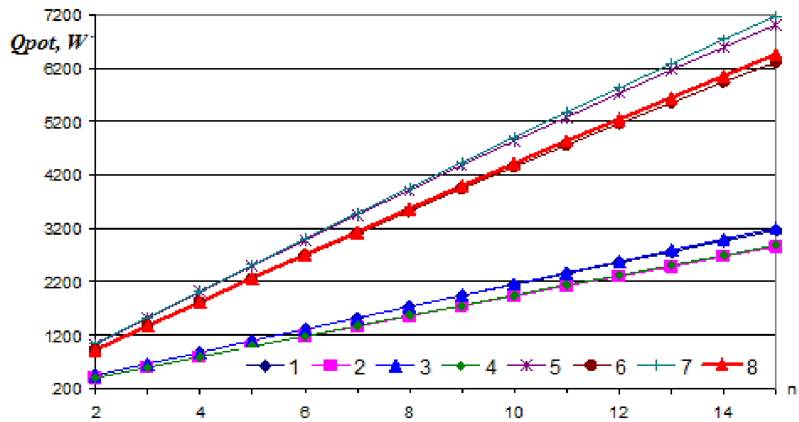

FIGURE 6. Total heat loss for eight variants of a computational experiment depending on the number of sections in the heat exchanger

CONCLUSION

Summarizing the results of the work, we note that an algorithm and program for thermal calculation of a multi-section heat exchanger have been developed. Using the developed program, a computational experiment was carried out for the quadratic resistance mode for various values of the number of heat exchanger sections and arc flow rates, nodal temperatures and pressures were determined depending on numerous initial parameters of the object.

It was revealed that the costs through the vertical radiator pipes are almost uniform: with an increase in the number of sections, as well as with distance from the entrance to the heat exchanger, the costs through the vertical pipes decrease.

The lowest temperature in the heat exchanger is formed in the lower part of the far section.

The developed algorithm for calculating a separate heat exchanger can be used when developing an algorithm for calculating a two-line heating network.

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