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Mathematical Models and Algorithms For Calculating a Two-Pipe Heating Network with Modern Heat Exchangers

Ziyomat Shirinov, Shohjaxon Ravshanov
Institute of Mechanics and Seismic Stability of Structures named after MT Urazbaev, Uzbekistan Academy of Sciences, Tashkent, Uzbekistan

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Abstract: The features of the layout and design of a water heating system are analyzed. The theoretical foundations of heat and mass transfer processes and methods of their application to solving practical problems with the involvement of the theory of similarity and dimensionality are discussed. Quasi-one-dimensional equations of momentum, mass and heat transfer in pipelines are derived. Hydrodynamic and energy flow regimes in pipelines and features of their use in calculating complex pipeline networks are discussed. It is shown that it is advisable to involve quasi-one-dimensional equations of momentum, heat and mass in calculating water heating systems.

Keywords: Quasi-one-dimensional, two-pipe heating, mass transfer.

Let us consider an elementary section of a pipeline with an internal radius $R(x)$ and a length Δx. At the entrance to the elementary section, the values of the variables are w_1, T_1, ρ_1 and p_1, and at the exit, w_2, T_2, ρ_2 they are and p_2. When deriving the equations, we use the provisions of the quasi-one-dimensional approach, i.e. changes in the indicators occur over time t and only along the coordinate x.

MASS CONSERVATION EQUATION

Entering the elementary section during and Δt time, the mass of the liquid is $\pi R_1^2 \rho_1 w_1 \Delta t$, and the mass of the liquid leaving it is $\pi R_2^2 \rho_2 w_2 \Delta t$. A positive difference $(\pi R_2^2 \rho_2 w_2 - \pi R_1^2 \rho_1 w_1)\Delta t$ leads to a decrease in the density of the liquid in the volume $\pi R^2 \Delta x$ ($R = D/2$ is the average value of the internal radius in the section) by $\rho^{(2)} - \rho^{(1)}$, where $\rho^{(2)}, \rho^{(1)}$ are the average values of the density of the liquid at the beginning and end of the time interval Δt. Thus, the mass balance in an elementary section of the pipeline is described by the dependence:

$$\left(\pi R_2^2 \rho_2 w_2 - \pi R_1^2 \rho_1 w_1\right)\Delta t = \pi R^2 \Delta x\left(\rho^{(2)} - \rho^{(1)}\right).$$

Let's divide both sides of the equation by $\pi R^2 \Delta t \Delta x$ and under conditions $\Delta t \to 0$ and $\Delta x \to 0$ we arrive at the differential equation for conservation of mass in a pipeline [9]:

$$\frac{\partial f\rho}{\partial t}+\frac{\partial f\rho w}{\partial x}=0.$$

If it is assumed that is $f=\pi R^2$ a constant value, then this equation is written as [10; 6]:

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho w}{\partial x}=0.$$

The change in the radius of the pipeline depending on the temperature is insignificant. This is explained by the fact that for most solids the coefficient of linear expansion $\alpha_l=\Delta l/l$ when heated by one degree is $\alpha_l\approx 10^{-6}..10^{-5}\ K^{-1}$. If we assume that the temperature changes by 100 degrees, then $\Delta l=(10^{-4}..10^{-3})l$. In this case, the length of the circumference of the cross-section is $2\pi R\left(1+\left(10^{-4}..10^{-3}\right)\right)$. That is, the greatest change in the circumference does not exceed 0.1%, and the cross-sectional area is no more than 0.2%. Accordingly, if the change f is not man-made (i.e. the pipe is not telescopic), then changes $R(x)$ and $f(x)$ depending on the water temperature can be neglected.

It is similarly proved that at moderate pressure values the values of the radius R and cross-sectional area f of the pipeline are practically independent of the liquid pressure, since the Young's modulus for solid materials is approximately 10^{+11} Πa (for steel 216 $г\Pi a$). That is, with an increase in pressure by 10^5 Πa (10 m of the water column) the length of the pipeline circumference will increase by approximately 0.1%. This change in the radius of the pipeline can be neglected in the conditions of our calculations.

CONSERVATION OF MOMENTUM EQUATION

Let us select an elementary section between the sections x and $x+\Delta x$. Let us formulate Newton's second law: the change in momentum on the section over time is

$$\frac{d}{dt}\int_x^{x+\Delta x}(f\rho w)dx=\sum F_i\cdot$$

As the first force factor F_1 we take the difference in pressure force in the inlet and outlet sections:

$$F_1=(fp)|_x-(fp)|_{x+\Delta x}.$$

The second force factor will be related to the force of gravity. If the center of the pipeline axis has a variable height $y(x)$, then

$$F_2=-g\int_x^{x+\Delta x}\frac{\partial(f\rho y)}{\partial x}dx,$$

where g is the acceleration of gravity.

The third force factor is the friction force, which, according to the Darcy- Weisbach hypothesis , is

$$F_3=-2\alpha\int_x^{x+\Delta x}\rho fwdx\cdot$$

The arbitrariness of the values Δx and Stokes' theorem allow us to obtain the equation

$$\frac{d(f\rho w)}{dx}=-\frac{\partial(fp)}{\partial x}-g\frac{d(f\rho y)}{dx}-2\alpha f\rho w.$$

We take into account the value of the resistance coefficient λ in the Darcy- Weisbach formula:

$$2\alpha=\frac{\lambda}{2D}|w_{cp}|$$

and we open the total differential in the form $\frac{d}{dt}=\frac{\partial}{\partial t}+w\frac{\partial}{\partial x}$.

As a result, the equation for conservation of momentum for a pipeline with variable transverse area takes the form

$$-\frac{\partial(fp)}{\partial x}=\frac{\partial(f\rho)}{\partial t}+\frac{\partial(f\rho w^2)}{\partial x}+g\frac{d(f\rho y)}{dx}+\frac{\lambda}{2D}f\rho|w|w.$$

If $f=const$, then

$$-\frac{\partial p}{\partial x}=\frac{\partial(\rho w)}{\partial t}+\frac{\partial(\rho w^2)}{\partial x}+\rho g\frac{dy}{dx}+\frac{\lambda}{2D}\rho|w|w.$$

EQUATION OF CONSERVATION OF HEAT

The amount of heat of a unit mass of liquid is expressed by the internal energy $\xi=c_B T$, where c_B is the average value of the specific heat capacity of water in the considered ranges of temperature and pressure values.

Let's make a balance ratio of the internal energy of heated water

$$\frac{d}{dt}\int_x^{x+\Delta x}(f\rho c_B T)dx=\lambda_T f\frac{\partial T}{\partial x}|_x-\lambda_T f\frac{\partial T}{\partial x}|_{x+\Delta x}+E_{BU}^f\Delta x+E_t f\Delta x-k_{cp}(T-T_{oc})\bar{f}_{op}\Delta x.$$

Here the first two terms to the right of the equal sign are the heat flux, according to Fourier's law, in the input and output cross-sections of an elementary section with a thermal conductivity coefficient of λ_T. The third term represents the intensity of internal heat sources per unit length of the pipeline. This may be the Joule-Thomson effect, evaporation, condensation, hydrate formation, and other phenomena [7; 16]. The fourth term is the dissipation of the kinetic energy of the liquid due to friction. The last term to the right of the equal sign represents the heat exchange of the liquid with the environment of the section with length Δx according to Newton's law [20; 8]. Here it was assumed that T_{oc} is the ambient temperature; k_{cp} is the effective value of heat exchange between the transported liquid and the environment through the pipeline and its insulation.

The value $\bar{f}_{op}\Delta x$ represents the area of the external surface of the pipeline through which heat is transferred. In conventional pipelines, one can take $\bar{f}_{op}=2\pi\bar{R}$, where $\bar{R}$ is the external radius of the pipeline. For cases where it is necessary to increase the intensity of heat exchange with the environment, finning (increasing the external surface) of the pipeline is performed, i.e. the required intensity of heat transfer is achieved by increasing the value of $\bar{f}_{op}$. (Remember that the value $\bar{f}_{op}$ is equal to the external perimeter of 1 m of the heat exchanger and is measured in m .)

Stokes' law and the arbitrariness of the value Δx allow us to write the equation of conservation and transfer of heat in the form:

$$\frac{\partial(f\rho\varepsilon)}{\partial t}+w\frac{\partial(f\rho\varepsilon)}{\partial x}=\frac{\partial}{\partial x}\left(\lambda_T f\frac{\partial T}{\partial x}\right)+fE_{BU}+fE_t-k(T-T_{oc})\overline{f}_{op}\cdot$$

Here the term of kinetic energy dissipation E_t due to liquid viscosity is mathematically described only for the laminar flow regime [10;6], and has a rather complex form. For other flow regimes there are approximate estimates. Therefore, in practice, they often turn to the equation of total energy, and not thermal energy in a separate form. Or take into account the dissipation of mechanical energy approximately. When taking into account the latter factor, the temperature of the agent will be overestimated.

EQUATION OF CONSERVATION OF TOTAL ENERGY

We write the equation of conservation of momentum in the form

$$\frac{\partial\rho w}{\partial t}+\frac{\partial\rho w^2}{\partial x}=-\frac{\partial fp}{\partial x}-g\frac{d(f\rho y)}{dx}-\frac{\lambda}{2\Delta}f\rho|w|w\cdot$$

We multiply the terms of this equation by w and add the result term by term using the heat conservation equation:

$$\frac{\partial f\rho\left(\varepsilon+\frac{w^2}{2}\right)}{\partial t}+\frac{\partial f\rho w\left(\varepsilon+\frac{w^2}{2}\right)}{\partial x}=$$

$$=-w\frac{\partial fp}{\partial x}-wg\frac{d(f\rho y)}{dx}+\frac{\partial}{\partial x}\left(\lambda_T f\frac{\partial T}{\partial x}\right)+fE_{BU}-k_{cp}(T-T_{oc})\overline{f}_{op}.$$

This equation took into account the dissipation of the kinetic energy of the liquid $fE_T=\frac{\lambda}{2D}f\rho|w|w^2$. That is, the equation does not include the energy of dissipation due to friction, since it was not formed, but rather passed from one kinetic form of the energy of a unit volume of liquid into another form - into the internal energy of the liquid.

Thus, the dynamic state of an elementary section of a pipeline with a variable diameter is described by quasi-one-dimensional equations [14]:

$$\begin{cases}\frac{\partial f\rho}{\partial t}+\frac{\partial f\rho w}{\partial x}=0,\\ \frac{\partial f\rho w}{\partial t}+\frac{\partial f\rho w^2}{\partial x}=-\frac{\partial fp}{\partial x}-\rho g\frac{dfy}{dx}-\frac{\lambda}{2D}f\rho|w|w,\\ \frac{\partial fp\left(\varepsilon+\frac{w^2}{2}\right)}{\partial t}+\frac{\partial f\rho w\left(\varepsilon+\frac{w^2}{2}\right)}{\partial x}=-w\frac{\partial fp}{\partial x}-\\ -w\rho g\frac{dfy}{dx}+\frac{\partial}{\partial x}\left(\lambda_T f\frac{\partial T}{\partial x}\right)+fE_{BU}-k_c(T-T_{oc})\overline{f}_{op}.\end{cases}$$

A similar system of equations for gas transportation is given in [14].

SOLUTION OF THE SYSTEM OF EQUATIONS OF STATE OF A FLUID IN A STATICALLY FUNCTIONING ELEMENTARY SECTION OF A PIPELINE WITH A FINITE LENGTH.

The hydraulic network consists of linear and point elements. A linear network element is an elementary section of a pipeline of a given length l. The internal diameter of the section D

and its resistance coefficient have constant values. At the entrance to the section, the values of pressure λ, velocity w_H and temperature of the liquid T_H are given p_H. The values of the constants used in the system of equations are also known. It is required to find the values of pressure p_K, velocity w_K and temperature T_K of the liquid at the exit from the section.

To solve the problem, we will make the following assumptions.

1. The live cross-section of the liquid is constant and equal to the cross-sectional area of the TP, i.e. $f = const$ the liquid does not have a free surface.
2. The liquid is slightly compressible , i.e. the change in the density of the liquid depending on changes in pressure, leveling height and temperature is negligible ($\rho \approx const$).
3. The leveling height of the TP axis $y(x)$ is known.
4. There are no internal heat sources, $E_{BИ} = 0$ i.e.
5. Heat exchange associated with longitudinal heat diffusion is negligible.
6. The ambient temperature and the reduced heat capacity of the liquid are constants ($T_{oc} = const$, $c_B = const$).
7. The pipe is cylindrical and has no ribs ($\bar{f}_{op} = \pi \bar{D}$; $f = \pi D^2 / 4$).

Using these assumptions, we transform the equations of the system.

According to assumptions 1 and 2, from the second equation we have $w = const$. So that $w_K = w_H = w$. In connection with this and according to assumption 3, the first equation takes the form

$$-\frac{dp}{dx} = \frac{\lambda Q^2 \rho}{2Df^2} + \rho g \frac{dy}{dx},$$

where $Q = S\,w$ is the constant volumetric flow rate of liquid for the section.

Taking into account the value of the pressure p_H at the entrance to the section, we integrate the last equation from 0 to l and obtain

$$-\left(p_K - p_H\right) = \frac{\lambda Q^2 \rho}{2Df^2} l + \rho g \left(y_K - y_H\right).$$

From here we find the pressure value at the end of the section

$$p_K = p_H - \frac{\lambda Q^2 \rho}{2Df^2} l - \rho g \left(y_K - y_H\right).$$

One of the main elements of this formula is the friction resistance coefficient λ. It is a kind of measure of the tangential friction stress in pipelines.

The friction drag coefficient depends significantly on the hydrodynamic flow regime and the cross-sectional shape of the pipeline.

Depending on the equivalent roughness $k_э$ and Reynolds number $Re = wD / \nu$ laminar and transitional flow regimes and three turbulent flow subregimes are expected [4;10]. This fact is proven by the results of Nikuradze's experimental studies conducted in round cylindrical pipes [10].

The consensus of experts on the value of the resistance coefficient for a round pipe in a laminar flow regime $\lambda = 24 / Re$ is achieved by the fact that this formula has a strict scientific basis and is appropriate when $Re < 2200$.

In some works, the Zaichenko formula is used to calculate the transitional flow regime (from laminar to turbulent) $\lambda = 0.0025 Re^{1/3}$, which is reliable for the interval $2200 < Re < 4000$.

These two formulas are appropriate if the equivalent roughness is small $k_э < 10D / \text{Re}$. Under the same condition, when $4000 < Re$ a smooth flow regime around the roughness is established, for which the Blasius formula is appropriate $\lambda = 0.3164 Re^{-1/4}$ According to the boundary layer theory, in this case the size of the equivalent roughness will be less than the thickness of the viscous sublayer [10].

A mixed regime of turbulent flow around roughness is expected if the size of the equivalent roughness is close to the value of the viscous sublayer thickness ($10 < \text{Re} k_э / D < 158$). For such cases, Leibenson proposed the formula $\lambda = 10^{-0.627} (k_э / D)^{0.127} Re^{-0.123}$ [15].

At $158 < \text{Re} k_э / D$ the development mode of roughness flow occurs , when the boundary layer thickness is less than the equivalent roughness, and the drag coefficient does not depend on the Reynolds number . To calculate the drag coefficient, the formula of B.A. Shifrinson is used $\lambda = 0.11 (k_э / D)^{0.25}$.

Note that there are other approximation relationships for the drag coefficient (Colebrook , VNIIGaz , A.D. Altshul ...). But the formulas given above allow a one-term representation of the drag coefficient [13 ; 17 ; 15]:

$$\lambda = \varsigma (k_э / D)^{\theta} Re^{n}.$$

In the third chapter of the dissertation, this formula is used in the process of hydraulic calculation of a heat exchanger and a two-line heating network.

The second main element of the equation of conservation of momentum in a pipeline is the local slope of the route from the horizon $dy / dx = \sin\alpha$, where $y(x)$ is the law of change of the leveling height depending on the distance ($\sin\alpha$ taken taking into account that the length of the elementary section acts as the hypotenuse of a right triangle). The energy " post-pass " flow regime, which is closely related to the value $\sin\alpha$, is rarely discussed in the literature.

At significant negative slopes ($\sin\alpha < \sin\alpha_{kr} < 0$) a post-saddle flow regime occurs . Unlike the usual energy regime, when the pressure drops downstream ($dp / dx < 0$), in the post-saddle regime the pressure increases downstream ($dp / dx > 0$). The reason for this is the transition of the potential energy of gravity to the kinetic energy of the transported medium. Moreover, part of the potential energy of gravity is spent on overcoming the force of frictional resistance, and the remaining part of the energy of gravity is converted into kinetic energy of the flow. In the works [13; 17; 15] this energy regime was discovered for main gas pipelines and substantiated on a scientific basis.

Since small diameter pipes are used in the water heating system, this paper considers problems with a normal energy regime, when $dp / dx < 0$.

Let's get back to our equations.

According to $w = const$ assumptions 4-7, the equation of conservation of total energy is transformed and takes the following form:

$$\rho w c_B \frac{dT}{dx} = -w\frac{dp}{dx} - w\rho g\frac{dy}{dx} - k_{cp}\left(T - T_{oc}\right)\frac{\overline{f}_{op}}{f}.$$

Taking into account $-\frac{dp}{dx} = \frac{\lambda Q^2 \rho}{2Df^2} + \rho g\frac{dy}{dx}$ we have

$$\rho w c_B \frac{dT}{dx} = w\frac{\lambda Q^2 \rho}{2Df^2} - k_{cp}\left(T - T_{oc}\right)\frac{\overline{f}_{op}}{f}.$$

From this follows an ordinary linear differential equation for the temperature of the liquid

$$\frac{dT}{dx} = \frac{\lambda Q^2}{2Df^2 c_B} - \frac{k_{cp}\overline{f}_{op}}{\rho Q c_B}\left(T - T_{oc}\right)$$

or

$$\frac{dT}{dx} + Sh\left(T - T_{oc} - \frac{\lambda Q^3 \rho}{2k_{cp}Df^2\overline{f}_{op}}\right) = 0.$$

Here, in keeping with the technical literature [11; 7], the constant $Sh = \frac{k_{cp}\overline{f}_{op}}{\rho Q c_B}$ was designated by the V.G. Shukhov coefficient.

The boundary condition for this equation is $T(0) = T_H$.

The solution we obtained is

$$T(l) = T_{oc} + \frac{\lambda Q^3 \rho}{2k_{cp}Df^2\overline{f}_{op}} + \left(T_H - T_{oc} - \frac{\lambda Q^3 \rho}{2k_{cp}Df^2\overline{f}_{op}}\right)\exp\left(-Sh\, l\right).$$

If we neglect in our solution the terms $\frac{\lambda Q^3 \rho}{2k_{cp}Df^2\overline{f}_{op}}$ that were formed when taking into account the pressure force in energy, we obtain the well-known formula of V.G. Shukhov [8]. These terms indirectly reflect the shares of the gravitational force in the law of temperature change. In this regard, the last solution represents a generalization of Shukhov's formula taking into account the work of the pressure force on a moving liquid in the field of the gravitational force.

Thus, if the values of velocity w_H, pressure p_H and temperature are given at the input to the elementary section T_H slightly compressible liquid, then at the end of the section the flow velocity remains the same, and the values of the pressure p_K and temperature T_K of the liquid are determined by the dependencies

$$p_K = p_H - \frac{\lambda Q^2 \rho}{2Df^2} l - \rho g\left(y_K - y_H\right),$$

$$T_K = T_{oc} + \frac{\lambda Q^3 \rho}{2k_{cp}Df^2\overline{f}_{op}} + \left(T_H - T_{oc} - \frac{\lambda Q^3 \rho}{2k_{cp}Df^2\overline{f}_{op}}\right)\exp\left(-Sh\, l\right).$$

Here, when determining the pressure drop, the resistance force and the force of gravity were taken into account, and when determining the change in temperature, heat exchange with the environment was taken into account and, unlike Shukhov's formula, the dissipation of mechanical energy was additionally taken into account according to the quadratic law of resistance.

To take into account the pressure loss due to local resistances caused by changes in the flow direction and cross-sectional area, in medium and low working pressure pipelines, it is proposed to use the formula

$$\Delta p = \varsigma \frac{\rho u^2}{2}$$

or corrections are made for the length of the calculated section in the form of $\Delta l = \varsigma D / \lambda$. The value of the coefficient ς for various conditions can be found in technical reference books and methodological manuals.

A similar correction is required for k_{cp}, since the known values are obtained only for the internal flow. In other cases, formulas are used that take into account both the external flow velocity and the Nussult number , which serves as a dimensionless heat transfer coefficient [20].

COMPUTATIONAL EXPERIMENT ON A HEAT EXCHANGER IN A SINGLE-PIPE HEATING NETWORK.

The following data (option 0) were taken as the basic information, which gave results closer to the experimental data: $l = 0.07\ \text{м}$, $h = 0.6\ \text{м}$, $\dot{h}_p = 0.15\ \text{м}$, $D_m = 0.03\ \text{м}$, $h_p = 0.15\ \text{м}$, $D_v = 0.01\ \text{м}$, $D_g = 0.03\ \text{м}$, $\dot{D}_p = 0.03\ \text{м}$, $D_p = 0.03\ \text{м}$, $\delta_m = 0.005\ \text{м}$, , $\delta_v = 0.12\ \text{м}$, $\delta_g = 0.03\ \text{м}$, $\dot{\delta}_g = 0.20\ \text{м}$, $\dot{\delta}_p = 0.05\ \text{м}$, $\delta_p = 0.05\ \text{м}$, $p_H = 110000.0\ \text{Па}$, $\rho = 1000.0\ \text{кг м}^{-3}$, $\lambda_m = 0.002$, $\lambda_v = 0.002$, $\lambda_g = 0.002$, $\dot{\lambda}_p = 0.002$, $c_B = 4190.0\ \text{Дж кг}^{-1}\ \text{К}^{-1}$, $g = 9.8\ \text{м с}^{-2}$, $\lambda_T = 200\ \text{Вт м}^{-1}\text{К}^{-1}$, $\alpha_B = 3500.0\ \text{Вт м}^{-2}\ \text{К}^{-1}$, $T_{oc} = 293.15\ \text{К}$. $T_H = 343.15\ \text{К}$ In addition, the values of the complexes $k_B = \frac{\alpha_{BT}}{\lambda_T}\left[\text{м}^{-1}\right]$, $\dot{F}_p = \frac{\pi \dot{D}_p^2}{4}\left[\text{м}^2\right]$, $k_H = \frac{\alpha_{TOc}}{\lambda_T}\left[\text{м}^{-1}\right]$ were $F_m = \frac{\pi D_m^2}{4}\left[\text{м}^2\right]$ used $F_v = \frac{\pi D_v^2}{4}\left[\text{м}^2\right]$. $F_g = \frac{\pi D_g^2}{4}\left[\text{м}^2\right]$ The $\alpha_H = 17.0\ \text{Вт м}^{-2}\ \text{К}^{-1}$ average $F_p = \frac{\pi D_p^2}{4}\left[\text{м}^2\right]$ velocity of the coolant at the entrance to the main line was $w = 0.4\ \text{м с}^{-1}$ and the volumetric flow rate of the liquid $Q = F_m w = 2.8274\ \text{л}/\text{с}$ was $k_{cp} = 7.5\ \text{Вт м}^{-2}\ \text{с}^{-1}$.

Additional calculations were performed with a change in the value of a separate indicator, when the other data remained as basic. The following were adopted as such indicators: for mass flow rate - $Q = 1{,}4137\ \text{л}/\text{с}$ (option 1), for the initial temperature of the coolant - $T_n = 323{,}15\ \text{К}$ (option 2), for the ambient temperature - $T_{oc} = 283{,}15\ \text{К}$ (option 3) and $T_{oc} = 303{,}15\ \text{К}$ (option 4), for the resistance coefficient - $\lambda = 0{,}005$ (option 5), for the heat transfer coefficient in sections with flow - $k_{cp} = 17{,}5\ \text{Вт}/\text{м}^2/\text{с}$ and set (option 6) $\alpha_{\text{в}} = 2500{,}0\ \text{Вт}/\text{м}/\text{К}$, $\alpha_{\text{н}} = 7{,}0\ \text{Вт}/\text{м}/\text{К}$ data for the option of three-layer heat exchange in vertical intermediate pipes (option 7), as well as for the thickness of the upper horizontal pipes - $\delta_{gv} = 0{,}10\ \text{м}$ (option 8).

In all variants, calculations were performed for the number of sections N from 2 to 20. The values N, the total length of the heat exchanger (in meters), and the flow rates (in l/s) were saved and printed: along the arc $AB\,(Q_A)$, along the arcs BB_1, B_1B_2, B_2C (Q_B), along the arc CD (q_K), along the arc BC (q_C),and also along the arc AD (q_D). Then we saved the pressure values (in Pa) in nodes A, B, in node C, calculated by the upper and middle arcs (PC and PB12C), and in the final node D, calculated by the bridge CD and the lower arc AD. We saved the nodal values T_B, T_C (end of the middle arc), T_{B_1}, T_{B_2}, T_{CB} (end of the upper arc), T_{D_C} (end of the output bridge), T_{D_A} (end of the lower arc) and T_K (at the outlet of the section with the heat exchanger). This list ends at the second loss of thermal energy in the section with the heat exchanger $Q_{тепл}\ (Bm)$. Repeated saving of individual indicators is associated with checking the obtained result according to the proposed algorithm [19].

The presented calculation options resulted in two variants of hydrodynamic indicators for a fixed number of heat exchanger sections: variant 0 corresponded to the basic data of our calculation and variant 1 – when the coolant flow rate was reduced by half (Fig. 2.6). The remaining calculation options repeated the results of the basic variant 0 (Fig. 2.6a), since according to the model, the thermophysical indicators of the coolant do not affect the hydrodynamic indicators of the coolant.

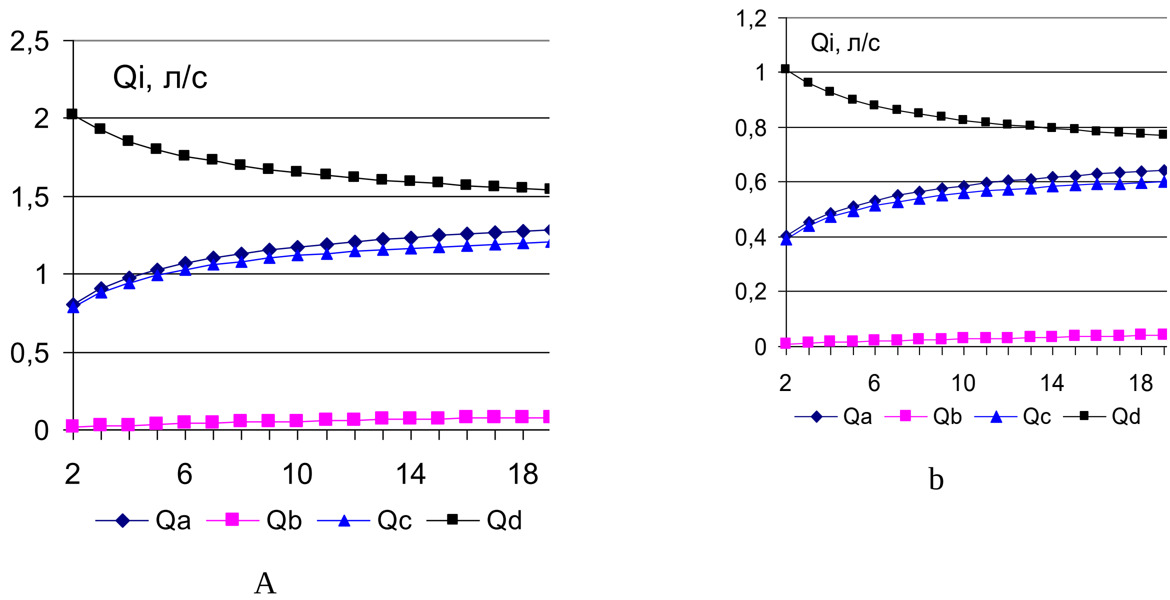

Fig. 1. Change in liquid flow rate along arcs $AB\ (Q_A)$, $BB_1B_2C\ (Q_B)$, $BC\ (q_C)$ and $AD\ (q_D)$ depending on the number of sections . a – $Q = 2.8274\ м^3\ c^{-1}$ (options 0, 1-8), b – $Q = 1.4137\ м^3\ c^{-1}$ (option 1)

With an increase in the number of sections, the flow rates along the arcs AB, BC and BB_1B_2C increase and the nature of their increase is close to an exponential law. Note that the flow rate along the long arc BB_1B_2C remains less than in other arcs. This corresponds to the nature of the transfer phenomenon, which is due to analogues of Kirchhoff's laws: the flow rate along a long and thin arc will be less than along a short and thick arc. According to these data, the flow rate of the coolant decreases along the main line as the number of sections increases. Fig. 2.6b shows the same values when the total flow rate of the coolant was halved (option 1). Despite the fact that the quadratic law of resistance was implemented, the flow rate curves turned out to be affine-like : in Fig. 2.6b their ordinates are two times less (within an accuracy of 0.0001 l/s) than in Fig. 2.6a [18].

The results of calculations of nodal pressures (without counting the change in leveling height) also had two variants, which are noted above. In variant 0, with a change N from two to twenty, the pressure drop at the transition of the section with the heat exchanger was from 0.38 to 2.20 Pa, and in variant 1 - from 0.10 to 0.55 Pa. Here, the quadratic law of resistance was clearly expressed. The pressure drops were insignificant, so their graphs were not provided [19].

Figures 2.7-2.9, related to the nodal temperatures of the coolant, show the values of the coolant temperature in the nodes $B\ (T_B)$, $B_1\ (T_{B_1})$, $B_2\ (T_{B_2})$, $C\ (T_C)$ and . In addition to them, the values of the coolant temperature in the approach to the section $D(T_K)$ along the upper arc (T_{CB}), in the approach to the section D along the arc $CD(TD\_C)$ and along the main line (TD_A) are shown C.

After the flows merged in the sections C, D the average temperature for the flow rates was taken.

The upper curves T_B, T_C, T_{D_A}, due to their small difference are presented as a single curve (Fig. 2.7). The second descending curve from the top refers to the outlet temperature of the coolant, which serves as a measure of heat loss in the jumpers and the heat exchanger itself. The next descending curve is the temperature at the end of the jumper CD. The first ascending curve from the top refers to the section B_1, the second – to the section B_2. With an increase in the number of sections along the arc, BC the temperature increases, which is due to taking into account the change in temperature in the vertical intermediate pipes.

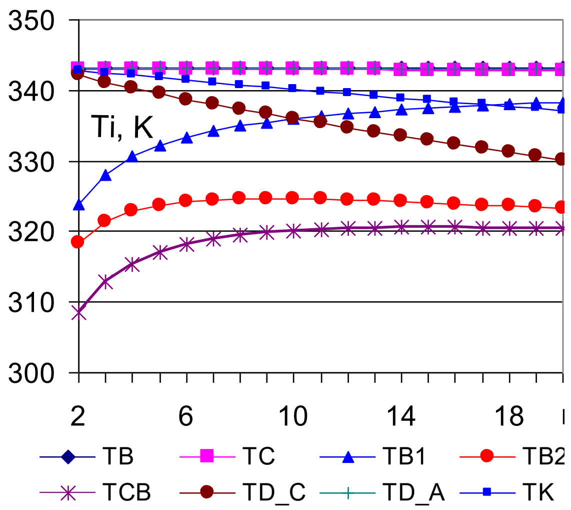

Fig. 2. Changes in the temperature of the coolant $B(T_B)$, $B_1(T_{B_1})$, $B_2(T_{B_2})$, $C(T_C)$ and $D(T_K)$, as well as in the approaches to the nodes C and D. Option 0

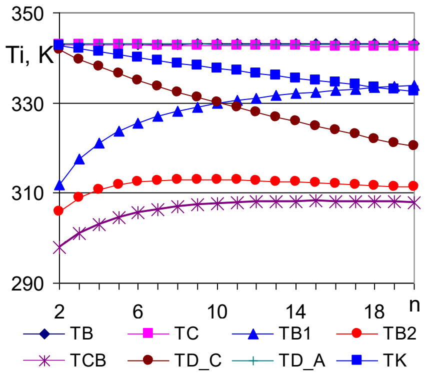

Fig. 3 Changes in the temperature of the coolant $B(T_B)$, $B_1(T_{B_1})$, $B_2(T_{B_2})$, $C(T_C)$ and $D(T_K)$, as well as in the approaches to the nodes C and D. Option 1

The outlet temperature of the coolant at $Q = 2.8274\ м^3 c^{-1}$ was 337.15 K, while at $Q = 1.4137\ м^3 c^{-1}$ was 332.61 K (Fig. 2.8). That is, at a lower coolant flow rate, the decrease in coolant temperature will be significant than at a higher flow rate. This corresponds to the nature of the Shukhov formula, where the denominator of the exponential function argument includes the volumetric flow rate $Q = fw$ at the transverse area of the pipeline f and the average w flow rate.

The graphs of the nodal temperatures and in the approaches to the merging of flows were obtained for variants 2-7, which are similar to the curves in Fig. 2.7. Comparison of a large number of indicators for nine variants of results is quite labor-intensive. Therefore, we will focus on individual indicators: on the flow rate of the coolant Q_A flowing through the heat exchanger; on the temperature T_K at the outlet and the section with the heat exchanger and the second heat loss in the section with the heat exchanger.

As the calculation results showed, with an increase in the number of sections, the flow rate of the coolant through the heat exchanger increases in the considered calculation options (Fig. 2.9).

Fig. 2.10 shows the values of the outlet temperature T_K of the coolant from the area with the heat exchanger depending on the number of sections N in the heat exchanger.

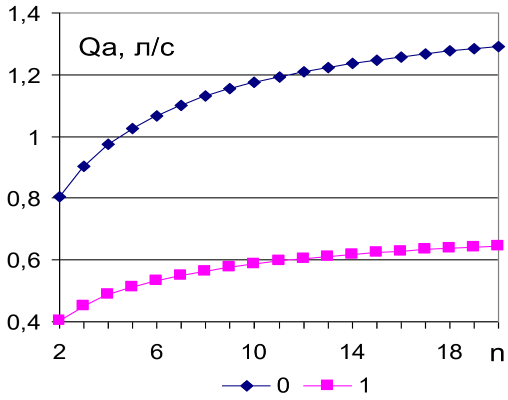

Fig. 4 . Change in coolant flow through the heat exchanger depending on the number of sections. Curve 0 – options 0, 2-8;

curve 1 – option 1

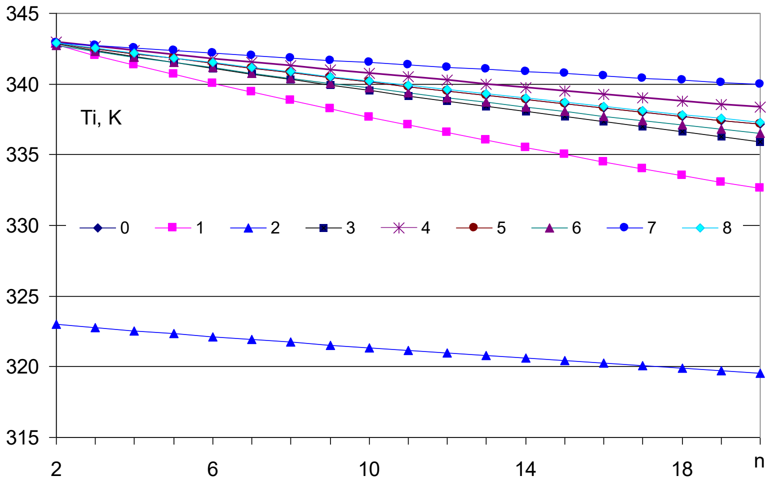

Fig. 5. Change in the outlet temperature of the coolant depending on the number of sections when changing individual initial data

The upper curve (7) refers to the variant of decreasing the heat transfer coefficient between the agent and the heat exchanger body (from $\alpha_B = 3500.0$ to $\alpha_B = 2500.0\ Bm\ м^{-2}\ K^{-1}$), as well as between the coolant body and the surrounding atmosphere (from $\alpha_H = 17.0$ to $\alpha_H = 7.0\ Bm\ м^{-2}\ K^{-1}$). The result decreases from 342.88 K (at $N = 2$) to 339.96 K (at $N = 20$).

The second curve from the top (4) refers to the case when the ambient temperature is $T_{oc} = 303.15\ K$. Since the difference between the temperatures of the contacting areas decreases, the decrease in the coolant temperature is insignificant.

Three variants (the basic calculation variant, the variant of the resistance coefficient of 0.005

and the variant of changing the wall thickness of the upper horizontal pipes from 0.2 m to 0.1 m) are presented by a practically coinciding curve. Below them are the graphs when k_{cp} 17.5 was taken (against $k_{cp} = 7.5\ Bm\,м^{-2}\,c^{-1}$) and when the ambient temperature was $283.15\ K$(against $293.15\ K$).

A twofold decrease in the total flow rate resulted in curve 2. When the initial temperature of the coolant decreased to $323.15\ K$(against $343.15\ K$) the result is curve 2, which is lower than all the curves for the output temperature.

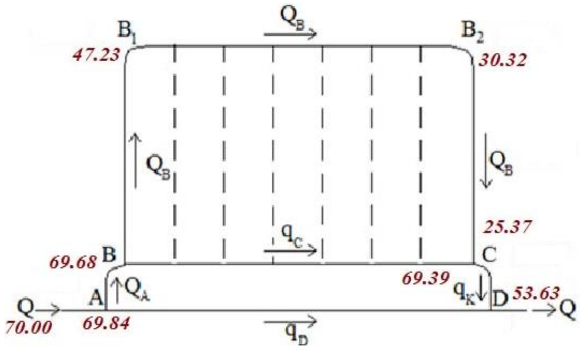

Fig. 6. Change in the temperature of the coolant in the heat exchanger using the proposed method

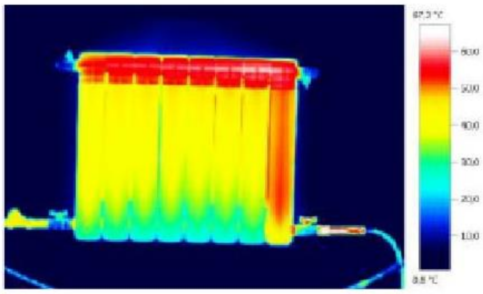

Fig. 7. Change in coolant temperature according to experimental data [2]

According to the experimental results, the coolant temperature was 40...80 ^{0} C [1,2], the volumetric flow rate was 0.13...0.22 m3 $^/$ h, and the air temperature was 20 ^{0} C. The TESTO-350 television device was used for filming [2]. In our results, the coolant temperature was 75 ^{0} C, the volumetric flow rate was 0.18 m3 $^/$ h, and the air temperature was 20 ^{0} C [18;19].

All the graphs turned out to be decreasing, and the decrease is practically linear. This justifies the engineering practice of calculating the capacity of the coolant by the number of sections (see [12]).

The smallest decrease in the outlet temperature of the coolant was obtained with a decrease in the average value of the heat transfer coefficient in the Shukhov formula, and the largest decrease was obtained with a decrease in the coolant flow rate. In general, in another interval, a change in the temperature of the coolant was obtained, which at the inlet had a temperature 10 degrees lower. These patterns, in particular, determined the nature of the change in the total heat loss in the section with the heat exchanger (Fig. 2.13).

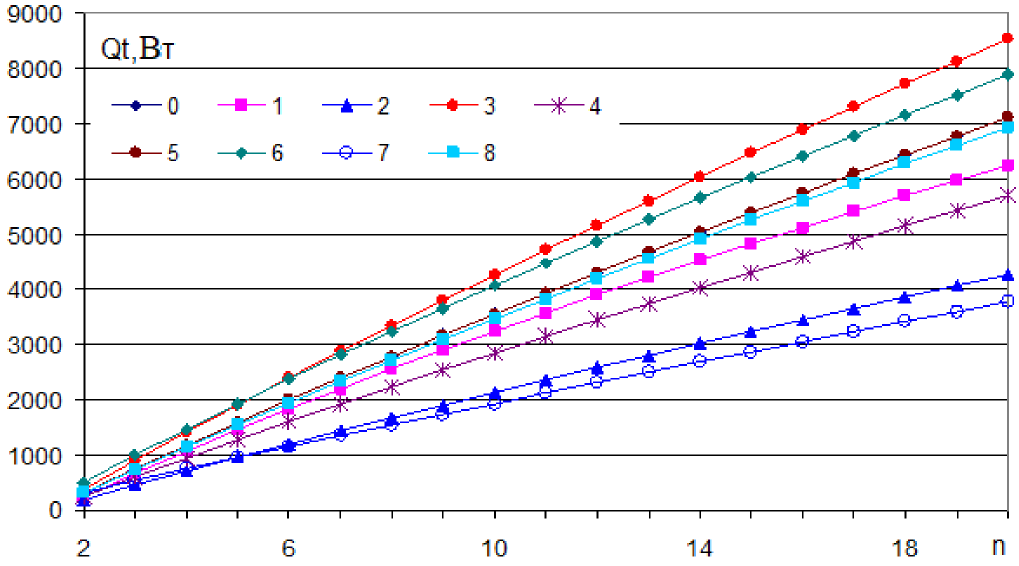

Fig. 8. Intensity of heat release Q (Вт) from the section with a heat exchanger depending on the number of sections

Note the almost linear increase in the volume of heat removed depending on the increase in the number of sections. From top to bottom are curves 3, 6, 0 (5), 8, 1, 4, 2 and 7. The list is headed by the option of low ambient temperature (curve 3). It is noteworthy that it is followed by the option of increasing the average value of the heat transfer coefficient along the perimeter of the heat exchanger.

Despite the applied hypothesis, the selected database and the obtained results adequately describe the heat and mass transfer processes in multi - section heat exchangers. This is also confirmed by the qualitative coincidence of the results of the computational experiment with the data [5], which are presented in Fig. 2.14. Accordingly, The proposed mathematical model and calculation algorithm can be used in hydraulic and thermal calculations of a single-pipe heating network.

CONCLUSION

Based on the hypothesis that the flow is along the outer contour of the heat exchanger, and in the intermediate vertical pipes the established heat exchange occurs according to the boundary conditions of the third kind, a quasi-one-dimensional mathematical model of a multi-section heat exchanger connected to a single-pipe network has been developed.

To determine the nodal pressures of the heat exchanger and the heat supply network, the quadratic law of resistance is used. In the first case, the change in the leveling height of the calculation nodes is not taken into account, and in the second case, the change in the leveling height of the calculation nodes is not taken into account.

An algorithm for hydraulic calculation of a heat exchanger has been developed using analogues of the first and second Kirchhoff laws, which ensure the unambiguity of calculation results for a multi-circuit network.

Based on Shukhov's formula, an algorithm for thermal calculation of a multi-section heat exchanger has been developed.

An algorithm has been developed for calculating a single-pipe heating network with a limited number of heat exchangers, the number of identical sections of which is known.

The success of the chapter is the choice of basic data for the multifactorial object, which were used in the course of the calculated experiment.

During the computational experiment, a number of features of heat and mass transfer in heat exchangers and the network as a whole were revealed . In particular, an increase in the number of heat exchanger sections leads to a virtually linear increase in the heat removed and an increase in the flow rate of the coolant through the heat exchanger; changes in the total flow rate and resistance coefficient lead to a change in nodal pressures; a decrease in the total flow rate of the coolant and an increase in the difference between the temperature of the coolant and the ambient temperature , as well as heat transfer coefficients, leads to a more intensive transfer of heat to the environment.

LITERATURES

1. Ivan Pachón : Numerical Analysis of a Household Radiator Test Room // Tesi di Laurea Highway in Engineering Energetica , Politecnico di Milano. Copyright December 2019.
2. Mar'ina ZG, Vereshchagin AY, Novozhilova AV, Komarevtsev MA and Isaeva KO Study of the thermal characteristics of the aluminum radiator ROYAL Thermo Evolution // IOP Conf. Series: Materials Science and Engineering 1211 (2022) 012005. // doi:10.1088/1757-899X/1211/1/012005
3. Novitsky NN, Alekseev AV, Grebneva OA, Lutsenko AV, Tokarev VV, Shalaginova ZI Multilevel modeling and optimization of large-scale pipeline systems operation // Energy, Volume 184, 1 October 2019, Pages 151-164 https://doi.org/10.1016/j.energy.2018.02.070
4. Hydraulic calculation of a two-pipe heating system with natural circulation of the coolant: method. instructions for performing calculation and graphic work / With the author: Yu.V. Vankov, A.R. Zagretdinov . - Kazan: Kazan . g os. energy univ., 2017. – 20 p.
5. Zhumaev Zh., Khamidov O.M., Shirinov Z.Z., Usmonova G.M. Experimental study of the efficiency of a heating system with a new hot water boiler (KUOV) / Khalkar « Innovation va Zamonaviy akhborot technologylarini taleem , fan va boshkaruv sohalarid koollash istiqbollari » Ilmiy-Amaly online conference materiallari . Samarkand , 2020. – 153-156 b.
6. Komina G.P., Proshutinsky A.O. Hydraulic calculation and design of gas pipelines: a tutorial; SPbGASU . – SPb ., 2010. – 148 p.
7. Korotaev Yu.P., Tagiev V.G., Gergedava Sh.K. System modeling of optimal operating modes of natural gas production facilities . – M.: Nedra, 1989. – 264 p.
8. Korotaev Yu.P., Shirkovsky A.I. Extraction, transport and underground storage of gas. – M.: Nedra, 1997. – 487 p.
9. Kudashev S.F., Kudasheva O.V., Dushutina O.V., Ravilov R.R. Intensification of heat transfer in a plate heat exchanger due to pulsation of the coolant flow // Modern science-intensive technologies, 2019, No. 10. - P.262-267.
10. Loitsyansky L.G. Mechanics of liquids and gases. – M.: Drofa, 2003. – 840 p.
11. Novoselov V.F., Golyanov A.I., Muftakhov E.M. Typical calculations in the design and operation of gas pipelines / Textbook for universities. - M.: Nedra, 1982. - 136 p.
12. Pokotilov V.V. Water heating systems. – Vienna: « HERZ Armaturen ", 2008. – 159 p.
13. Sadullaev R., Vagapov I.Kh., Zainiev N.Z., Khuzhaev I.K., Khurramova R.I. Calculation of the main gas pipeline taking into account the terrain // Gas Industry, Moscow, 2003. No. 8. – P. 58-59 .

14. Seleznev V.E., Aleshin V.V., Pryalov S.N. Modern computer simulators in pipeline transport. Mathematical methods of modeling and practical application. Ed. V.E.Seleznev // M.: MAKS Press, 2007. – 200 p.

15. Xujayev I.K. Development of mathematical models of diffusion combustion and gas transportation through pipelines: Diss … Doctor of Technical Sciences. – Tashkent, IMiSS AS RUz , 2009. – 336 p.

16. Xujayev I.K., Bazarov O.Sh. Theory and methods for calculating low- and super - compressible real liquids in pressure pipelines. - Tashkent: Voris . - 2021. - 153 p.

17. Xujayev I.K., Sadullaev R., Arifzhanov A.Sh. On control of processes of transportation of compressible and incompressible media through parallel pipelines // Automation in industry – Moscow. – No. 6, 2008. – P.19-20.

18. Xujayev I.K., Shirinov Z.Z. Hydraulic and thermal calculation of a multi-section heat exchanger operating in a single-pipe heat supply network. // Mechanics of Muammolari , 2023, No. 4. p. 43-54.

19. Xujayev I.K., Shirinov Z.Z. Simplified method for calculating a section of a single-thread heating network with a modern heat exchanger // Universum : technical sciences, 2024. 8(125)., pp. 25-37.

20. Yudaev B.N. Heat transfer. Textbook for higher educational institutions. – M.: Higher school, 1973. – 360 p.