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Development of a mathematical model for describing the spatial dynamics of temperatures and humidity levels in grain storage facilities

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ABSTRACT

This article presents the development of a bi-dimensional mathematical model designed for controlling and predicting thermal and humidity transfer processes during grain storage. The model takes into account the impact of pests on these processes by introducing specific parameters related to their influence. The mathematical model is based on two-dimensional differential equations that describe the dynamics of heat and moisture distribution. An implicit finite difference scheme method is used to solve the model. The article provides a detailed explanation of the theoretical foundations of the model, methods for determining the parameters, and numerical computation algorithms. Furthermore, a series of computational experiments were conducted based on the developed model, and the results were analyzed to confirm the model's consistency with the physical nature of the processes.

Keywords: grain products, heat exchange, moisture exchange, pests, mathematical model of a physical process, storage process, forecasting, parametric analysis, computational algorithms

1. INTRODUCTION

In modern grain storage systems, it is essential to apply mathematical tools to interpret, regulate, and predict various physical phenomena. One of the urgent challenges is the development of reliable models and computational algorithms that can reflect the influence of pests on the thermal and moisture behavior of the storage environment, and consequently, mitigate negative consequences.

This study introduces a two-dimensional computational scheme designed to simulate temperature and humidity variations in grain storage conditions while incorporating pest-related parameters. The model investigates how heat and moisture propagate within the bulk of stored grain, enabling early detection of environmental shifts caused by biological activity. The main objective of the study is to support product quality preservation and improve the overall efficiency of storage operations.

In [1], a mathematical model was presented to forecast the transport of thermal energy and moisture in porous structures, and numerical simulations were carried out using computer software. The model incorporates variables such as the emission of heat and humidity from natural materials and the fluctuations of environmental thermal-moisture conditions. It is also capable of determining such changes at different locations in solar-irradiated porous media.

Reference [2] discusses a multidimensional modeling framework complemented with a computational algorithm and visualization tools for analyzing moisture and temperature migration. It examines the effects of heat release from natural porous materials alongside changes in environmental air conditions. Moreover, the article explores the behavior of materials during storage and drying stages.

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In [3], researchers focus on the coupled transfer of energy and mass within porous substances. Experimental investigations reveal temperature-dependent material properties, including conductivity and heat capacity under various saturation levels. Sorption characteristics such as moisture uptake and vapor permeability are evaluated. A validated model for describing the hygrothermal behavior of biobased materials is also presented.

Study [4] offers a new model addressing porous bodies in partially isolated environments based on state equations, consistent with thermodynamic principles that were previously overlooked.

Article [5] outlines a theoretical analysis of heat and mechanical contact between materials of varying texture. A predictive model is formulated for assessing thermal resistance under different engineering conditions.

In [6], researchers highlight challenges and approaches to modeling thermal conditions in industrial settings, utilizing modern computation platforms for solving practical problems.

The classification of moisture and heat transport phenomena within multilayer grain structures is discussed in [7]. It is concluded that thin-layer drying under high-humidity air conditions is inadequately described by existing theories. Therefore, a prototype closed system capable of handling four-foot-deep grain layers was proposed to collect sufficient data for analyzing prolonged drying and cooling behavior.

Reference [8] evaluates moisture migration techniques applied to agricultural products using sorption-related methods. The study highlights how the efficiency of mass exchange depends on diffusion coefficients, moisture retention potential, and thermal gradients.

In [9], an integrated modeling system is proposed for describing internal and external temperature and moisture interactions in heterogeneous porous domains, including exchange processes with the surrounding environment.

Study [9] explores the spatial distribution of steel structures during grain drying and investigates the interdependence between thermal conduction and humidity migration. It centers on modeling corn drying processes under varying physical damage levels using consistent layout conditions.

In work [10] presents a simulation model targeting natural drying and open-area storage of agricultural products. The model includes factors such as solar influence, inner energy and humidity generation, environmental heat-moisture interactions, and spatio-temporal climate variations.

The drying process is characterized by thermal and moisture interaction between the airflow and the material being dried, as well as by the complex behavior of internal moisture redistribution. Article [8] provides a theoretical framework for potato dehydration, where moisture variation is analyzed as a function of internal mass transfer. This framework is used to determine key drying characteristics under microwave exposure.

In study [10], a computational model, algorithmic procedure, and simulation outcomes were presented to describe thermal and humidity dynamics in porous substances. This approach considers internal energy and vapor emission from natural porous items such as raw cotton, agricultural seeds, and their derivatives. The modeling software used enables estimation of temperature and humidity variation at multiple locations inside the material, which is useful for preventing quality degradation due to external heating and spontaneous ignition, and also serves as a tool for decision-making.

Study [10] explores the thermal-moisture exchange challenges observed during dehydration using infrared heating and heat pumps, supported by mathematical modeling of temperature and moisture migration equations. Experiments were conducted using purple yam to determine the effect of infrared energy on both drying intensity and heating speed.

2. PROBLEM STATEMENT

Open grain storage structures typically feature a rectangular geometry and are directly influenced by ambient environmental conditions. The formulated computational scheme describes how temperature and humidity evolve within the grain bulk, incorporating internal thermal energy exchange, moisture distribution, and pest activity. The mathematical formulation is grounded in the principles of thermo-hygrometric transport equations:

$$\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial x}\left(a_{eff}\left(Q(T)\right)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial z}\left(a_{eff}\left(Q(T)\right)\frac{\partial T}{\partial z}\right)+g\left(T,Q\right)+g_{pest}\left(T,Q\right); \tag{1}$$

$$\frac{\partial Q}{\partial \tau}=\frac{\partial}{\partial x}\left(D_{eff}\left(Q(T)\right)\frac{\partial Q}{\partial x}\right)+\frac{\partial}{\partial z}\left(D_{eff}\left(Q(T)\right)\frac{\partial Q}{\partial z}\right)+Q_{pest}\left(T,Q\right); \tag{2}$$

With initial conditions:

$$T\left(x,z,\tau\right)\big|_{\tau=0}=T_0\left(x,z\right); \tag{3}$$

$$Q\left(x,z,\tau\right)\big|_{\tau=0}=Q_0\left(x,z\right) \tag{4}$$

And prescribed boundary constraints

$$\mu\frac{\partial T}{\partial x}\bigg|_{x=0}=\psi_1\left(T-T_{ex}\right); \tag{5}$$

$$\mu\frac{\partial T}{\partial x}\bigg|_{x=L_x}=\psi_1\left(T-T_{ex}\right); \tag{6}$$

$$\mu\frac{\partial T}{\partial z}\bigg|_{z=0}=\psi_1\left(T-T_{ex}\right); \tag{7}$$

$$\mu\frac{\partial T}{\partial z}\bigg|_{z=L_z}=\psi_1\left(T-T_{ex}\right); \tag{8}$$

$$\omega\frac{\partial Q}{\partial x}\bigg|_{x=0}=\psi_2\left(Q-Q_{ex}\right); \tag{9}$$

$$\omega\frac{\partial Q}{\partial x}\bigg|_{x=L_x}=\psi_2\left(Q-Q_{ex}\right); \tag{10}$$

$$\omega\frac{\partial Q}{\partial z}\bigg|_{z=0}=\psi_2\left(Q-Q_{ex}\right); \tag{11}$$

$$\omega \frac{\partial Q}{\partial z}\bigg|_{z=L_z} = \psi_2 \left(Q - Q_{ex}\right); \tag{12}$$

Where: T and Q – The values of temperature and humidity of the porous body; a_{eff} — is the effective thermal diffusivity dependent on the moisture content $Q(T)$; $g(T,Q(T))$ — is a function that could represent external or internal sources/sinks of heat (possibly dependent on temperature and moisture); $g_{pest}(T,Q(T))$ — accounts for pest influences on heat transfer; D_{eff} — is the diffusivity coefficient governing moisture redistribution as influenced by internal content $Q(T)$; $Q_{pest}(T,Q(T))$ — is the pest-related term affecting moisture transfer; μ - Thermal conductivity coefficient; ω - Moisture conductivity coefficient; ψ_1 - Heat transfer coefficient; T_{ex} – Ambient temperature; ψ_2 - Moisture transfer coefficient; Q_{ex} – Ambient humidity.

The equations (1) and (2) defines the relationship between moisture (Q) and temperature (T):

$$Q(t) = Q_0 + kT.$$

3. NUMERICAL SOLUTION OF THE PROBLEM

To find a numerical solution for equations (1)–(12), the finite-difference approach is employed, where the continuous domain is substituted with a discrete grid in both space and time [15-16]:

$$\Omega_{xz\tau} = \left\{ \left(x_m = m\Delta x,\ z_l = l\Delta z,\ \tau_n = n\ \Delta\tau \right);\ m = \overline{1, N_x};\ l = \overline{1, M_z},\ n = \overline{0, N_\tau},\ \ \Delta\tau = 1/N_\tau \right\},$$

Following this, the differential components in equation (1) are transformed into difference expressions Ox:

$$\frac{1}{2}\frac{T_{m,l}^{n+1/2} - T_{m,l}^{n}}{\Delta\tau/2} + \frac{1}{2}\frac{T_{m+1,l}^{n+1/2} - T_{m+1,l}^{n}}{\Delta\tau/2} = = \frac{1}{\Delta x^2}\left(a_{eff,m+0,5,l} T_{m+1,l}^{n+1/2} - \left(a_{eff,m+0,5,l} + a_{eff,m-0,5,l} \right) T_{m,l}^{n+1/2} + a_{eff,m-0,5,l} T_{m-1,l}^{n+1/2} \right) +$$

$$+ \frac{1}{\Delta z^2}\left(a_{eff,m,l+0,5} T_{m,l+1}^{n} - \left(a_{eff,m,l+0,5} + a_{eff,m,l-0,5} \right) T_{m,l}^{n} + a_{eff,m,l-0,5} T_{m,l-1}^{n} \right) +$$

$$+ \frac{1}{2} G_{m,l}^{n+1/2} + \frac{1}{2} G_{pest,m,l}^{n+1/2};$$

And grouping similar members, we get:

$$\frac{a_{eff,m-0,5,l}}{\Delta x^2} T_{m-1,l}^{n+1/2} - \left(\frac{a_{eff,m+0,5,l} + a_{eff,m-0,5,l}}{\Delta x^2} + \frac{1}{\Delta\tau} \right) T_{m,l}^{n+1/2} + \left(\frac{a_{eff,m+0,5,l}}{\Delta x^2} - \frac{1}{\Delta\tau} \right) T_{m+1,l}^{n+1/2} =$$

$$= -\left(\frac{1}{\Delta\tau} - \frac{a_{eff,m,l+0,5} + a_{eff,m,l-0,5}}{\Delta z^2} \right) T_{m,l}^{n} - \frac{1}{\Delta\tau} T_{m+1,l}^{n} -$$

$$- \frac{a_{eff,m,l+0,5}}{\Delta z^2} T_{i,l+1}^{n} - \frac{a_{eff,m,l-0,5}}{\Delta z^2} T_{m,l-1}^{n} - \frac{1}{2} G_{m,l}^{n+1/2} - \frac{1}{2} G_{pest,m,l}^{n+1/2}.$$

Let us define the necessary notations:

$$a_{m,l} = \frac{a_{eff,m-0,5,l}}{\Delta x^2};\quad b_{m,l} = \frac{a_{eff,m+0,5,l} + a_{eff,m-0,5,l}}{\Delta x^2} + \frac{1}{\Delta\tau};\ c_{m,l} = \frac{a_{eff,m+0,5,l}}{\Delta x^2} - \frac{1}{\Delta\tau};$$

$$d_{m,l} = \left(\frac{1}{\Delta\tau} - \frac{a_{eff,m,l+0,5} + a_{eff,m,l-0,5}}{\Delta z^2} \right) T_{m,l}^{n} + \frac{1}{\Delta\tau} T_{m+1,l}^{n} + \frac{a_{eff,m,l+0,5}}{\Delta z^2} T_{m,l+1}^{n} + + \frac{a_{eff,m,l-0,5}}{\Delta z^2} T_{m,l-1}^{n} + \frac{1}{2} G_{m,l}^{n+1/2} + \frac{1}{2} G_{pest,m,l}^{n+1/2}$$

This yields a tri-diagonal system of algebraic expressions:

$$a_{m,l}T_{m-1,l}^{n+1/2}-b_{m,l}T_{m,l}^{n+1/2}+c_{m,l}T_{m+1,l}^{n+1/2}=-d_{m,l}. \tag{13}$$

To proceed, the boundary condition in equation (5) is discretized and Ox obtain:

$$\mu\frac{-3T_{0,l}^{n+1/2}+4T_{1,l}^{n+1/2}-T_{2,l}^{n+1/2}}{2\Delta x}=-\psi_1\left(T_{ex}-T_{0,l}^{n+1/2}\right) \tag{14}$$

Substituting from (13) at m=1 we derive:

$$a_{1,l}T_{0,l}^{n+1/2}-b_{1,l}T_{1,l}^{n+1/2}+c_{1,l}T_{2,l}^{n+1/2}=-d_{1,l} \tag{15}$$

By putting $T_{2,l}^{n+\frac{1}{3}}$ from (15) into (14), we find $T_{0,l}^{n+\frac{1}{3}}$:

$$T_{0,l}^{n+1/2}=\frac{4\mu c_{1,l}-b_{1,l}\mu}{3\mu c_{1,l}-a_{1,l}\mu+2\Delta x\psi_1 c_{1,l}}T_{1,l}^{n+1/2}+\frac{d_{1,l}\mu+2\Delta x\psi_1 c_{1,l}T_{ex}}{3\mu c_{1,l}-a_{1,l}\mu+2\Delta x\psi_1 c_{1,l}}$$

Where the running coefficients $\alpha_{0,l},\beta_{0,l}$ are calculated using:

$$\alpha_{0,l}=\frac{4\mu c_{1,l}-b_{1,l}\mu}{3\mu c_{1,l}-a_{1,l}\mu+2\Delta x\psi_1 c_{1,l}} \text{ and } \beta_{0,l}=\frac{d_{1,l}\mu+2\Delta x\psi_1 c_{1,l}T_{ex}}{3\mu c_{1,l}-a_{1,l}\mu+2\Delta x\psi_1 c_{1,l}}.$$

Similarly, approximating the boundary condition (6) by Ox, we obtain:

$$\mu\frac{T_{N-2,l}^{n+1/2}-4T_{N-1,l}^{n+1/2}+3T_{N,l}^{n+1/2}}{2\Delta x}=-\psi_1\left(T_{ex}-T_{N,l}^{n+1/2}\right) \tag{16}$$

Using a directional sweep strategy for the values at *N, N−1,* and *N−2,* the solution is derived we find $T_{N-1,l}^{n+1/2}$ and $T_{N-2,l}^{n+1/2}$:

$$T_{N-1,l}^{n+1/2}=\alpha_{N-1,l}T_{N,l}^{n+1/2}+\beta_{N-1,l}; \tag{17}$$

$$T_{N-2,l}^{n+1/2}=\alpha_{N-2,l}\alpha_{N-1,l}T_{N,l}^{n+1/2}+\alpha_{N-2,l}\beta_{N-1,l}+\beta_{N-2,l} \tag{18}$$

By putting $T_{N-1,l}^{n+1/2}$ from (17) and $T_{N-2,l}^{n+1/2}$ from (18) into (16), we find $T_{N,l}^{n+1/2}$:

$$T_{N,l}^{n+1/2}=\frac{\left(4\beta_{N-1,l}-\alpha_{N-2,l}\beta_{N-1,l}-\beta_{N-2,l}\right)\mu-2\psi_1\Delta xT_{ex}}{\alpha_{N-2,l}\alpha_{N-1,l}\mu-4\alpha_{N-1,l}\mu+3\mu-2\psi_1\Delta x} \tag{19}$$

Temperature $T_{N-1,l}^{n+1/2}$ sequence, $T_{N-2,l}^{n+1/2}$, ..., $T_{1,l}^{n+1/2}$ via backward computation with respect to decreasing index by decreasing the value of *i* sequences:

$$T_{m,l}^{n+1/2}=\alpha_{m,l}T_{m+1,l}^{n+1/2}+\beta_{m,l}, \quad m=\overline{N-1,1},\ l=\overline{0,M} \tag{20}$$

Similarly, equation (2) is reformulated through discrete finite-difference expressions:

$$\frac{1}{2}\frac{Q_{m,l}^{n+1/2}-Q_{m,l}^{n}}{\Delta\tau/2}+\frac{1}{2}\frac{Q_{m+1,l}^{n+1/2}-Q_{m+1,l}^{n}}{\Delta\tau/2}=$$

$$=\frac{1}{\Delta x^2}\left(D_{eff,m+0,5,l}Q_{m+1,l}^{n+1/2}-\left(D_{eff,m+0,5,l}+D_{eff,m-0,5,l}\right)Q_{m,l}^{n+1/2}+D_{eff,m-0,5,l}Q_{m-1,l}^{n+1/2}\right)+$$

$$+\frac{1}{\Delta z^2}\left(D_{eff,m,l+0,5}Q_{m,l+1}^{n}-\left(D_{eff,m,l+0,5}+D_{eff,m,l-0,5}\right)Q_{m,l}^{n}+D_{eff,m,l-0,5}Q_{m,l-1}^{n}\right)+\frac{1}{2}Q_{pest,m,l}^{n+1/2};$$

Again, reorganizing terms with similar structure, the result becomes:

$$\frac{D_{eff,m-0,5,l}}{\Delta x^2}Q_{m-1,l}^{n+1/2}-\left(\frac{D_{eff,m+0,5,l}+D_{eff,m-0,5,l}}{\Delta x^2}+\frac{1}{\Delta\tau}\right)Q_{m,l}^{n+1/2}+\left(\frac{D_{eff,m+0,5,l}}{\Delta x^2}-\frac{1}{\Delta\tau}\right)Q_{m+1,l}^{n+1/2}=$$

$$=-\left(\frac{1}{\Delta\tau}-\frac{D_{eff,m,l+0,5}+D_{eff,m,l-0,5}}{\Delta z^2}\right)Q_{m,l}^{n}-\frac{1}{\Delta\tau}Q_{m+1,l}^{n}-\frac{D_{eff,m,l+0,5}}{\Delta z^2}Q_{m,l+1}^{n}-$$

$$-\frac{D_{eff,m,l-0,5}}{\Delta z^2}Q_{m,l-1}^{n}-\frac{1}{2}Q_{pest,m,l}^{n+1/2}.$$

Let's introduce the following notations:

$$\bar{a}_{m,l}=\frac{D_{eff,m-0,5,l}}{\Delta x^2};\ \ \bar{b}_{m,l}=\frac{D_{eff,m+0,5,l}+D_{eff,m-0,5,l}}{\Delta x^2}+\frac{1}{\Delta\tau};\ \bar{c}_{m,l}=\frac{D_{eff,m+0,5,l}}{\Delta x^2}-\frac{1}{\Delta\tau};$$

$$\bar{d}_{m,l}=\left(\frac{1}{\Delta\tau}-\frac{D_{eff,m,l+0,5}+D_{eff,m,l-0,5}}{\Delta z^2}\right)Q_{m,l}^{n}+\frac{1}{\Delta\tau}Q_{m+1,l}^{n}+\frac{D_{eff,m,l+0,5}}{\Delta z^2}Q_{m,l+1}^{n}+$$

$$\frac{D_{eff,m,l-0,5}}{\Delta z^2}Q_{m,l-1}^{n}+\frac{1}{2}Q_{pest,m,l}^{n+1/2}$$

A tridiagonal structure of algebraic equations with respect to the desired variables:

$$\overline{a_{m,l}}Q_{m-1,l}^{n+1/2}-\overline{b_{m,l}}Q_{m,l}^{n+1/2}+\overline{c_{m,l}}Q_{m+1,l}^{n+1/2}=-\overline{d_{m,l}} \tag{21}$$

Next, the boundary expression from equation (9) is discretized with the second order of accuracy by Ox and obtain:

$$\omega\frac{-3Q_{0,l}^{n+1/2}+4Q_{1,l}^{n+1/2}-Q_{2,l}^{n+1/2}}{2\Delta x}=-\psi_2\left(Q_{ex}-Q_{0,l}^{n+1/2}\right) \tag{22}$$

From the system of equations (21) for $m=1$, we obtain:

$$\overline{a_{1,l}}Q_{0,l}^{n+1/2}-\overline{b_{1,l}}Q_{1,l}^{n+1/2}+\overline{c_{1,l}}Q_{2,l}^{n+1/2}=-\overline{d_{1,l}} \tag{23}$$

By putting $Q_{2,l}^{n+1/2}$ from (23) into (22), we find the value $Q_{0,l}^{n+1/2}$:

$$Q_{0,l}^{n+1/2}=\frac{4\omega\overline{c_{1,l}}-\overline{b_{1,l}}\omega}{3\omega\overline{c_{1,l}}-\overline{a_{1,l}}\omega+2\Delta x\psi_2\overline{c_{1,l}}}Q_{1,l}^{n+1/2}+\frac{\overline{d_{1,l}}\omega+2\Delta x\psi_2\overline{c_{1,l}}Q_{ex}}{3\omega\overline{c_{1,l}}-\overline{a_{1,l}}\omega+2\Delta x\psi_2\overline{c_{1,l}}}; \tag{24}$$

Where from relation (24) the running coefficients are determined using:

$$\overline{\alpha_{0,l}} = \frac{4\omega\overline{c_{1,l}} - \overline{b_{1,l}}\omega}{3\omega\overline{c_{1,l}} - \overline{a_{1,l}}\omega + 2\Delta x\psi_2\overline{c_{1,l}}}\ ;\ \overline{\beta_{0,j}} = \frac{d_{1,j}\omega + 2\Delta x\psi_2 c_{1,j}Q_{ex}}{3\omega c_{1,j} - a_{1,j}\omega + 2\Delta x\psi_2 c_{1,j}}.$$

Similarly, approximating the boundary condition (10) by Ox and we get:

$$\omega\frac{Q_{N-2,l}^{n+1/2} - 4Q_{N-1,l}^{n+1/2} + 3Q_{N,l}^{n+1/2}}{2\Delta x} = -\psi_2\left(T_{ex} - T_{N,l}^{n+1/2}\right) \quad (25)$$

A directional sweep technique is then applied *N, N-1* and *N-2,* we find the value of $Q_{N-1,l}^{n+1/2}$ and $Q_{N-2,l}^{n+1/2}$:

$$Q_{N-1,l}^{n+1/2} = \overline{\alpha_{N-1,l}}Q_{N,l}^{n+1/2} + \overline{\beta_{N-1,l}}; \quad (26)$$

$$Q_{N-2,l}^{n+1/2} = \overline{\alpha_{N-2,l}}\,\overline{\alpha_{N-1,l}}Q_{N,l}^{n+1/2} + \overline{\alpha_{N-2,l}}\,\overline{\beta_{N-1,l}} + \overline{\beta_{N-2,l}} \quad (27)$$

By putting $Q_{N-1,l}^{n+1/2}$ from (26) and $Q_{N-2,l}^{n+1/2}$ from (27) into (25), we find $Q_{N,l}^{n+1/2}$:

$$Q_{N,l}^{n+1/2} = \frac{\left(4\overline{\beta_{N-1,l}} - \overline{\alpha_{N-2,l}}\,\overline{\beta_{N-1,l}} - \overline{\beta_{N-2,l}}\right)\omega - 2\psi_2\Delta x Q_{ex}}{\overline{\alpha_{N-2,l}}\,\overline{\alpha_{N-1,l}}\omega - 4\overline{\alpha_{N-1,l}}\omega + 3\omega - 2\psi_2\Delta x} \quad (28)$$

Moisture Sequence Values $Q_{N-1,l}^{n+1/2}$, $Q_{N-2,l}^{n+1/2}$, ..., $Q_{1,l}^{n+1/2}$ is determined by the method of backward sweeping by decreasing *i* sequences:

$$Q_{m,l}^{n+1/2} = \overline{\alpha_{i,j}}Q_{m+1,l}^{n+1/2} + \overline{\beta_{m,l}},\ \text{where}\ m = \overline{N-1,1},\ l = \overline{0,M} \quad (29)$$

Next, similarly, we approximate equation (1) by Oz, then equation (2) and obtain the change in temperature :

$$T_{m,M}^{n+1} = \frac{\left(4\overline{\overline{\beta}}_{m,M-1} - \overline{\overline{\alpha}}_{m,M-2}\overline{\overline{\beta}}_{m,M-1} - \overline{\overline{\beta}}_{m,M-2}\right)\mu - 2\psi_1\Delta z T_{ex}}{\overline{\overline{\alpha}}_{m,M-2}\overline{\overline{\alpha}}_{m,M-1}\mu - 4\overline{\overline{\alpha}}_{m,M-1}\mu + 3\mu - 2\psi_1\Delta z},$$

And change in humidity :

$$Q_{m,M}^{n+1} = \frac{\left(4\overline{\overline{\overline{\beta}}}_{m,M-1} - \overline{\overline{\overline{\alpha}}}_{m,M-2}\overline{\overline{\overline{\beta}}}_{m,M-1} - \overline{\overline{\overline{\beta}}}_{m,M-2}\right)\omega - 2\psi_2\Delta z Q_{ex}}{\overline{\overline{\overline{\alpha}}}_{m,M-2}\overline{\overline{\overline{\alpha}}}_{m,M-1}\omega - 4\overline{\overline{\overline{\alpha}}}_{m,M-1}\omega + 3\omega - 2\psi_2\Delta z}.$$

Temperature $T_{m,M-1}^{n+1}$, $T_{m,M-2}^{n+1}$, ..., $T_{m,1}^{n+1}$ and humidity values $Q_{m,M-1}^{n+1}$, $Q_{m,M-2}^{n+1}$, ..., $Q_{m,1}^{n+1}$ are determined sequentially by the method of backward sweeping to decrease the value of the index l :

$$T_{m,l}^{n+1} = \overline{\overline{\alpha}}_{m,l}T_{m,l+1}^{n+1} + \overline{\overline{\beta}}_{l,m}\ \text{and}\ Q_{m,l}^{n+1} = \overline{\overline{\overline{\alpha}}}_{m,l}Q_{m+1,l}^{n+1} + \overline{\overline{\overline{\beta}}}_{m,l},\ \text{where}\ m = \overline{0,N},\ l = \overline{M-1,1}.$$

The constructed computational framework and algorithmic developed for monitoring and forecasting thermo-hygrometric behavior are a powerful tool for optimizing grain storage conditions and increasing the efficiency of managing these processes. In particular, the purpose of this model is not only to describe the dynamics energy and moisture distribution patterns, but also to provide the ability to forecast changes in these parameters depending on environmental conditions, moisture content and pest impact.

4. RESULTS AND DISCUSSION

In this work, a planar mathematical framework temperature and moisture interaction dynamics throughout the storage period of grain commodities products was developed. The constructed model incorporates natural factors (air temperature and humidity) and the influence of pests, which allows predicting the quality and safety of the product under various storage conditions.

Below are the results of computational experiments that show how grain temperature and moisture change over time depending on storage conditions.

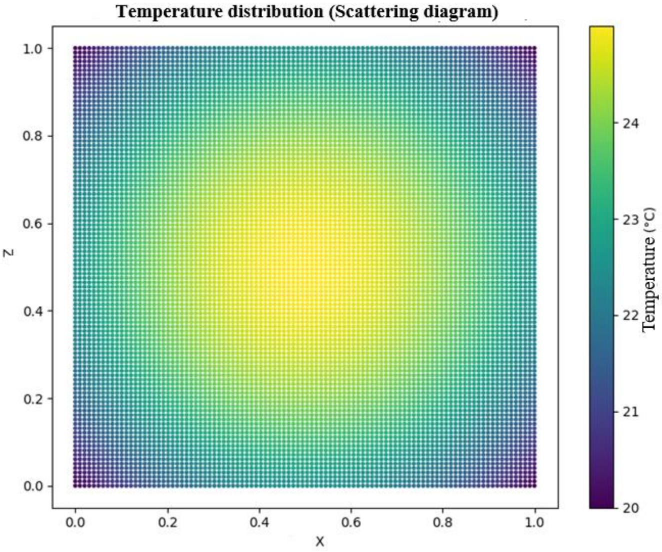


Figure 1. Temperature distribution $T_0 = 20^0 C,\ T(x,z,0) = 25^0 C$.

Figure 1 The first image shows the temperature distribution as a scatter plot on a two-dimensional plane. The X and Z axes represent the spatial coordinates on the plane from 0 to 1. The color of the dots indicates the temperature distribution.

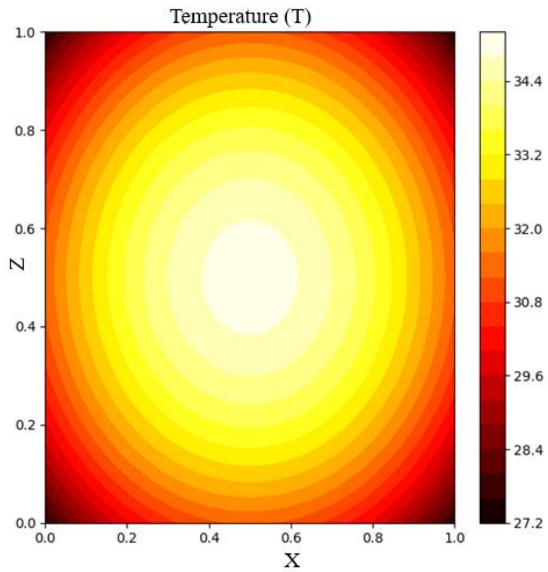
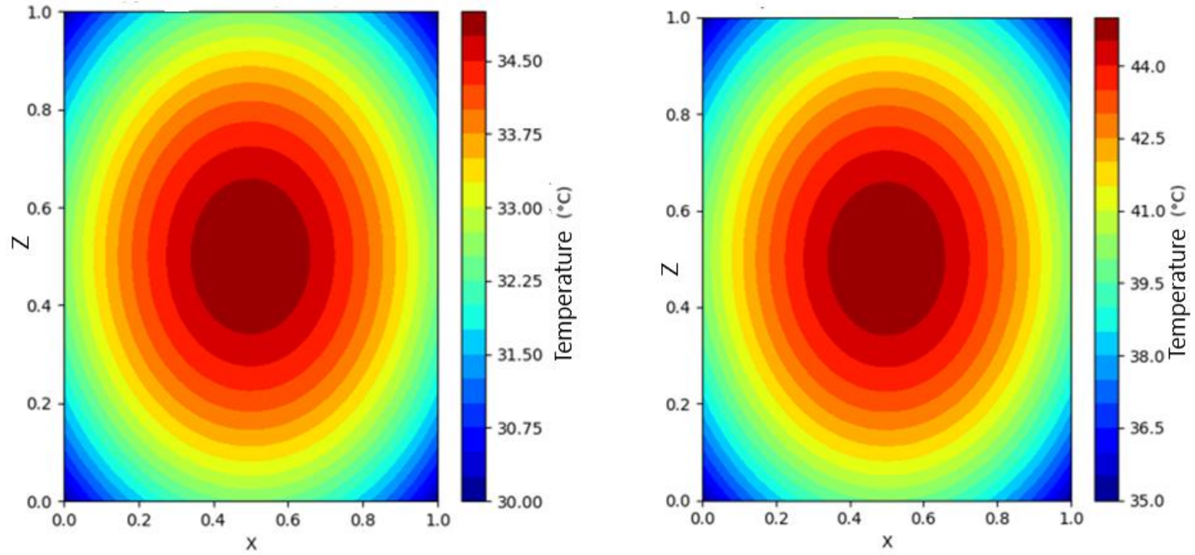


Figure 2. Temperature distribution $T_0 = 27^0 C,\ T(x,z,0) = 35^0 C.$

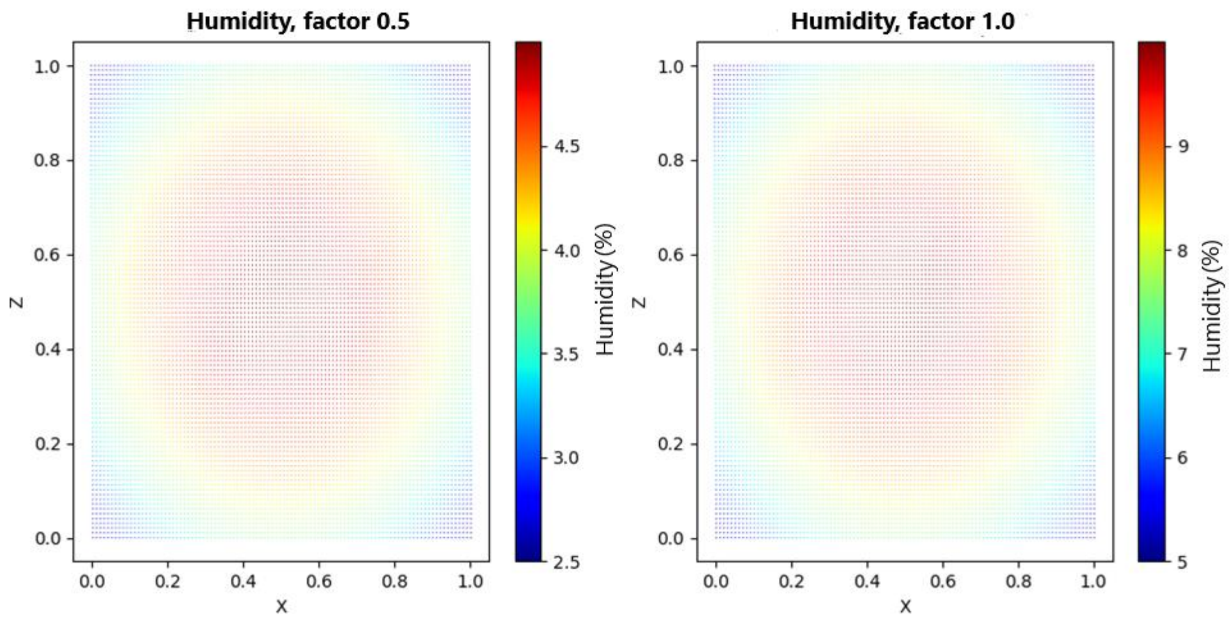
Figure 2 is a contour plot showing the temperature distribution. The temperature increases from the center to the edges in concentric circles. The color scale shows the temperature range from approximately 27 to 35 degrees. The X and Z axes probably represent spatial coordinates.



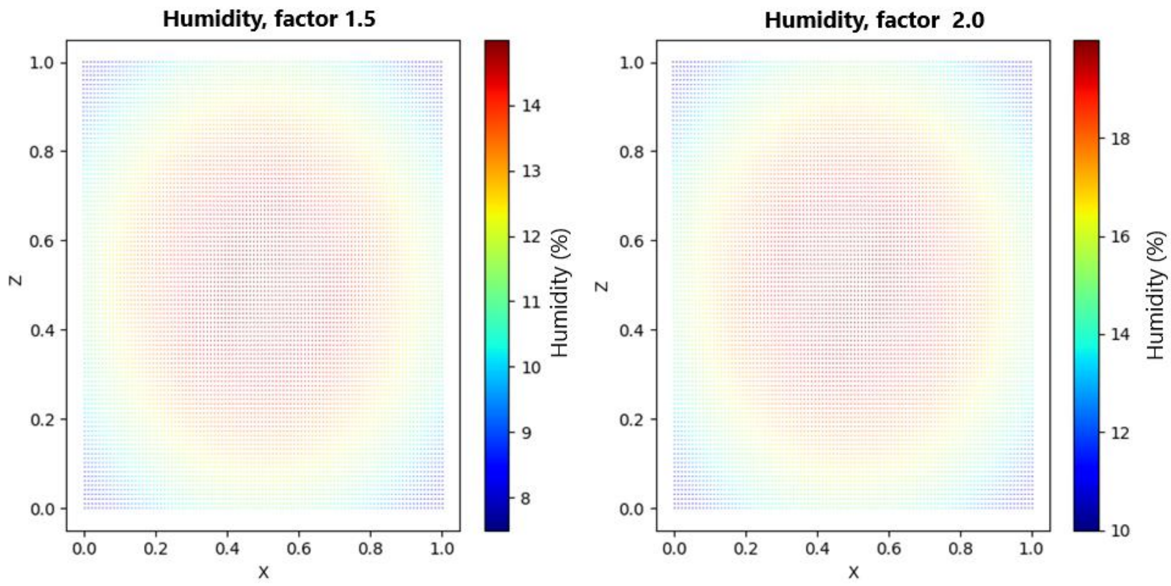
a b

Figure 3. Contour diagram of temperature distribution: a) $T_0 = 30^0 C,\ T(x,z,0) = 35^0 C.$; b) $T_0 = 35^0 C,\ T(x,z,0) = 44^0 C.$

Figure 3 is a contour diagram showing the temperature distribution. The diagram shows that the temperature increases from the center to the edges in concentric circles. The color scale in Figure 3.a shows the temperature range from 30℃ to 35℃, and in Figure 3.b from 35℃ to 44℃. The center of the diagram has the lowest temperature values, while the edges have the highest.



a b



c d

Figure 4. Contour diagram of moisture distribution: a) $Q_0 = 2.5\%,\ Q(x,z,0) = 4.5\%.$; b) $Q_0 = 5\%,\ Q(x,z,0) = 9\%.$; c) $Q_0 = 8\%,\ Q(x,z,0) = 14\%.$; d) $Q_0 = 10\%,\ Q(x,z,0) = 18\%.$

Figure 4 contains four contour plots showing the distribution of humidity depending on some factor (probably a coefficient or a model parameter). Each plot shows humidity as a percentage (%) on the XZ plane, where X and Z are normalized spatial coordinates (from 0 to 1). The color gradient displays the humidity level: low values are blue, high values are red. In all cases, the distribution of humidity is symmetrical, with maximum humidity in the center and a gradual decrease towards the edges. The plots visually demonstrate the influence of the factor on the distribution of humidity.

The graphs show the temperature distribution in the central and peripheral areas. They clearly show temperature differences in the storage environment. Typically, the temperature in the center of grain products may be higher, since heat has a stronger effect on the central parts due to external conditions (e.g. air temperature). These changes depend on storage conditions (ventilation, air humidity) and pest activity. For example, pests can locally increase the temperature, which leads to accelerated decomposition of the grain.

5. CONCLUSION

This article develops a mathematical model of changes moisture levels and internal energy transmission transfer and the impact of external temperature on grain preservation dynamics, while also accounting for the influence of pests, temperature and humidity. The model is based a discrete approximation technique based on the finite-difference scheme, which approximates the boundary conditions for spatial and temporal variables with double accuracy and ensures absolute stability. The model is aimed at monitoring tracking and forecasting thermal and moisture-related interactions in grain products, considering environmental influences, water content, and pest-related variables, moisture content and the influence of pests, in order to improve the efficiency of their storage conditions. This model and its results are important for maintaining product quality in agriculture and efficient resource management. The graphical results of this model are presented below.

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