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«AMALIY MATEMATIKA VA AXBOROT TEXNOLOGIYALARINING ZAMONAVIY MUAMMOLARI» XALQARO ILMIY-AMALIY ANJUMAN

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O'ZBEKISTON RESPUBLIKASI
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«AMALIY MATEMATIKA VA AXBOROT TEXNOLOGIYALARINING ZAMONAVIY MUAMMOLARI»
XALQARO ILMIY-AMALIY ANJUMAN
TEZISLAR TO'PLAMI

ABSTRACTS
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ТЕЗИСЫ
МЕЖДУНАРОДНОЙ НАУЧНО-ПРАКТИЧЕСКОЙ КОНФЕРЕНЦИИ «СОВРЕМЕННЫЕ ПРОБЛЕМЫ ПРИКЛАДНОЙ МАТЕМАТИКИ И ИНФОРМАЦИОННЫХ ТЕХНОЛОГИЙ»

2021 YIL 15 APREL
BUXORO

ЎЗБЕКИСТОН РЕСПУБЛИКАСИ
ОЛИЙ ВА ЎРТА МАХСУС ТАЪЛИМ ВАЗИРЛИГИ
БУХОРО ДАВЛАТ УНИВЕРСИТЕТИ
АХБОРОТ ТЕХНОЛОГИЯЛАРИ ФАКУЛЬТЕТИ

АМАЛИЙ МАТЕМАТИКА ВА АХБОРОТ ТЕХНОЛОГИЯЛАРИНИНГ ЗАМОНАВИЙ МУАММОЛАРИ

ХАЛҚАРО МИҚЁСИДАГИ ИЛМИЙ-АМАЛИЙ АНЖУМАН

МАТЕРИАЛЛАРИ

2021 йил, 15-апрель

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CHIZIQLI INTEGRAL TENGLAMALAR SISTEMASINI SONLI USULDA YECHISH.

Atoyev Dilshod Dilmurodovich
BuxDU Amaliy matematika va dasturlash texnologiyalari kafedrasi o'qtuvchisi

Tayanch so'zlar: *Volterra tenglamasi, integral tenglama, yadro, chekli yig'indi,kvadratur formula.*

Maqolada Volterra tenglamasini sonli yechish usullaridan foydalanib taqribiy yechimi aniqlangan. Mathcad matematik tizimi orqali algoritm tuzilgan, aniq va taqribiy yechimlar taqqoslangan. Tuzilgan algoritm chiziqli integral tenglamalar sistemasiga tadbiqi qo'llanilgan.

Ikkita noma'lum funksiya uchun yozilgan tenglamalar sistemalarini yechish usullaridan birini qaraylik. Bu usul noma'lumning soni ikkitadan ortiq bo'lganda ham yaroqlidir. Bu sistemani ketma-ket yaqinlashish usuli bilan yechamiz.

1-misol. Ushbu chiziqli integral tenglamalar sistemasi yechamiz:

$$\left.\begin{aligned} u(x) &= x+\lambda\int_{\rho x}^{x}\upsilon(t)\,dt,\\ \upsilon(x) &= x+\lambda\int_{\rho x}^{x}u(t)\,dt, \end{aligned}\right\} \tag{1}$$

bunda

$$0<\rho<1.$$

Sistemalarning yechimini quyidagi ikkita funktsional qator ko'rinishida izlaymiz:

$$\left.\begin{aligned} u(x) &= u_0(x)+\lambda u_1(x)+\lambda^2u_2(x)+\ldots+\lambda^n u_n(x)+\ldots,\\ \upsilon(x) &= \upsilon_0(x)+\lambda\upsilon_1(x)+\lambda^2\upsilon_2(x)+\ldots+\lambda^n\upsilon_n(x)+\ldots, \end{aligned}\right\} \tag{2}$$

bunda $u_i(x)$ va $\upsilon_i(x)$ lar – aniqlanishi lozim bo'lgan noma'lum funksiyalardir $(i=0,1,2,3,\ \ldots)$

Faraz qilaylik, (1) sistemaning yechimi (2) qatorlardan iborat bo'lsin. U holda (2) ni (1) ga qo'yish natijasida quyidagi ayniyatlar hosil bo'ladi:

$$\left.\begin{aligned} u_0(x)+\lambda u_1(x)+\lambda^2u_2(x)+\ldots &\equiv x+\lambda\int_{\rho x}^{x}\left[\upsilon_0(t)+\lambda\upsilon_1(t)+\lambda^2\upsilon_2(t)+\ldots\right]dt,\\ \upsilon_0(x)+\lambda\upsilon_1(x)+\lambda^2\upsilon_2(x)+\ldots &\equiv x+\lambda\int_{\rho x}^{x}\left[u_0(t)+\lambda u_1(t)+\lambda^2u_2(t)+\ldots\right]dt, \end{aligned}\right\}$$

Har bir tenglikning ikki tomonidagi bir xil darajali $\lambda^m(m=0,1,2,\ldots)$ larning koeffisentlarini o'zaro tenglashtirib, ketma-ket u_i va υ_i larni topamiz [4]:

$u_0(x)=x, \qquad \upsilon_0(x)=x;$

$u_1(x)=\int_{\rho x}^{x}\upsilon_0(t)dt=\int_{\rho x}^{x}tdt=\frac{1}{2}(1-\rho^2)x^2, \qquad \upsilon_1(x)=\int_{\rho x}^{x}u_0(t)dt=\int_{\rho x}^{x}tdt=\frac{1}{2}(1-\rho^2)x^2.$

$\frac{1}{2}(1-\rho^2)=A_1$ deb belgilasak, $\quad u_1(x)=A_1x^2, \upsilon(x)=A_1x^2;$ shuningdek,

$$u_2(x)=\int_{\rho x}^{x}\upsilon_1(t)\,dt=A_1\int_{\rho x}^{x}t^2dt=A_1A_2x^3, \qquad \upsilon_2(x)=\int_{\rho x}^{x}u_1(t)\,dt=A_1\int_{\rho x}^{x}t^2dt=A_1A_2x^3,$$

$$A_2=\frac{1}{3}(1-\rho^2)$$

va hokazo $u_n(x)=\upsilon_n(x)=A_1A_2\ldots A_nx^{n+1}, \qquad A_n=\dfrac{1}{n+1}(1-\rho^{n+2}), \quad n=1,2,3,\ \ldots$

Endi topilgan ifodalarni (2) qatorlarga qo'ysak izlanayotgan yechim kelib chiqadi:

$$\left.\begin{aligned} u(x) &= x\left[1 + A_1(\lambda x) + A_1A_2(\lambda x)^2 + \ldots + A_1A_2\ldots A_n(\lambda x)^n + \ldots\right], \\ \upsilon(x) &= x\left[1 + A_1(\lambda x) + A_1A_2(\lambda x)^2 + \ldots + A_1A_2\ldots A_n(\lambda x)^n + \ldots\right]. \end{aligned}\right\} \qquad (3)$$

Faraz qilaylik, $\rho = 0$ bo'lsin, u holda (1) dan Volterra tenglamalarining sistemasi hosil bo'ladi. O'sha sistemaning yechimini hosil qilish uchun (3) da $A_n = \frac{1}{n+1}$ deb olish kerak $n = (1, 2, 3, \ldots)$.

Ushbu chiziqli integral tenglamalar sistemasining yechimi (3) qator ko'rinishiga kelib qoldi, $\rho = 0$ deb Volterra tenglamalar sistemasini chekli n da hisoblab yechimini aniqlaymiz. Buning uchun yuqorida keltirilgan Volterra funksiyasini sistema uchun quyidagicha o'zgartiramiz:

```
Voltera(K, f, f1, a, b, h) := | x1 ← a
                              | i ← 1
                              | while  xi ≤ b
                              |    | x(i+1) ← xi + h
                              |    | i ← i + 1
                              | u1 ← f(x1)
                              | v1 ← f1(x1)
                              | for  i ∈ 2 .. (b − a)/h + 1
                              |    | s ← 0
                              |    | s1 ← 0
                              |    | for  j ∈ 2 .. i − 1          if  i > 2
                              |    |    | s ← s + K(xi, xj) · vj
                              |    |    | s1 ← s1 + K(xi, xj) · uj
                              |    | ui ← 1 / (1 − h/2 · K(xi, xi)) · (f(xi) + h/2 · K(xi, x1) · v1 + h · s)
                              |    | vi ← 1 / (1 − h/2 · K(xi, xi)) · (f1(xi) + h/2 · K(xi, x1) · u1 + h · s1)
                              | augment(u, v)
```

Tenglamalar sistemasini sonli usulda volterra funksiyasi va (3) qatordan yechimlarini aniqlaymiz. Topilgan yechimlarni solishtirib, grafik usulda tasvirlaymiz:

$\mathbf{K(x,t) := 1}$ $\qquad$ $\mathbf{f(x) := x}$ $\qquad$ $\mathbf{f1(x) := x}$

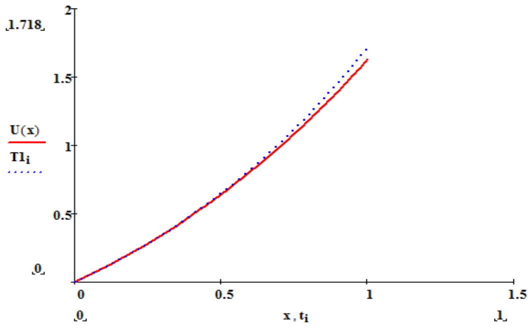
$\mathbf{Voltera(K, f, f1, 0, 1, 0.025)} =$

	1	2
1	0	0
2	0.025316455696203	0.025316455696203
3	0.051273834321423	0.051273834321423
4	0.077888361772598	0.077888361772598
5	0.105176674728866	0.105176674728866
6	0.133155831051116	0.133155831051116
7	0.161843320444815	0.161843320444815
8	0.191257075392785	0.191257075392785
9	0.221415482364755	0.221415482364755
10	0.252337393310698	0.252337393310698
11	0.284042137445146	0.284042137445146
12	0.316549533329833	0.316549533329833
13	0.349879901262234	0.349879901262234
14	0.384054075977733	0.384054075977733
15	0.419093419673372	0.419093419673372
16	0.455019835361306	...

$\mathbf{U(x)} =$

0
0.025314242230283
0.051263987047155
0.077859873204643
0.105112689740567
0.133033377908934
0.16163303313574
0.190922906998438
0.220914409229372
0.251619109743429
0.283048740690202
0.315215198530956
0.348130546140668
0.381807014935442
0.416257007025598
...

$$A(n) := \left| \begin{array}{l} s \leftarrow 1 \\ \text{for } \; i \in 1..n \\ \quad s \leftarrow s \cdot \dfrac{1}{n+1} \\ s \end{array} \right. \qquad U(x) := f(x) + \sum_{i=1}^{25} \left(A(i) \cdot f(x)^{i+1} \right) \qquad x := 0, 0.025..1$$



Xulosa. U(x) – (3) qatordan aniqlangan yechim. T1 – Volterra funksiyasi yechimi.
Grafigdan ko'rinadiki yechimlar xatoligi kichik, agar h qadamni kichraytirsak bu xatolik yanada kichrayadi. Yuqoridagi misollardan ko'rinadiki Volterra funksiyasini Volterra tenglamasiga qo'llanilsa kichik xatolikdagi taqribiy yechim olinadi.

Foydalanilgan adabiyotlar

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MODELLING CONTAMINANT TRANSPORT IN SATURATED AQUIFERS

Khaydarov Odil Shermakhmatovich

ABSTRACT

Underground water chemical pollution originates many diseases, for this reason such saturated aquifers are often used directly as drinking water sources or connected with shallow wells used for drinking water. In addition, underground aquifer have important roles for industry and packaging, for agriculture and living, and for health. In addition, underground aquifer have important roles for industry and packaging, for agriculture and living, and for health. We wish to investigate modelling of pollutant transport (the words contaminant and pollutant are used interchanging in this paper via saturated aquifers.

The important point to note here is the form of to predict the concentration distribution, retardation factor, and hydrodynamic dispersion tensor, and volumetric fluid injection rate of the source, and spatial as well as temporal, and in the saturated aquifer. The paper also deals with one of the methods of controlling the pollutant movement namely by pumping wells. A simulation model is developed to determine the number, location and rate of fluid transfer of a number of transfer of substances near the source of pollution so that the concentration is within the acceptable limits at the point of interest such as a city which gets its water supply from groundwater.